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Liquidation Mechanisms and Price Impacts in DeFi

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Abstract

This paper examines the price impacts of liquidations in decentralized finance (DeFi) lending and how they vary with fixed-spread and auction-based liquidation mechanisms. Using a theoretical framework, we show that the impact of these mechanisms depends on the liquidator participation cost, which determines the level of competition. Auctions mitigate the price impact of liquidations when the participation cost is low, but amplify them when it is high. Empirical analysis of Ethereum blockchain data shows that auction-based liquidations lead to smaller price drops by increasing competition, which raises collateral prices and reduces liquidation volumes. These findings underscore the importance of liquidation design in promoting market stability and mitigating fire-sale risks in DeFi lending.

Topics: Digital currencies and fintech
JEL codes: D47, D44, G33, G20

Résumé

Dans cette étude, nous examinons les effets produits sur les prix par les liquidations dans le secteur des prêts de la finance décentralisée, et comment ces effets varient selon qu'ils proviennent d'une liquidation par écart fixe ou par adjudication. À partir d'un cadre théorique, nous montrons que l'effet de ces mécanismes de liquidation dépend du coût de participation assumé par les liquidateurs, lequel détermine le niveau de concurrence. Le processus d'adjudication atténue l'effet d'une liquidation sur les prix lorsque le coût de participation est bas, mais l'amplifie lorsque ce coût est élevé. D'après notre analyse empirique de la chaîne de blocs d'Ethereum, les liquidations par adjudication limitent les baisses de prix en intensifiant la concurrence, ce qui fait monter les prix des garanties et réduit les volumes des liquidations. Ces constats soulignent l'importance des mécanismes de liquidation lorsqu'il s'agit de promouvoir la stabilité des marchés et d'atténuer les risques de vente en catastrophe dans le secteur des prêts de la finance décentralisée.

Sujets : Monnaies numériques et technologies financières
Codes JEL : D47, D44, G33, G20

1 Introduction

Asset liquidation comes in different forms. One approach involves an *immediate* sale, where the distressed seller quickly disposes of assets to the first available buyer, often at a discounted price. Factors such as binding margin constraints or demands from capital providers necessitate the immediacy of disposal. For example, during a margin call, a leveraged investor is forced to liquidate their stock holdings for immediate liquidity, which demands a liquidity premium. The result is a transaction price significantly below the asset's fundamental value. Another common approach to liquidation is through an auction process, where assets are sold to the highest bidder. Unlike immediate sales, auctions allow for greater bidder competition. Nevertheless, studies have shown that even in auction settings, liquidation prices tend to be discounted from fundamental values. For example, mortgage foreclosures use auctions to liquidate the underlying properties, with foreclosure prices consistently below market values (Campbell et al., 2011; Park and Bang, 2014; Donner, 2020).[1] Bankruptcy proceedings of distressed firms also often involve liquidation auctions. Sweden, for example, employs an automated bankruptcy auction system, where Eckbo and Thorburn (2008) find evidence of fire-sale discounts in piecemeal liquidations of distressed firms.[2]

In a frictionless market, the mis-pricing caused by fire sales would be corrected almost immediately through arbitrage. However, empirical studies have shown that fire sales negatively impact asset prices for a significant period of time, even in the most liquid of markets (Coval and Stafford, 2007; Falato et al., 2021; Guren and McQuade, 2020). It remains unclear, however, whether the mechanism of liquidation *per se* plays a role in its price impacts. This question is challenging to address, as it is rare to observe variation in liquidation mechanisms for the same set of assets.

This paper addresses this gap by leveraging the unique context of decentralized finance (DeFi). In this context, two distinct mechanisms are employed to liquidate the same set of collateral assets across different lending protocols. To the best of our knowledge, this is the first study to analyze the relationship between asset liquidation mechanisms and their impact on asset prices.

We first present a simple theoretical framework that models the incentives of potential liquidators under different liquidation mechanisms and derive the resulting market outcomes. Using block-by-block transaction-level data from the Ethereum blockchain, we then empirically document the temporary price impacts of collateral liquidations under different liquidation mechanisms, and compare how these impacts differ across mechanisms. Our study

[1]Depending on country and methodology, the foreclosure discount ranges from 8% to 27%.

[2]In the United States, large Chapter 11 bankruptcy cases often involve auction mechanisms in one form or another, as noted by Baird and Rasmussen (2003).

sheds light on how liquidator incentives shape the liquidation outcomes, contributing to the observed price volatility in cryptocurrencies. These insights have potential implications for policy discussions surrounding asset liquidations, particularly in the context of bankruptcy liquidation auctions where fire-sale risks are a major concern (Eckbo and Thorburn, 2008, 2009; Bhattacharyya and Singh, 1999; Hansen and Thomas, 1998). Our results suggest that the relative fixed cost of participation—specifically, the cost relative to liquidation revenue—is a crucial factor in mitigating fire-sale risks, and that the comparison between mandatory auctions and alternative mechanisms critically hinges on this cost factor.

In DeFi, lending is one of the most important services.[3] Compared to lending in traditional finance, DeFi lending protocols are plagued by the anonymity of borrowers and thus the infeasibility of credit checks. As a result, virtually all DeFi lending is collateralized (excluding flash loans), with a typical haircut ranging from 20% to 40%. However, since the collateral assets are cryptocurrencies such as Ether (ETH) or Bitcoin (BTC), the collateral value is subject to large fluctuations due to the extreme price volatility of cryptocurrencies. The price volatility can lead to a sudden depreciation of collateral value below the borrowed amount and erode the solvency of the protocols.

The solution to this problem is found in enforcing collateral liquidation on borrowing positions that become under-collateralized (that is, with a loan-to-value (LTV) ratio over a certain threshold).[4] This is similar to the margin call in leveraged trading, except that the real-time LTV for every borrowing position is *public* information. As a result, any blockchain user has the opportunity to monitor LTVs, identify positions that violate the LTV threshold, and initiate the liquidation process on these under-collateralized positions. In theory, this means that anyone can become a *liquidator*; however, in practice, liquidation in DeFi is a highly *specialized* activity due to the level of expertise required to set up bots with sophisticated configurations (Dos Santos et al., 2022; Lehar and Parlour, 2023). These specialized bots can mass liquidate under-collateralized positions within seconds, since there is virtually no capital constraint due to the use of *flash loans*. This invention unique to DeFi eliminates credit risk and allows for unconstrained flow of capital for any arbitrage trade, including liquidation. Given the instantaneity and ease of funding, the DeFi liquidation process can inflate the market supply of cryptocurrencies that serve as collateral assets, putting downward pressure on asset prices and amplifying negative shocks (Lehar and Parlour, 2023; Sasi-Brodesky and Nassr, 2023).

Interestingly, the liquidation mechanism—that is, how the liquidation price and quantity

[3]As of October 2024, the total value locked in DeFi lending protocols exceeds $33 billion, making up 37% of value locked in DeFi, according to DefiLlama.

[4]Note that there is typically a short grace period (a few hours) before liquidation, which allows the borrower to repay the borrowed asset and lower the LTV to a safe level.

are determined—varies across different lending protocols in DeFi. Currently, there are two primary liquidation methods: the fixed-spread mechanism and the auction mechanism. In a fixed-spread liquidation, the collateral is liquidated at a prespecified fixed discount to the market price.[5] In an auction, the liquidation price is determined by the highest bid. Fixed-spread liquidation is used in lending platforms such as Aave and Compound, while the auction is primarily used by MakerDao.[6] Our goal is to compare the *amplification* effects of the two liquidation mechanisms; that is, how an initial price shock is amplified due to the collateral liquidation channel.

For this purpose, we develop a tractable model that features a risky cryptocurrency (the collateral asset), a lending protocol, and a centralized exchange. The model is static. Buyers and sellers with heterogeneous valuations trading in the centralized exchange determine the initial equilibrium price of the cryptocurrency. In addition to the cryptocurrency traded on the exchange, a smaller proportion is locked in a lending protocol as collateral to secure loans of varying sizes. We disturb the initial equilibrium state by engineering an exogenous negative demand shock that depresses the price of the cryptocurrency. This renders some loans under-collateralized and thus liquidable. We then solve the liquidators' problem under two alternative assumptions on the liquidation mechanism to derive the total quantity of liquidated collateral. This determines the new supply function and therefore the new equilibrium price of the cryptocurrency. If one liquidation mechanism leads to a larger quantity of liquidated collateral and therefore a lower price in the new equilibrium, we conclude that it has a larger amplification effect.

Specifically, we define liquidators as specialized agents who can use flash loans to purchase collateral from liquidations and immediately sell it in the centralized market to make a profit. Potential liquidators first make an entry decision to become active liquidators, where entry incurs a fixed cost. Upon entry, an active liquidator is randomly matched with a liquidation opportunity through a search and matching process. Unmatched eligible loans are *not* liquidated.

If a liquidation opportunity is matched with more than one liquidator, they must compete for the opportunity, and how they compete depends on the liquidation mechanism being used. Under the fixed-spread mechanism, active liquidators compete by bidding on tips to the settlement agent, with the highest bidder winning the transaction priority and buying the collateral at fixed discount. Under the auction mechanism, they compete by bidding on

[5]The fixed discount is determined through the governance system of the lending protocol, alongside other parameters. Although this parameter can undergo discrete changes over time, such adjustments occur infrequently.

[6]MakerDAO operates on a different business model than Aave or Compound, as it also issues the decentralized stablecoin, DAI. Section 2 describes the difference in greater detail.

the price of the collateral, and the highest bidder wins the right to buy the collateral at the winning price. In both cases, the liquidation sells just enough collateral to recover the total amount of debt, with the remaining collateral (if any) returned to the protocol.

Under the two alternative liquidation mechanisms, we solve for the liquidators' equilibrium entry decision and bidding strategy, which determine the number of loans liquidated (extensive margin) and the quantity of collateral liquidated for each loan (intensive margin). These two margins jointly determine the total quantity of liquidated collateral and, consequently, the amplification effect. We find that the auction and the fixed-spread mechanism have opposing effects on these two margins.

The auction mechanism leads to a higher number of liquidated loans compared to the fixed-spread mechanism (higher extensive margin). This is because there are more active liquidators under the auction mechanism, resulting in a lower chance of unmatched liquid able loans and thus a higher number of liquidations. The auction mechanism leads to more entry because of its higher expected profits for active liquidators. We refer to this as the *entry effect*.

In contrast, the auction mechanism lowers the amount of collateral liquidated for each loan (lower intensive margin). Since liquidators are directly competing on the liquidation price, the auction mechanism generally has a higher liquidation price than the fixed-spread mechanism, and therefore a smaller amount of collateral needs to be liquidated. We refer to this as the *competition effect*. Note that although liquidators also compete under the fixed-spread mechanism, it is the settlement agent that benefits from the competition, not the protocol. By making the liquidation price immune to market forces, the fixed-spread mechanism opens the door for other parties to extract surplus from market participants.

The net outcome of the two effects depends on the entry cost. If the entry cost is sufficiently low, entry is high enough under both mechanisms to ensure that almost all liquidable loans are matched and liquidated. Because there is little difference in the extensive margin for the two mechanisms, the overall outcome is determined by the competition effect. This leads to a smaller total quantity of liquidated collateral and, therefore, a smaller amplification effect under the auction mechanism.

If the entry cost is sufficiently high, the reverse is true. The difference in entry is more pronounced, leading to more liquidations under the auction mechanism. The entry effect overpowers the competition effect, and therefore the auction mechanism has a larger amplification effect.

In an intermediate range of entry cost, the comparison hinges on the size of the initial demand shock. A larger shock generally leads to a lower equilibrium price, and therefore a lower expected profit since expected profits are proportional to the price. Thus, for larger shocks, the expected profits are too low to attract a sufficient number of active liquidators, leading to a predominant entry effect. This is similar to the case with high entry cost, where the fixed-spread mechanism has a smaller amplification effect. Conversely, the case of smaller shocks is similar to the case with low entry cost, and the auction mechanism has a smaller amplification effect.

In our empirical analysis, we draw on transaction-level data from the Ethereum blockchain to examine the price impact of Aave (fixed-spread) and Maker (auction) liquidations. Specifically, we focus on liquidations backed by wrapped Ether (ETH) and identify the price impact of a liquidation as the within-the-block change in the price of ETH. ETH prices are inferred from swap trades right before and after the liquidation transaction in the same block (referred to as the pre- and post-liquidation market price).[7] We find an average 0.054% price drop for Aave and 0.01% price drop for Maker liquidations, suggesting a smaller price impact for Maker.

We then examine whether this pattern is driven by confounding factors such as liquidation sizes, market conditions, initial shocks, debt tokens, or spillover from adjacent liquidations. We regress the price change of a liquidation on the type of liquidation mechanism, controlling for liquidation revenue, the pre-liquidation ETH price, fixed effects on the date and debt token, as well as the characteristics of the liquidation wave it is in. Our results reveal that the auction mechanism is associated with a significant reduction of the negative price impact.

According to our theory, this result is due to a dominating competition effect, which leads to a higher liquidation price and consequently a lower liquidation discount for the auction mechanism. Indeed, the liquidation discount for Maker is on average 1.8%, significantly lower than Aave's fixed discount of 5%. When the liquidation discount is included as an explanatory variable in the price change regression, the coefficient on the liquidation mechanism is no longer significant. This suggests that Maker's smaller price impact is driven by its lower liquidation discount.

In the model, the lower discount reduces the price impact through the quantity channel. We use aggregate data to provide supportive evidence on the quantity channel. We document that Maker liquidates a smaller quantity of ETH per unit of ETH deposited compared to Aave. This is consistent with our finding of a smaller price impact. Overall, our empirical results align with the model predictions in the case of relatively low entry cost.

While there is a young though rapidly growing literature on DeFi, most of the focus is on decentralized exchanges or decentralized stablecoins. Chiu et al. (2023) provide the first theoretical study of DeFi lending protocols. They examine the sources of fragility of DeFi

[7]Note that even liquidations within the same block can have different price impacts, since their positions in the block vary.

lending caused by its features such as information frictions, oracle problems, and the rigidity of smart contracts. Two empirical studies show the destabilizing effect of DeFi liquidation. Lehar and Parlour (2022) focus on collateral liquidation in Aave and Compound (fixed-spread protocols), finding empirical evidence that such liquidations have negative price impacts on the price of collateral assets across crypto exchanges. In a similar spirit, Sasi-Brodesky and Nassr (2023) focus on USDC debt collateralized by ETH (or WETH) in Aave and Compound, finding a positive correlation between the amount of debt liquidated and ETH's price volatility. Our paper provides the first empirical evidence on the price effect of auction liquidations in Maker, which have been excluded from previous studies.[8] We further compare the price impact of auction liquidations with that of fixed-spread liquidations and investigate the underlying channel. In addition, we contribute to the DeFi literature by providing a theoretical framework that explicitly models liquidators' incentives and strategies.

This paper is also related to the literature on fire sales by providing new evidence on fire-sale discounts and the price impact of fire sales. Most empirical studies on fire sales either do not directly observe the transaction price during the fire sale, or do not have the market price from frequent trades to reliably estimate the fundamental value. One notable exception is Dinc et al. (2017), who observe both by studying the firm's sales of minority equity stakes in third-party public companies. They find that the seller receives an average fire-sale discount of about 8%. This is likely to be an underestimate because those sales are *voluntary*; that is, potential firms can choose not to sell and thus these unrealized sales are unobserved in their data. The unique setting of DeFi lending not only allows us to observe both the transaction price and before-and-after prices on a highly frequent basis, but also limits the cases of unrealized sales due to DeFi's forced liquidation model. This allows us to more accurately estimate fire-sale discounts. We estimate that in auction liquidations, the fire-sale discount is on average 2% for ETH. This is smaller than the discount found in traditional finance, likely due to the use of flash loans that eliminate the capital constraint for liquidators.

The rest of the paper is organized as follows. Section 2 describes the industry background in detail and how it motivates our model. Section 3 introduces the model and characterizes the equilibrium under the two mechanisms. Section 4 compares the implications of the two mechanisms on price volatility. Section 5 provides empirical analysis, and Section 6 concludes.

[8] Sasi-Brodesky and Nassr (2023) include liquidations in Maker in their descriptive analysis, but not in the main regression analysis of its effect on price volatility.

2 Background: Lending and Liquidation Protocols in Decentralized Finance

We first provide a broad overview of DeFi protocols on the Ethereum blockchain that provide borrowing and lending services, as well as the collateral liquidation mechanisms. See Chiu et al. (2023) and Heimbach and Huang (2023) for more institutional details on DeFi.

2.1 Lending Protocols

Two types of DeFi protocols support Crypto borrowing and lending. The first type is based on a *pool*-to-peer lending model; most of the largest lending protocols, such as Aave and Compound, fall into this category. In this model, the protocol holds "liquidity pools" for a set of crypto assets. The assets in the pools are deposited by depositors who receive interest over time. Anyone who wants to borrow from one of the pools must first deposit one or more types of crypto assets as collateral. This model does not create new cryptocurrencies or stablecoins.

The second type is based on a "*vault*" model and is closely coupled with the decentralized issuance of stablecoins. The most prominent example is MakerDAO, the issuer of the stablecoin DAI. In MakerDAO, anyone can deposit any acceptable crypto assets into a permissionless "vault", which in turn allows the depositor to borrow against the value of the crypto assets locked in it. The borrowed asset is DAI, a stablecoin, and DAI is *mined* through the creation of a borrowing position. A vault is different from a pool in that it does not pay interest to the deposited assets.

In both types of lending protocols, repayments must be made in the same borrowed asset. Unlike a traditional loan, a borrower does not face a fixed time period (that is, loan maturity) to repay the loan and repayments (partial or full) are accepted at any time.

Because of the high price volatility of crypto assets, a high haircut is applied to the collateral, which is often referred to as over-collateralization in DeFi lending. As a result, the maximum loan quantity is a fraction of the value of collateral, and this fraction (that is, the maximum loan-to-value (LTV) ratio) typically ranges from 66.7% to 82.5%, depending on the type of collateral asset and borrowed asset. For example, at LTV=66.7%, for every 1 ETH of collateral, a borrower can take a loan worth at most 0.667 ETH. In this calculation, the value of ETH relative to the value of the borrowed asset is based on the price feed provided by the oracles.

After loan origination, the LTV ratio fluctuates because the prices of crypto assets change over time. The protocol defines a threshold for LTV below which a loan becomes *under-*

collateralized and can be liquidated. This threshold is often referred to as the liquidation threshold and can be equal to or slightly higher than the maximum LTV ratio. For example, the threshold is equal to the maximum LTV ratio in MakerDAO, but can be 5–7.5% higher in Aave and Compound.

Under-collateralization in DeFi is analogous to default in traditional finance. In a repo contract, the lender has the right to liquidate the collateral pledged if default occurs. Similarly, under-collateralization in DeFi triggers the transfer of collateral from the borrower to the lender. The lender liquidates just enough collateral to recover (a specific fraction of or all of) the debt plus any predetermined liquidation fees, and transfers the remaining collateral (if any) back to the borrower.

The liquidation threshold is frequently reached because the price of crypto assets is highly volatile. In theory, borrowers can make repayments manually to avoid liquidation. This is, however, impractical for an average user because it requires constant monitoring of the collateral price and transaction fees. An average user typically does not have the infrastructure for such intense monitoring (Qin et al., 2021).[9]

2.2 Liquidation Mechanisms

If a DeFi loan is under-collateralized, it is eligible for liquidation. An eligible loan is not automatically up for liquidation until someone starts the liquidation process. In principle, anyone can participate in this process, but in practice, this is a highly specialized activity (Lehar and Parlour, 2022). It requires substantial expertise, which creates the entry barrier. Those who participated in liquidations, often called *liquidators*, typically operate bots (that is, automated tools that perform a blockchain price lookup, price observation, and liquidation attempt if the liquidation threshold is triggered) (Qin et al., 2021). Such bots are professionally set up and maintained, and typically involve *flash loans*, which we discuss next. All of these are knowledge barriers to participating in liquidations.

One crucial feature that separates DeFi from traditional finance is the availability of flash loans, which essentially eliminates the capital constraint and credit risk in certain transactions. For example, if a trader wants to do an arbitrage operation in traditional finance, they need to first come up with some funding to buy the asset in one market and then sell the asset in another market. If they do not have the funds, they can borrow from some lender. During this transaction, both the trader and the lender face risks. Prices could change during the operation, which can lead to a loss for the trader. Because of the potential loss, the trader may not be able to repay the loan and the lender faces credit risks.

[9] New tools have recently been created to help borrowers monitor their LTV ratios to avoid liquidations. These tools are not perfect, and liquidations still occur in blockchains.

Due to the risk, the trader may only be able to borrow a limited amount of capital for this transaction. Flash loans alleviate this problem. They allow traders to borrow, trade, and repay in a single transaction. If repayment cannot be made, the whole transaction will not happen. This guarantees repayment and significantly reduces the risks faced by liquidators and lenders. Consequently, flash loans greatly alleviate the capital constraint for engaging in liquidation trading.[10]

DeFi lending protocols specify the process through which the collateral of an under-collateralized loan is sold to liquidators; this is referred to as the liquidation mechanism. DeFi lending protocols adopt two primary liquidation mechanisms. One mechanism uses a fixed liquidation spread and the other is based on an auction process.

Fixed-Spread Liquidation The pool-to-peer lending protocols, such as Aave, Compound, and dYdX, allow any liquidator to identify an under-collateralized loan and purchase its collateral asset at a predetermined discount from the current market price provided by the oracle. The amount of collateral that can be purchased depends on the amount of debt. For example, in Aave, each liquidation aims to recover up to 50% of the total outstanding debt (loan value plus any liquidation fee). In dYdX, liquidation can recover all the total outstanding debt. The discount directly provides an economic incentive for liquidators to engage in monitoring and liquidation because it implies profitable arbitrage opportunities. We illustrate this process using a simplified example abstracting from liquidation fees or transaction costs.

Example. *Consider a lending protocol with a maximum LTV and a liquidation threshold of 75%. Suppose the initial price of ETH is 2800 USD. By depositing 1 ETH, the borrower can take out a loan with a maximum value of 2100 USD. Suppose the borrower actually borrows 2000 USDC, which is worth 2000 USD. Suppose then the price of ETH drops by 10.7% to 2500 USD. The current LTV is* $\frac{2000}{2500} = 80\% > 75\%$, *which is above the liquidation threshold. The loan then becomes under-collateralized, and the collateral is up for liquidation. Suppose the protocol uses a 15% fixed-spread liquidation mechanism and allows all outstanding loans to be recovered through liquidation. A liquidator can submit a transaction to repay 2000 USD by purchasing the collateral asset at a price of* $\frac{2500}{1+15\%} = 2174$ *USD/ETH. Therefore, the liquidator can obtain* $\frac{2000}{2174} = 0.92$ *ETH and sell it for* $0.92 \times 2500 = 2300$ *USD, making a profit of* $2300 - 2000 = 300$ *USD.*

When multiple potential liquidators are aware of a liquidation opportunity, the allocation is based on a first-come, first-serve basis. The liquidator whose transaction is settled first

[10]The transaction fee is the only capital that a liquidator needs to provide out of their own pocket.

realizes the liquidation profit. Therefore, potential liquidators have incentives to increase the fees paid to settlement agents on the blockchain for settlement priority. This is known as the *gas premium*.[11] Settlement agents prioritize the completion of the transaction with the highest fee.

Auction Liquidation MakerDAO's collateral liquidation is the most well-known example of an auction-based liquidation mechanism.[12] In contrast with fixed-spread liquidation, the individual who identifies the under-collateralized position in auctions is not necessarily the one who performs the liquidation trade. Technically speaking, kicking off a liquidation auction for an under-collateralized loan and bidding in that liquidation auction invokes two separate functions from Maker's liquidation module.[13] The one who kicks off the auction immediately receives a small financial reward. Once the auction begins, interested liquidators submit their bids. The auction format was originally based on an English auction, and then transitioned to a new descending auction format in April 2021.

The old format can be regarded as an English auction where bidders bid on the unit price for the collateral. The bidder with the highest bid wins the auction and purchases collateral at a unit price equal to the highest bid. But the quantity of collateral that the winner can get depends on the total value of the collateral, calculated as the quantity of collateral multiplied by the highest bid. If it is less than the debt value, the winning bidder gets all the collateral. If it is more than the debt value, the winning bidder can purchase just enough to pay back the debt. The rest of the collateral is returned to the borrower.[14]

There are three main criticisms of this auction format. First, bidding is costly because every bid needs to be recorded on the blockchain even if it does not lead to a transaction, requiring a non-negligible fee. Second, auctions takes time to conclude, leading to price

[11] Recently, transactions can also be settled through private networks such as the Flashboy relay. In this case, traders can directly share profits with settlement agents in the form of private transfers Lehar and Parlour (2023).

[12] Angle is another DeFi lending protocol that uses an auction-based liquidation mechanism.

[13] In principle, they can be performed by the same trader. In reality, the address for kick-off rewards is most likely to be different from the address of the subsequent bidders, implying there are specialized bots performing each function.

[14] In practice, this format is often referred to as a two-stage "tend-dent" English auction. The first stage is called the "tend" phase, where bidders bid the amount of DAI they are willing to pay if they can obtain the entire collateral. There are two possible outcomes of this phase. First, the highest bidder is willing to pay only part of the debt. In this case, the auction ends and the highest bidder pays their bid and obtains the entire collateral. Second, the highest bidder is willing to pay the full amount and the auction moves on to the next stage. In the second stage (the "dent" phase), bidders compete by taking a smaller amount of collateral while repaying the full amount of debt. Bidders essentially bid the unit price for the collateral, and the bidder with the highest bid (that is, taking the least amount of collateral) wins the auction. The auction terminates in this stage if there are no bids after the last bid within a certain time frame, or if the total duration of the auction exceeds a certain threshold.

risks. Third, these single-unit auctions may require bidders to have a large amount of DAI to participate. Bidders cannot use flash loans to alleviate this problem because auctions take time to conclude. The "Black Thursday" event for MakerDAO (12–13 March 2021) demonstrates these shortcomings, when a single liquidator obtained $8.32 million in crypto assets at no cost. This is possible when no other competitors participate in the tend phase.

This event prompted MakerDAO to switch to a descending multi-unit auction, which works as follows. An auction starts with an initial asking price for the collateral that is higher than the market price by a known percentage. The asking price goes down by 1% every 90 seconds. At every asking price, bidders can submit the quantities they would like to purchase. Once a bid is submitted, it is executed immediately. Then the system updates the information on the remaining debt and collateral, broadcasts the information to everyone, and the auction continues. It ends if the debt is fully recovered or the collateral is depleted or a deadline is reached. In most cases, an auction ends with a full recovery of debt and some remaining collateral is returned to the borrower. In relatively rare cases, an auction ends with no remaining collateral, and the debt is not fully recovered. The Maker also sets two conditions that will trigger a restart of the auction: (1) when the duration of the auction exceeds a certain time frame, and (2) when the current market price changes too much compared to the initial price.

Compared with the old format, the new auction format makes major improvements in several aspects. First, every bid leads to a transaction, which lowers the bidding cost and shortens the duration of the auction. Second, because a bid is executed immediately, flash loans can be used, which relaxes the capital constraints. Finally, the lowest asking price can be set by choosing the deadline of the auction to prevent the price from being too low.

Interestingly, although these are multi-unit auctions, 92% of them end up with only one bidder paying all the debt.[15] This is consistent with the fact that there is virtually no capital constraint to bid, so the winner can take as much collateral as they can. Thus, it is similar to the canonical Dutch auction with an indivisible good, which is equivalent to first-price sealed-bid auctions. Therefore, we model this as a single-unit first-price sealed-bid auction.

3 Model

This section develops a parsimonious model that allows us to study the implications of liquidation mechanisms. The model has two key components: (1) a crypto exchange modelled as a competitive market that determines the market price of the cryptocurrency, and (2) a set of DeFi loans with heterogeneous LTV. If a negative demand shock or a positive supply

[15] Based on bid-level data collected from May 2021 to May 2022.

shock occurs, the market price of the cryptocurrency falls. It then triggers the liquidation of DeFi loans, which in turn increases the supply of the cryptocurrency to the exchange and further reduces the market price. This continues until a new equilibrium is reached. We then study how the new equilibrium price of the cryptocurrency depends on the liquidation mechanism (that is, the fixed-spread mechanism and the auction).

Suppose there is a cryptocurrency (for example, ETH) with a fixed supply of $1+\alpha$. There is a continuum of buyers and a continuum of sellers, each having a unit measure. Each buyer has no cryptocurrency but is interested in buying one unit. Each seller owns one unit of the cryptocurrency. A buyer's valuation for the cryptocurrency follows a distribution F_b with a support $[\underline{v}^b, \bar{v}^b]$ and a seller's valuation follows F_s with a support $[\underline{v}^s, \bar{v}^s]$. They trade in an exchange, which is a competitive market. A buyer is willing to buy if the market price p is lower than their valuation. Therefore, the demand of cryptocurrency in the exchange is $D(p) = 1 - F_b(p)$. Similarly, a seller is willing to sell if the market price is higher than their valuation. Therefore, the supply of cryptocurrency from the sellers is $S(p) = F_s(p)$.

Apart from the exchange, a measure of α cryptocurrencies is locked in a DeFi lending protocol, with each unit serving as collateral for a loan.[16] The protocol allows borrowing of up to θp_0^*, where $\theta < 1$ is the maximum LTV and p_0^* is the equilibrium price of the cryptocurrency in the exchange, satisfying $S(p_0^*) = D(p_0^*)$. In the initial equilibrium, we assume that the loan size, l, is heterogeneous and follows an exogenous distribution F_L.[17] All loans are well-collateralized, with the largest loan size equal to θp_0^*. This concludes the characterization of the initial equilibrium.

To analyze how the two liquidation mechanisms may have different amplification effects of an initial price shock, we engineer an unexpected negative shock to buyer valuations that shifts $F_b(\cdot)$ to $\tilde{F}_b(\cdot)$, with $F_b(\cdot)$ dominating $\tilde{F}_b(\cdot)$ in the first-order stochastic sense. This leads to a demand function $\tilde{D}(p)$ that is lower than $D(p)$ at every p, resulting in a drop in the price and liquidations of DeFi loans. Let $Q(p)$ be the quantity of liquidated collateral, which depends on the price because more loans are liquidated if the price is lower. The liquidated collateral is immediately supplied to the exchange and the resulting supply of the cryptocurrency is $\tilde{S}(p) = S(p) + Q(p)$. The new equilibrium price solves $\tilde{D}(p) = \tilde{S}(p)$. The liquidation mechanism affects the new equilibrium $\tilde{p}^*$ by affecting $Q(p)$, which is the focus of the rest of this section. We then compare the new equilibrium price under the fixed-spread mechanism (p_f^*) and the auction mechanism (p_a^*) with p_0^*. If one mechanism leads to a higher price drop, we conclude that it amplifies the negative shock. It is worth noting that although we focus on a negative demand shock, the analysis applies to other shocks that lower the

[16] The collateral is held by the lending protocol and not available for lending.

[17] We assume these loans are not used for purchasing the asset.

equilibrium price.

3.1 Supply from Loan Liquidations

For any market price p, a loan larger than θp is eligible for liquidation.[18] A continuum of potential liquidators play a two-stage game. In the first stage, they decide whether or not to participate in liquidations of loans of size l, trading off the expected profit with the fixed cost of participation C, which is also referred to as the entry cost. If a liquidator decides to participate, they actively search for a liquidation event. Active liquidators are randomly matched with liquidation events such that the number of liquidators n in a liquidation event of a loan with size l follows a Poisson distribution:

$$\Pr(n = k) = \mu(k; \eta_l) = \frac{\eta_l^k e^{-\eta_l}}{k!} \quad \text{for } k = 0, 1, 2, \cdots, \tag{1}$$

where λ_l denote the density of participating liquidators given loan size l and $\eta_l = \lambda_l / f_L(l)$ is the liquidator-to-liquidation event ratio.

In the second stage, liquidators matched to the same liquidation event compete for the collateral. The liquidation mechanism determines how they compete. In a fixed-spread liquidation mechanism, liquidators compete for transaction priority by bidding on the "tips" paid to settlement agents.[19] The liquidator offering the highest tip has the highest settlement priority and thus wins the liquidation. In an auction-based liquidation mechanism, liquidators compete by bidding higher liquidation prices. Therefore, liquidator competition benefits miners under the fixed-spread mechanism, whereas under auctions, the benefits accrue to DeFi borrowers.

We use backward induction to solve this two-stage game under the two different mechanisms. This gives us the expected quantity of liquidated collateral from a loan of size l when the market price is p, denoted as $q(p, l)$. Integrating over all liquidable loans (that is, loans with a size between θp and θp_0^*), we obtain the expected quantity of liquidated collateral:

$$Q(p) = \alpha \int_{\theta p}^{\theta p_0^*} q(p, l) dF_L(l). \tag{2}$$

[18] Here, we assume that the maximum LTV at origination and the liquidation threshold are the same; however in practice, there might be a small difference. For example, the maximum LTV in Aave is 75%, while the liquidation threshold is 80%. In Compound, it is 82.5% vs. 90%. MakerDAO has the same value for the maximum LTV and the liquidation threshold.

[19] In Ethereum, the total transaction cost = gas unit (limits) × (base fee + tip). For simplicity, we assume that all liquidation trades use the same gas unit, which is normalized to 1. The base fee is part of the fixed participation cost. Tips, also known as priority fees, are an additional payment to Ethereum miners to encourage a faster settlement.

We next present details on solving for $q(p, l)$ and characterize the equilibrium under two alternative assumptions on the liquidation mechanism. To facilitate comparison, we use subscripts f and a to denote quantities under the fixed-spread mechanism and the auction mechanism, respectively, from this point on.

Fixed-Spread Liquidation

In a fixed-spread liquidation model, the collateral asset of an eligible loan can be purchased at a *predetermined* discount from the market price, denoted by γ. Then, at $p < p_0^*$, the profit from liquidating a loan of size l absent from the transaction cost is:

$$\pi_f(\gamma p, l) = \min\left\{\frac{l}{\gamma p}, 1\right\}(p - \gamma p R). \tag{3}$$

Here, $\min\{l/\gamma p, 1\}$ is the quantity of liquidated collateral. If $\gamma p > l$, the total value of the collateral exceeds that of the loan, which occurs if the price is sufficiently high. In this case, a quantity of $l/\gamma p$ collateral is sold to the liquidator, which generates just enough revenue to repay the debt (*partial liquidation*). If $\gamma p < l$, the entire 1 unit of collateral is liquidated, even though it is still not enough to repay the debt (*complete liquidation*).

For each unit of collateral obtained, the liquidator gets $p - \gamma p R$, where p is the revenue obtained from selling the collateral in the exchange and $\gamma p R$ is the liquidation cost. It equals the total payment at the liquidation γp multiplied by $R = 1 + r$, where r is the funding cost. Since the liquidator uses flash loans to obtain the funding, r can be regarded as the interest rate on the flash loan. We focus on the case where $\gamma R < 1$.[20]

In the second stage, matched liquidators compete for the collateral by bidding on tips via a first-price sealed-bid auction. A liquidator does not know the number of competitors, but knows the distribution of the number of matched liquidators. A liquidator then chooses the tip t to maximizes the expected profit:

$$\Pi_f(p, l; \eta_l) = \max_t \ \mathbb{E}_m\left\{[\pi_f(\gamma p, l) - t]\prod_{j=1}^{m}\Pr(t > t_j)\right\} \tag{4}$$

where m is the number of competitors and $\prod_{j=1}^{m}\Pr(t > t_j)$ is the winning probability of liquidator i. Because n follows a Poisson distribution η_l, m also follows the same Poisson distribution by the property of a Poisson game with population uncertainty (Myerson, 1998). Therefore, the expected profit implicitly depends on η_l. This problem is closely related to

[20] In practice, R ranges from 1 to 1.003. On the lower end, dYdX does not charge any interest rate for a flash-loan-like function. On the higher end, Uniswap V2 charges the highest interest rate (0.3%) based on users' borrowed assets (Wang et al., 2021). The fixed discount ranges from 5% to 15%.

Burdett and Judd (1983) and the following lemma holds.

Lemma 1. *If $\eta_l \in (0, \infty)$, the bidding game has a unique equilibrium that features a mixed strategy. A liquidator's bid follows a distribution H supported on $[0, \bar{t}]$, where $\bar{t} = \pi_f(\gamma p, l)\,(1 - e^{-\eta_l})$ and*

$$H(t) = \frac{1}{\eta_l}\left[\log \pi_f(\gamma p, l) - \log\left(\pi_f(\gamma p, l) - t\right)\right]. \tag{5}$$

Proof. See Appendix A. □

We now outline the proof of this lemma. First, following the argument of Burdett and Judd (1983), liquidators use mixed strategies in the unique equilibrium and all the strategies used in equilibrium yield the same expected pay-off. Because the liquidator who bids the lowest t can win only if no other liquidator is matched with the liquidation, the lowest bid is 0. Otherwise, the liquidator with the lowest bid can obtain a higher profit by deviating to 0. Therefore, the lower bound of the support of H is $\underline{t} = 0$. At $t = \underline{t}$, the expected pay-off is $\mu(0; \eta_l)\pi_f(\gamma p, l)$. If a liquidator bids $t \in (0, \bar{t}]$, they can win even if other liquidators arrive. The liquidator faces m competitors with probability $\mu(m; \eta_l)$. In this case, they win only if all the m competitors bid less than t, which occurs with probability $H(t)^m$. Because m can range from 0 to ∞, the expected profit is $\sum_{m=0}^{\infty} \mu(m; \eta_l)\left[\pi_f(\gamma p, l) - t\right] H(t)^m$. We can then equate it with the profit under $t = 0$ to obtain an equation in $H(t)$:

$$\sum_{m=0}^{\infty} \mu(m; \eta_l)\left[\pi_f(\gamma p, l) - t\right] H(t)^m = \mu(0; \eta_l)\left[\pi_f(\gamma p, l) - t\right]. \tag{6}$$

This equation uniquely determines $H(t)$ for each t, and $\bar{t}$ follows from $H(\bar{t}) = 1$.

It is worth noting that the highest tip, $\bar{t}$, is a fraction $(1 - e^{-\eta_l})$ of the total liquidation profit, resembling a profit-sharing regime. This profit share increases with the level of competition, as measured by η_l. More liquidators lead to high profits to the settlement agent; that is, competition leads to higher rent extracted by the settlement agent, as opposed to reducing the amount of collateral liquidated.

Now consider the first stage of the game. Because of the equi-profit condition,

$$\Pi_f(p, l; \eta_l) = \mu(0; \eta_l)\pi_f(\gamma p, l) = e^{-\eta_l}\pi_f(\gamma p, l). \tag{7}$$

Then by free-entry condition, the equilibrium liquidator-to-liquidation-event ratio under price p and loan size l, $\eta_f^*(p, l)$, satisfies

$$e^{-\eta_f^*(p,l)}\pi_f(\gamma p, l) = C. \tag{8}$$

This equation has a finite solution if and only if $\pi_f(\gamma p, l) > C$. Otherwise, $\eta_f^*(p, l) = \infty$ and liquidators do not participate. This occurs if p and/or l are too low, or C is too high.

Proposition 1. *If $\pi_f(\gamma p, l) \leq C$, there is no successful liquidation. If $\pi_f(\gamma p, l) > C$, the liquidator-to-liquidation event ratio for loans with size l is*

$$\eta_f^*(p, l) = \log \pi_f(\gamma p, l) - \log C. \tag{9}$$

Moreover, $\bar{t} = \pi_f(\gamma p, l) - C$ and

$$H(t) = \frac{\log \pi_f(\gamma p, l) - \log(\pi_f(\gamma p, l) - t)}{\log \pi_f(\gamma p, l) - \log C}. \tag{10}$$

Proof. See Appendix A. □

Supply from Liquidation Proposition 1 allows us to calculate $Q(p)$ under the fixed-spread mechanism, denoted as $Q_f(p)$. Given p, the expected quantity of liquidated collateral from a loan of size l is

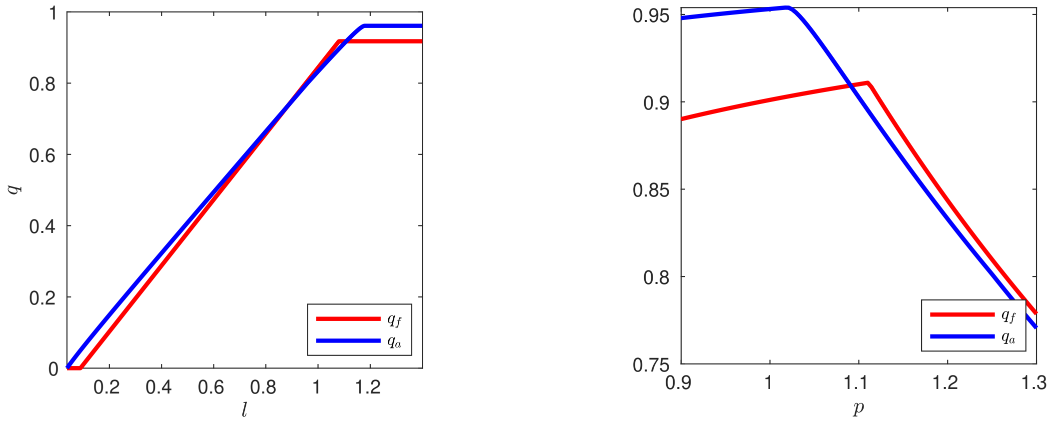
$$q_f(p, l) = \left[1 - \mu(0; \eta_f^*(p, l))\right] \min\left\{\frac{l}{\gamma p}, 1\right\}, \tag{11}$$

where $1 - \mu(0; \eta_f^*(p, l)) = 1 - e^{-\eta_f^*(p,l)}$ is the probability of liquidation (that is, the probability that at least one liquidator is matched with the liquidation event), and $\min\{l/\gamma p, 1\}$ is the liquidated quantity of collateral conditional on liquidation. If $\pi_f(\gamma p, l) > C$, (8) implies $e^{-\eta_f^*(p,l)} = C/\pi_f(\gamma p, l)$. Using this observation, we can write

$$q_f(p, l) = \begin{cases} 0 & \text{if } \pi_f(\gamma p, l) \leq C \\ \min\left\{\frac{l}{\gamma p}, 1\right\} - \frac{C}{(1-\gamma R)p} & \text{if } \pi_f(\gamma p, l) > C \end{cases}. \tag{12}$$

After some algebra, we can show that if $p < C/(1 - \gamma R)$, revenue from a full liquidation is not sufficient to cover the participation cost. Therefore, no liquidator participates and $q_f(p, l) = 0$ regardless of l. If $p > C/(1 - \gamma R)$, then

$$q_f(p, l) = \begin{cases} 0 & \text{if } (1 - \gamma R)\frac{l}{\gamma} \leq C \\ \frac{l}{\gamma p} - \frac{C}{(1-\gamma R)p} & \text{if } C < (1 - \gamma R)\frac{l}{\gamma} < (1 - \gamma R)p \\ 1 - \frac{C}{(1-\gamma R)p} & \text{if } \gamma p \leq l \end{cases}. \tag{13}$$



(a) q as a function of l

(b) q as a function of p

Figure 1: q_f and q_a

We then obtain $Q_f(p) = \int_{\theta p}^{\theta p_0^*} q_f(p, l) dF_l(l)$ for any F_l.

Figure 1a plots $q_f(p, l)$ as a function of l for a given p in red if $p > C/(1-\gamma R)$. It is 0 if l is sufficiently low and is constant if l is sufficiently high. If l is low, liquidating a small amount of collateral is sufficient to pay back the loan. This implies a low revenue from liquidation, which is not sufficient to cover the participation cost. Therefore, no one participates in the liquidation and $q_f(p, l)$ is 0. If l is sufficiently high, all collateral will be liquidated and the revenue is independent of the loan size. Therefore, the liquidation quantity and participation do not depend on the loan size. If l is intermediate, the amount of liquidated collateral and the liquidation revenue both increase with l. Therefore, $q_f(p, l)$ increases with l.

The red curve in Figure 1b shows $q_f(p, l)$ as a function of p given the value of l. It is non-monotone in general. Intuitively, a higher p has two effects. On the one hand, it increases the revenue from liquidation. This induces more participation and a more successful liquidation event (extensive margin). On the other hand, it decreases the amount of liquidated collateral needed to pay back the loans within a liquidation (intensive margin). The former effect increases $q_f(p, l)$, while the latter effect decreases it. If p is low, all collateral are liquidated in a successful liquidation and then the extensive margin dominates. If p is high, the intensive margin dominates.

Auctions

We next study the case where the collateral is liquidated via a first-price sealed-bid auction with reservation price δp. In the second stage, liquidators bid on the price per unit of the collateral, with the highest bid winning the auction. In contrast with a standard first-price auction, the total revenue in this case is capped by the loan size, as the liquidation cannot raise more than the amount of the underlying debt. Specifically, if the winning liquidator bids $b < l$, they pay l in exchange for $l/b < 1$ units of collateral. Otherwise, the liquidator pays b and gets 1 unit of the collateral (that is, the total available amount). A liquidator's profit upon winning is

$$\pi_a(b,l) = \min\left\{\frac{l}{b}, 1\right\}(p - bR). \tag{14}$$

The liquidator chooses b to maximize the expected profit:

$$\max_{b \geq \delta p} \mathbb{E}_m\left[\pi_a(b,l)\prod_{j=1}^{m}\Pr\left(b > b_j\right)\Big|\eta_l\right]. \tag{15}$$

Similar to (4), the bidder trades off the profit conditional on winning and the probability of winning. The difference is that the bid affects the quantity of collateral obtained from the liquidation in an auction but not in a fixed-spread liquidation. We can show that in the equilibrium, a range of bids are used and they all yield the same profit. Let $G(b)$ be the equilibrium bid distribution, which is again determined by an equal-profit condition. Given η_l, it satisfies the following equal-profit condition:

$$e^{-\eta_l}\pi_a(\delta p, l) = \pi_a(\delta p, l)\sum_{n=0}^{\infty}\frac{\eta_l^n e^{-\eta_l}}{n!}G(b)^n \tag{16}$$

Then the following result holds.

Lemma 2. *Given $\eta_l \in (0,\infty)$, the unique equilibrium features mixed strategy. A liquidator's bid follows a distribution G supported on $[\delta p, \bar{b}]$, where $\pi_a(\bar{b}, l) = e^{-\eta_l}\pi_a(\delta p, l)$ and*

$$G(b) = \frac{1}{\eta_l}\left[\log \pi_a(\delta p, l) - \log \pi_a(b,l)\right]. \tag{17}$$

Proof. See Appendix A. □

Note that under both mechanisms, the ratio of the lowest realized profit to the highest realized profit ($\pi_a(\bar{b}, l)/\pi_a(\delta p, l)$ in auction and $(\pi_f(\gamma p, l) - \bar{t})/\pi_f(\gamma p, l)$ in fixed spread) is $e^{-\eta_l}$, which decreases with η_l. Intuitively, a higher liquidator-to-eligible-loan ratio results in more competition, which depresses the winning profit.

Similarly, the participation decision in the first stage of the game determines the liquidator-to-eligible-loan ratio through the free-entry condition and the results are summarized in the following proposition:

Proposition 2. *If $\pi_a(\delta p, l) \leq C$, there is no successful liquidation. Otherwise, the equilibrium the liquidator-to-eligible-loan ratio is given by*

$$\eta_a^*(p,l) = \log \pi_a(\delta p, l) - \log C \tag{18}$$

and the bids follow the distribution G supported on $[\delta p, \bar{b}]$, where $\pi_a(\bar{b}, l) = C$ and

$$G(b) = \frac{\log \pi_a(\delta p, l) - \log \pi_a(b, l)}{\log \pi_a(\delta p, l) - \log C}. \tag{19}$$

Proof. See Appendix A. □

Supply from liquidation If a liquidation is successful, the amount of liquidated collateral is determined by the highest bid, which has a distribution of

$$G^*(b) = \frac{1}{1 - e^{-\eta_a^*(p,l)}} \sum_{n=1}^{\infty} \frac{\eta_a^*(p,l)^n e^{-\eta_a^*(p,l)}}{n!} G(b)^n.$$

The expected amount of collateral liquidated for loan l is

$$q_a(p,l) = [1 - \mu(0; \eta_a^*(p,l))] \int_{\delta p}^{\bar{b}} \min\{l/b, 1\} \, dG^*(b), \tag{20}$$

where $1 - \mu(0; \eta_a^*(p,l)) = 1 - e^{-\eta_a^*(p,l)}$ is the probability of a successful liquidation, and $\int_{\delta p}^{\bar{b}} \min\{l/b, 1\} \, dG^*(b)$ is the expected liquidated quantity in a successful liquidation. We next derive the closed-form expression for $q_a(p,l)$. If $\eta_a^*(p,l) \in (0, \infty)$,

$$G^*(b) = \frac{e^{-\eta_a^*(p,l)} \pi_a(\delta p, l)}{\left[1 - e^{-\eta_a^*(p,l)}\right] \pi_a(b,l)} = \frac{C}{\left[1 - e^{-\eta_a^*(p,l)}\right] \pi_a(b,l)}, \tag{21}$$

where the first equality follows from (16) and the second equality uses (18). We can combine (20) and (21), and integrate with respect to b to obtain a closed-form expression for $q_a(p,l)$.

We again distinguish two cases. First, if $(1 - \delta R)p \leq C$, then $q_a(p,l) = 0$. In this case, the price of the collateral is too low such that the revenue from liquidation cannot cover the participation cost even if all the collateral is liquidated. As a result, no liquidator participates. Second if $(1-\delta R)p > C$, then

$$q_a(p,l) = \begin{cases} 0 & \text{if } l < \frac{\delta C}{1-\delta R} \\ \frac{C+Rl}{p} - \frac{C}{p(1-\delta R)} + \frac{C}{p}\log\frac{(1-\delta R)l}{\delta C} & \text{if } \delta p > l > \frac{\delta C}{1-\delta R} \\ \frac{C+Rl}{p} - \frac{C}{p(1-\delta R)} + \frac{C}{p}\log\frac{p-Rl}{C} & \text{if } \delta p < l < \frac{p-C}{R} \\ 1 - \frac{C}{p(1-\delta R)} & \text{if } \frac{p-C}{R} < l \end{cases}. \tag{22}$$

In this case, liquidators participate unless the size is too low. We can then compute $Q_a(p) = \int_{\theta p}^{\theta p_0^*} q_a(p,l)dF_l(l)$ for any distribution F_l.

The blue curves in Figure 1 show $q_a(p,l)$ as functions of l and p, respectively. Similar to $q_f(p,l)$, $q_a(p,l)$ is increasing in l and non-monotone in p. Interestingly, it may be lower or higher than $q_f(p,l)$ in this example. In particular, $q_a(p,l)$ is higher than $q_f(p,l)$ if p is lower, and is lower than $q_f(p,l)$ if p is high. Intuitively, a high p makes liquidations attractive, which induces more competition. Higher competition increases prices in auctions, but has no effect on prices in fixed-spread liquidations. As a result, less collateral is liquidated in auctions than in fixed-spread liquidations.

4 Mechanism Comparison

We now compare the amplification effect of a negative demand shock under the two mechanisms. First, notice because $\pi_a(\delta p, l) < \pi_f(\gamma p, l)$ if and only if $\delta > \gamma$, Propositions 1 and 2 imply that $\eta_a^*(p,l) < \eta_f^*(p,l)$ if and only if $\delta > \gamma$ for any p and l; that is, on the extensive margin, an eligible loan is less likely to be liquidated under auctions than under a fixed-spread mechanism if and only if $\delta > \gamma$. On the intensive margin, auctions lead to higher liquidation prices than fixed-spread liquidations if $\delta > \gamma$ because the reserve price reservation price is higher than the fixed-spread liquidation price. Higher prices lead to less liquidated collateral. As a result, a smaller supply of cryptocurrency is added to the market in auctions than in fixed-spread liquidations if $\delta \geq \gamma$.

Proposition 3. *If $\delta \geq \gamma$, $Q_a(p) \leq Q_f(p)$ for all $p < p_0^*$, which implies $p_a^* \geq p_f^*$.*

We next analyze the more interesting and realistic case with $\delta < \gamma$. Now there are two offsetting effects. On the one hand, more liquidators participate in auctions than in fixed-spread liquidations, as a low reservation price implies a high potential profit. This increases the number of loans liquidated. We refer to this effect as the entry effect. Conversely, a larger number of participants lead to more competition in auctions, which drives up the transaction price and reduces the amount of liquidated collateral per liquidation event. We refer to this as the competition effect.

The entry effect results in a higher supply from liquidations under the auction mechanism than under the fixed-spread mechanism, while the competition effect has the opposite implication. The comparison depends on which effect dominates. Our next result shows that for a given (p, l), which effect dominates depends on the entry cost C.

Lemma 3. *Suppose $\delta < \gamma < R^{-1}$. For any positive p and l, the following holds:*

1. *If $Rl < p$, there exists a cut-off $> C \in (0, \min\{p, l/\gamma\} - Rl)$ such that the expected quantity of liquidated collateral is lower under the auction mechanism than under the fixed-spread mechanism if and only if $C < C^*_{p,l}$.*

2. *If $Rl > p$, the expected amount of liquidated collateral is weakly higher under the auction mechanism than under the fixed-spread mechanism for all $C \geq 0$.*

This lemma shows that for a given p-l pair, there are two possibilities. First, if the the market price is too low, fixed spread leads to a weakly lower liquidation quantity. In this case, liquidation is not profitable under fixed spread due to the pre-specified discount but can still be profitable under auctions. Then the entry effect dominates and auctions lead to more liquidation. Second, if the market price is sufficiently high, auctions can lead to less liquidated collateral than fixed spread. This happens if and only if the entry cost is lower than a threshold, resulting in the prevalence of the competition effect.

Proposition 4. *Suppose $\underline{v} = \min\{\underline{v}^s, \underline{v}^b\}$ is positive. If $\delta < \gamma < R^{-1}$ and $f(l) > 0$ on $(0, \theta p^*)$, then there exists two cut-offs $0 < \underline{C} \leq \overline{C} < \infty$ such that*

1. *If $C < \underline{C}$, $Q_a(p) < Q_f(p)$ for all $p \geq \min\{\underline{v}_s, \underline{v}_b\}$.*

2. *If $C > \overline{C}$, $Q_a(p) > Q_f(p)$ for all $p \geq \min\{\underline{v}_s, \underline{v}_b\}$.*

3. *If $C \in (\underline{C}, \overline{C})$,$Q_a(p) < Q_f(p)$ only if p is not too small.*

This proposition is a consequence of Lemma 3. It shows that if the participation cost is low, the auction mechanism leads to less added supply of cryptocurrency from liquidations than the fixed-spread mechanism, regardless of the current price. If the participation cost is high, the reverse is true. And if the participation cost is in an intermediate range, auctions can lead to less added supply from liquidations only if the current price is not too low.

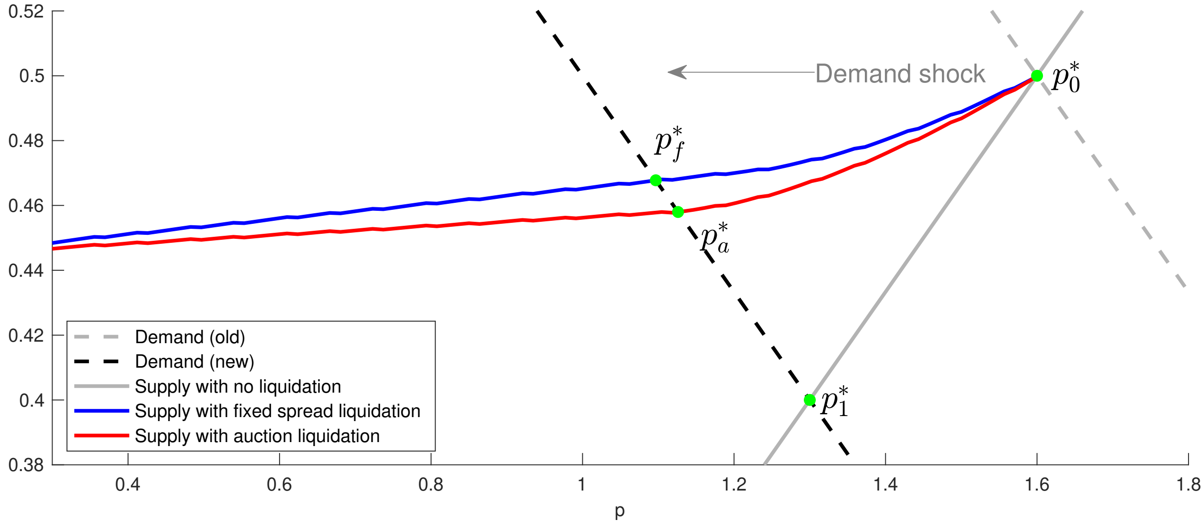
Intuitively, if the participation cost is low, the probability of successful liquidations is close to 1 under both mechanisms. The difference in liquidation quantity mainly results from the amount of liquidated collateral in each successful liquidation event. Then, the auction mechanism leads to less liquidated collateral because the competition effect drives up liquidation price and lowers the liquidated quantity. If the participation cost is sufficiently high, it exceeds the profit under fixed-spread mechanism, which is no more than a predetermined fraction of the market price. In this case, liquidators do not participate in fixed-spread liquidations. They may, however, still participate in auctions because they can significantly lower their bids to obtain sufficient profit to cover the participation cost. Therefore, auctions lead to more liquidated collateral than the fixed-spread mechanism does. If the participation cost is in an intermediate range, the probability of a successful liquidation is close to 1 only if the market price is sufficiently high. Only in this case, the auction mechanism may lead to less liquidated collateral because of the competition effect.

Proposition 4 implies that the supply function under the auction mechanism, $\tilde{S}_a(p)$, is lower than that under the fixed-spread mechanism, $\tilde{S}_f(p)$ for all $p \in [\underline{v}, p_0^*)$ if the participation cost is sufficiently low. The reverse is true if the participation cost is sufficiently high. And if the participation cost is in an intermediate range, $\tilde{S}_a(p) < \tilde{S}_f(p)$ only if p is not too small. This ranking in the supply function immediately implies the ranking of the amplification effect of a negative shock, which we illustrate through a numerical example.

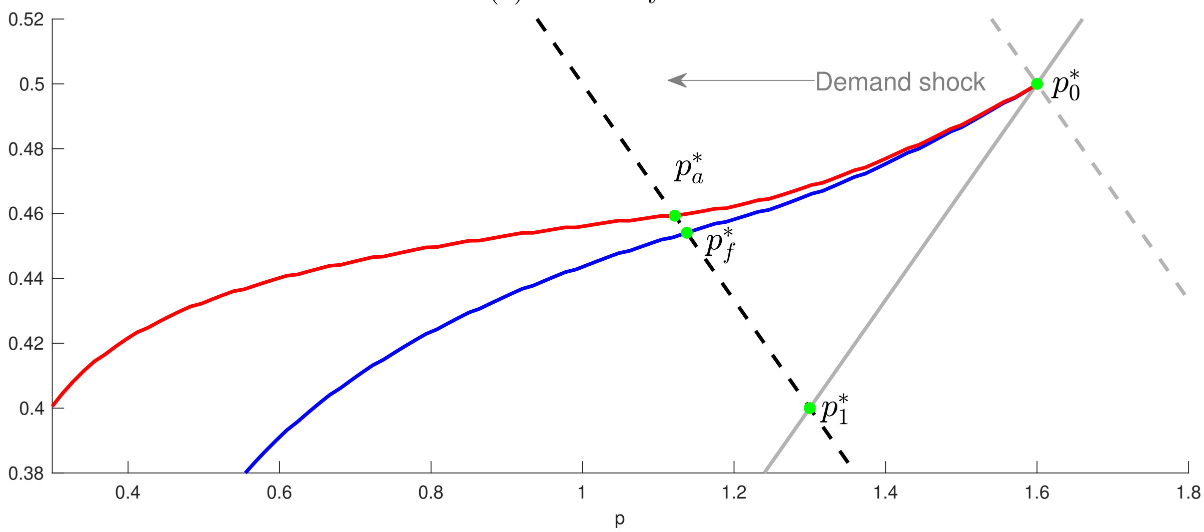
Numerical Example

We set $\gamma = 0.9$ and $\delta = 0.6$ based on the fixed discount and reservation price commonly observed in DeFi. The original demand and supply is given by $D(p) = 1 - (p - 0.1)/3$ and $S(p) = (p - 0.1)/3$, which implies $F_b(p) = F_s(p) = (p - 0.1)/3, p \in [0.1, 3.1]$. The original equilibrium price is therefore $p^* = 1.6$. The negative demand shock changes the demand function to $\tilde{D}(p) = 0.8 - p/3$. Other parameters are set as follows: $\theta = 2/3$, $R = 1.01$, $\alpha = 0.4$ and $l \sim U[0.1, 1]$.

Case (a): Low entry cost If the entry cost is as low as $C = 10^{-5}$, Figure 2a illustrates the impact of a negative demand shock. The horizontal axis is the price of the cryptocurrency and the vertical axis is the quantity. Before the shock hits, the equilibrium price is p_0^*, which is the intersection of the demand curve before the shock (dashed grey curve) and the supply. The blue and the red curves are the supply function under the fixed-spread mechanism and the auction mechanism, respectively. The blue curve is above the red curve for all p in the relevant range, implying that if the participation cost is low, auctions lead to a lower supply of cryptocurrency than the fixed-spread mechanism. As a result, the new equilibrium price under auctions is p_a^*, which is higher than that under the fixed-spread mechanism, p_f^*. For reference, p_1^* is the new equilibrium price if there are no liquidations. It is higher than p_a^*



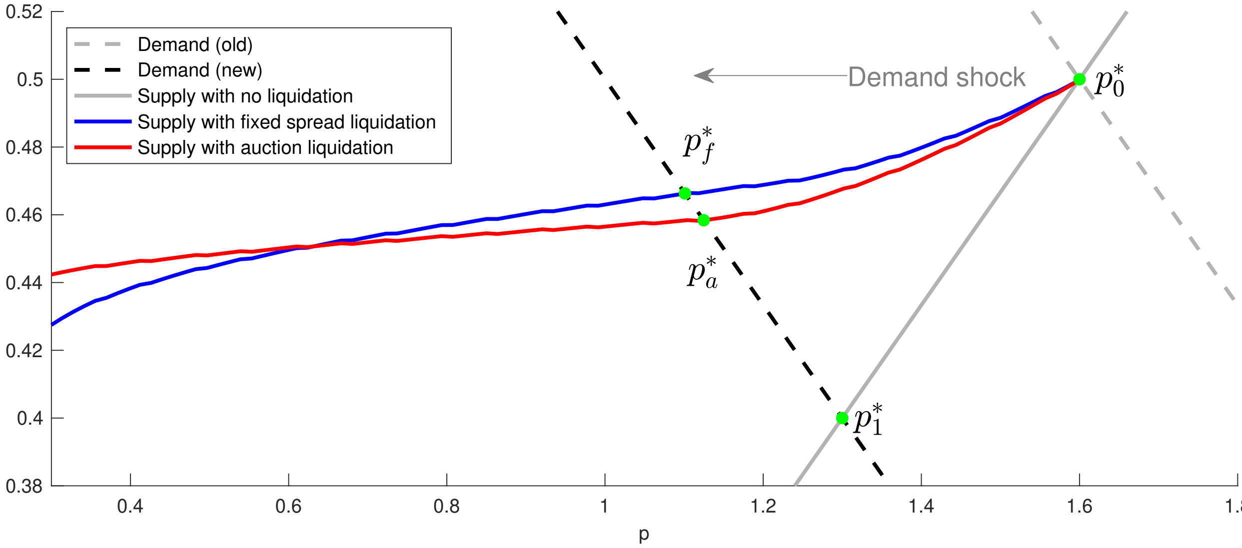
(a) Low entry cost



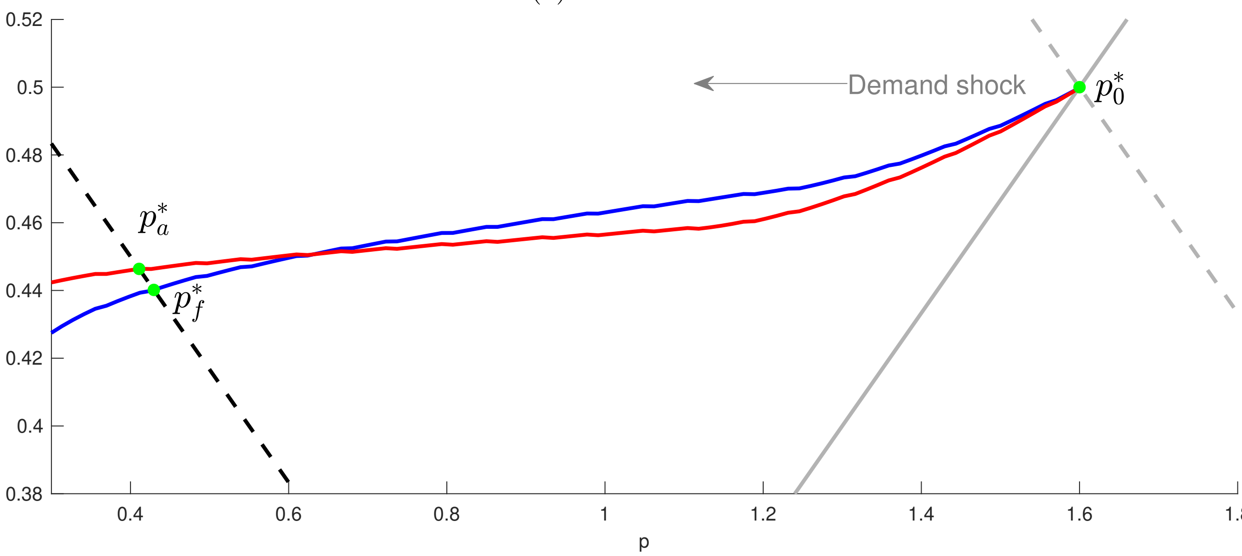
(b) High entry cost

Figure 2: Market outcomes with high and low entry costs

Figure 2a and 2b show the equilibrium market outcomes when the entry cost is low ($C = 10^{-5}$) and high ($C = 10^{-2}$), respectively. In both figures, p_0^* represents the initial equilibrium price before the demand shock, and p_1^*, p_a^*, and p_f^* denote the new equilibrium price after the demand shock in the case of no liquidation, auction liquidations, and fixed-spread liquidations, respectively.



(a) Small shock



(b) Large shock

Figure 3: Market outcomes with medium entry costs and different demand shocks

Figure 3a and 3b show the equilibrium market outcomes at an intermediate value of the entry cost ($C = 10^{-3}$) in response to a small and large demand shock, respectively. The small demand shock refers to a shift in demand function from $D(p) = 1-(p-0.1)/3$ to $\tilde{D}(p) = 0.8-(p-0.1)/3$ and a large demand shock represents the case when $\tilde{D}(p) = 0.55-(p-0.1)/3$.

and p_f^*, implying that liquidations amplify negative demand shocks. In this example, the price drop is 2% higher under the fixed-spread mechanism than under auctions.

Case (b): High entry cost Figure 2b shows the case with $C = 10^{-2}$, which is high under this parametrization. In all $p < p_0^*$, the auction mechanism leads to a higher supply of cryptocurrency than the fixed-spread mechanism. The auction mechanism leads to a price drop about 1.1% larger than the fixed-spread mechanism.

Case (c): Medium entry cost If the participation cost is intermediate at $C = 10^{-3}$, Figures 3a and 3b illustrate the cases with a small and a big negative demand shock, respectively. The small demand shock moves the demand to $\tilde{D}(p) = 0.8 - (p - 0.1)/3$ (as in the previous cases), while the large shock moves the demand to $\tilde{D}(p) = 0.55 - p/3$. Consistent with the theory, the auction mechanism leads to a lower supply only if p is sufficiently large, where participation is sufficiently high. Therefore, the ranking of the amplification effect depends on the size of the shock. If the shock is small, the new demand intersects with the supply in a region where auctions lead to lower supply than the fixed-spread mechanism, as in Figure 3a. Then, auctions lead to a higher price than the fixed-spread mechanism. With a large demand shock, the reverse is true, as shown in 3b. The new equilibrium price falls into the region where the auctions lead to more supply than the fixed-spread mechanism because the former leads to more participation than the latter.

Corollary 1. *Suppose $\underline{v} = \min\{\underline{v}^s, \underline{v}^b\}$ is positive. If $\delta < \gamma < R^{-1}$ and $f(l) > 0$ on $(0, \theta p^*)$, then there exit two cut-offs $0 < \underline{C} \leq \overline{C} < \infty$ such that:*

1. *If $C < \underline{C}$, auctions amplify negative demand shocks less than the fixed-spread mechanism.*

2. *If $C > \overline{C}$, auctions amplify negative demand shocks more than the fixed-spread mechanism.*

3. *If $C \in \left(\underline{C}, \overline{C}\right)$, auctions amplify negative demand shocks less than the fixed-spread mechanism only if the shock is not too big.*

So far, we have focused on the case where all loans use the same liquidation mechanism. In practice, loans can use different liquidation mechanisms. It is therefore interesting to study how the composition of liquidation mechanisms impacts the amplification effect. Suppose a λ fraction of loans use auctions as the liquidation mechanism and the rest use the fixed-spread mechanism. Then we can express the total supply with liquidation as $\tilde{S}(p) = S(p) + \lambda Q_a(p) + (1-\lambda) Q_f(p)$. Following the same argument as above, we have the following result, which shows how an increase in the share of auction liquidations affects the amplification effect. It is useful for our empirical analysis in Section 5.

Corollary 2. *Suppose* $\underline{v} = \min\{\underline{v}^s, \underline{v}^b\}$ *is positive and the loan size distribution is the same under different liquidation mechanisms. If* $\delta < \gamma < R^{-1}$ *and* $f(l) > 0$ *on* $(0, \theta p^*)$*, then two cut-offs exist* $0 < \underline{C} \leq \overline{C} < \infty$ *such that:*

1. *If* $C < \underline{C}$*, a higher share of auction liquidations reduces the amplification effect of a negative demand shock.*

2. *If* $C > \overline{C}$*, a higher share of auction liquidations increases the amplification effect of a negative demand shock.*

3. *If* $C \in \left(\underline{C}, \overline{C}\right)$*,a higher share of auction liquidations reduces the amplification effect of a negative demand shock only if the shock is not too big.*

Notice that all the results in this section deal with a negative demand shock. The results also carry over to a positive supply shock where the valuations of the sellers become lower or where the total quantity of the cryptocurrency increases.

5 Empirical Analysis

This section empirically examines the relationship between liquidation mechanism and its price impact. We focus on the Ethereum blockchain, which hosts by far the largest DeFi ecosystem. Using transaction-level blockchain data, we identify the price change associated with each single liquidation and examine the relationship between the liquidation mechanism and the magnitude of price changes.

5.1 Sample Construction

We retrieve on-chain data from Dune Analytics and collect collateral liquidation transactions that occur between 2021/04/26–2024/05/26 on Aave and Maker, focusing on the set of cryptocurrencies that are accepted as collateral by all three platforms.[21] Aave is the leading

[21] Dune Analytics is a blockchain data platform with publicly available query functionality. Dune's core dataset comes directly from blockchain data. The platform structures the raw on-chain data into a queryable form. Despite the open data approach, Dune's platform itself is proprietary. Other empirical work using Dune Analytics includes Azar et al. (2024).

platform that uses a fixed-spread liquidation mechanism, while Maker liquidation is auction-based. Together they account for the majority of liquidations on Ethereum.[22]

For each liquidation transaction, we use the following approach to identify the change in the market price of the affected collateral before and after the liquidation. Given the liquidation i, let a_i indicate the type of cryptocurrency sold as collateral. We first identify the last DEX trade involving the sale of a_i prior to liquidation i within the same block and infer the price of a_i from this swap, denoted as p_i^0. We then find the first swap trade selling a_i after liquidation i in the same block and record the implied price as p_i^1.[23] We require that the two trades occur on the same DEX. We then calculate the price change from p_i^0 to p_i^1 as $\Delta p_i = (p_i^1 - p_i^0)/p_i^0 \times 100\%$.

This approach identifies price changes for $12,806$ liquidations (47.3% of all liquidations from Aave and Maker). Liquidation transactions for which we cannot identify price changes are mostly backed by unpopular cryptocurrencies that are traded less often. Among the identified liquidations, three types of collateral cover 98% of the observations: Wrapped Ether (90%), Wrapped Bitcoin (4%), and Link (4%).[24] We refer to Wrapped Ether as ETH for short hereafter. For cleaner identification, we include only ETH-backed liquidations for the regression analysis, although results are robust to the inclusion of wrapped Bitcoin and Link. In addition, we focus on Aave liquidations with stablecoins as debt tokens, which accounts for 95% of the observations. This avoids fluctuations in the values of risky debt tokens affecting our results.[25]

Liquidations are heavily concentrated on days that are marked by negative news.[26] Two-thirds of the liquidations in the sample occur on days with at least seven other liquidation events from both Aave and Maker. We keep these observations for our regression analysis since our identification strategy relies on within-day variations in price impacts between Aave and Maker liquidations.

On days marked by cascading liquidations, these events tend to happen in rapid waves,

[22] We exclude Compound in the sample because the quantity of liquidated collateral on Compound is not accurate in Dune. This does not materially impact our sample coverage since the number of Compound liquidations is small $(3,894)$, compared to $27,072$ from Aave and Maker combined.

[23] The price is determined by dividing the dollar amount of the swap trade, measured by the value of the asset purchased, by the quantity of the asset sold (a_i).

[24] Cryptos that are not native to the Ethereum blockchain (e.g., Bitcoin) or are not a token (e.g., Ether) are wrapped so that smart contracts can handle them using a standard token interface, called ERC-20.

[25] We winsorize the price change and drop a few observations with abnormal values for the collateral amount, since extreme values are likely influenced by other transactions unrelated to liquidations.

[26] For example, June 13, 2022, witnessed the largest number of liquidation events in our sample. This spike was likely triggered by heightened expectations of a rate hike following an unexpected surge in inflation, as indicated by the CPI report released the preceding Friday. On that day, the U.S. stock market officially entered a bear market, and the highly correlated crypto market experienced significant losses. Bitcoin endured its largest price drop since 2011 in a broad sell-off, forcing lenders like Celsius to halt withdrawals.

with only seconds separating them. The size and duration of these waves, as well as the specific position of a liquidation within a wave, can affect the associated price impacts, as documented in Lehar and Parlour (2022). Here, we define liquidations of a given collateral to be part of the same *wave* if their occurrences are less than one minute apart. Notably, 98% of the waves involving multiple liquidations are exclusive to either Aave or Maker. To ensure accurate estimation of the differential price impacts between Aave and Maker, we focus our analysis on these homogeneous waves, excluding the mixed wave that might introduce confounding spillover effects between the two types of liquidations.

5.2 Variable Definition and Summary Statistics

Table 1: Summary Statistics

	Aave (Fixed-spread)	Maker (Auction)	Total
Pre-Liquidation Price p^0($)	1938.9	1981.7	1943.1
	(677.1)	(672.0)	(676.6)
Price Change Δp (%)	-0.0581	-0.0124	-0.0536
	(0.361)	(0.355)	(0.360)
Liquidation Discount (%)	5	1.834	4.690
	–	(2.389)	(1.201)
Liquidation Revenue (k)	39.28	145.4	49.67
	(185.0)	(488.2)	(234.8)
Wave Length (min)	1.312	1.160	1.297
	(1.669)	(1.396)	(1.645)
Wave Size	14.46	11.72	14.19
	(17.65)	(10.96)	(17.13)
Observations	7060		

This table presents the mean and standard deviation (in parentheses) for the variables of interest, grouped by protocol. The sample period is 2021/04/26–2024/05/26. Price change Δp_i is the percentage change in the collateral price before and after liquidation i. Collateral value (in thousands USD) is calculated as the amount of collateral liquidated times the price of the collateral before the liquidation (p_i^0). Wave length (in minutes) is defined as the time elapsed from the first liquidation to the last liquidation in the wave. Wave size is the total number of liquidations in the wave.

Table 1 presents the summary statistics of the key variables. The pre-liquidation price (p^0) of ETH has a mean of $1,943, since most of the liquidations in our sample occur during the downturn of ETH around June 2022 and May–June 2021. The difference in p^0 across Aave and Maker is not statistically significant, which assures the similarity in market conditions for the two types of liquidations.

The average price change (Δp) has the expected sign (negative), with an unconditional mean of -0.053%. In terms of magnitude, the average price change associated with Aave liquidations is larger than Maker liquidations, and the difference in means is statistically significant.

Aave's liquidation discounts are system-wide parameters determined by its governance DAO, varying with different types of collateral. These parameters can vary over time with discrete changes. For ETH, however, the liquidation discount is fixed at 5% throughout our sample. For Maker liquidations, we infer the liquidation discount by $(p_i^0 - p_i^l)/p_i^0 \times 100\%$ (that is, the percentage reduction in the liquidation price, denoted as p_i^l, compared to the pre-liquidation market price). Here p_i^l is directly observed, as it is the price at which a liquidator places a bid and purchases the collateral. Maker's liquidation has an average liquidation discount of 1.83%, which is significantly lower than Aave's fixed discount. This is consistent with the competition effect in our model, which predicts a higher liquidation price and lower liquidation discount for the auction format.

The two protocols, however, differ significantly in the size of liquidations. Here we calculate the liquidation revenue as the product of the liquidated quantity of collateral and the liquidation price.[27] For Aave, the liquidation revenue is at most 50% of the borrowed amount due to Aave's liquidation rule, with a mean revenue of \$39.28K per liquidation. For Maker, the liquidation revenue covers the amount borrowed in most cases, and the mean revenue is significantly higher, at \$145.4K. Overall, Maker accounts for 29% of all liquidation revenues, while accounting for 10% of liquidations.

To capture the duration and magnitude of liquidation waves, we measure wave length as the time difference between the first and last liquidation within a wave, and wave size as the total number of liquidations in the wave. On average, liquidations occur in waves that last about one minute with a dozen other liquidations. Aave waves are slightly larger than Maker waves, which aligns with the overall higher frequency of Aave liquidations in the sample.

5.3 Regression Analysis

Although the mean comparison from Table 1 suggests a smaller negative price impact for the auction mechanism, other confounding factors are at play, such as the size of the liquidation, the market condition, the initial shock that triggers the liquidation wave, and the character-

[27]For Aave, we infer the liquidation price by $p_i^l = p_i^0 \times (1 - discount/100\%)$.

istics of the wave. To control for these factors, we use the following regression equation to estimate the effect of liquidation mechanism:

$$\Delta p_i = \alpha + \beta Auction_i + \gamma X_i' + \delta_t + \mu_d + \eta_b + \epsilon, \tag{23}$$

where $Auction = 1$ if and only if it is a Maker liquidation and the control variable X_i includes the liquidation revenue, the pre-liquidation price (p^0), the length and size of the wave, and i's position in the wave. The liquidation revenue, which is essentially repayment for the borrowed amount, controls for the size of the liquidation. In other words, we compare liquidations that generate the same amount of repayments. The pre-liquidation price controls for the market condition, and the wave characteristics account for any cumulative impact of cascading liquidations that occur in a short time.

There are three sets of fixed effects. First, δ_t is the date fixed effect, which is used to control for the common shock that triggers the liquidation. Second, μ_d is the DEX fixed effect, where d indicates the DEX on which the two swap trades used to determine p_i^0 and p_i^1 occur.[28] This encompasses factors that influence the pricing of the collateral cryptocurrency on a specific DEX. Finally, η_b is the debt-token fixed effect, with b indicating the borrowed token (for example, DAI, USDC, etc). This controls for any changes in conditions that are specific to the borrowed token.

We can therefore interpret the key parameter, β, as the difference in Δp between an auction and fixed-spread ETH liquidation that repays the same amount of debt with a similar market condition, as part of similar liquidation waves on the same day.

Table 2 presents regression results. Column (1) shows the baseline result of regression equation (23) without adding control variable X. Column (2) includes the liquidation revenue and pre-liquidation price. Column (3) further adds the length and size of the wave and the liquidation's position in the wave. The coefficient on *Auction* is positive and significant in all three columns, with a point estimate of 0.055% in the last specification. Given our mean Δp at -0.058% for Aave liquidations, our estimate suggests that the auction mechanism is associated with a price change of -0.008%, implying a significant dampening effect of the negative price impact. Coefficients for liquidation revenue have a negative sign as expected, suggesting that larger liquidations are associated with larger negative price impacts. The length and the size of the wave have opposing effects, while a liquidation's position in a wave does not have a significant effect.

Our regression result suggests that the auction mechanism ameliorates the price impact of liquidations. According to our theory, this is due to a dominating competition effect, which

[28] In our robustness check where multiple types collateral are included in the sample, we modify this term to reflect the collateral-DEX fixed effect.

Table 2: The effect of liquidation mechanism on price changes

Dependent Variable:	Price change Δp (%)		
	(1)	(2)	(3)
Auction	0.0435***	0.0561***	0.0554***
	(7.65)	(5.69)	(5.85)
Liquidation Revenue ($K)		-0.000137**	-0.000132**
		(-4.48)	(-4.31)
Pre-Liquidation Price ($)		-0.000254**	-0.000264**
		(-3.94)	(-4.02)
Wave Length (min)			-0.0246***
			(-13.85)
Wave Size			0.00223***
			(8.56)
Position in Wave			0.0000734
			(0.11)
Date FE	Y	Y	Y
DEX FE	Y	Y	Y
Debt Token FE	Y	Y	Y
Observations	7060	7060	7060
R^2	0.029	0.040	0.044

This table shows estimated results of regression equation (23). The estimation sample consists of liquidations that belong to either a pure auction wave or a pure non-auction wave. *Auction* is a binary variable that equals 1 if it is a Maker liquidation. Definitions of other variables are provided in the note in Table 1. Date FE refers to the date fixed effect, and DEX FE is the fixed effect on the DEX on which the swap trades used to determine prices occur. Standard errors are clustered by DEX with t- statistics in parentheses. One, two, and three stars indicate significance at the 5%, 1%, and 0.1% level, respectively.

predicts a higher liquidation price and consequently a lower liquidation discount for the auction mechanism. To test whether the liquidation discount is the underlying mechanism, we conduct two additional regression analysis. First, we regress liquidation discount on *Auction* with the same set of control as in Equation (23) to confirm if auction liquidations are associated with lower discount. Second, we add liquidation discount as an explanatory variable in Equation (23) and check if the addition of this variable could explain the effect of *Auction* on price changes.

Table 3 presents the results from the two regressions. The first regression confirms the pattern on liquidation discounts observed in Table 1; namely, the auction mechanism is associated with a reduction in liquidation discount by 3.2 percentage points. The second regression reveals that when liquidation discount in included as an explanatory variable, the liquidation mechanism is no longer positive and become statistically insignificant, while liquidation discount is significant with the expected sign; that is, the higher the discount, the lower the liquidation price and consequently more negative Δp. This is consistent with our hypothesis that the auction mechanism affects Δp through liquidation discounts.

Ultimately, a lower discount reduces price impact through the quantity channel. For a fixed repayment amount, a lower discount corresponds to a higher liquidation price, reducing the quantity of collateral that must be sold. This results in a smaller shift in the supply curve and, consequently, a smaller impact on price. Directly testing the quantity channel within the current framework is challenging because the quantity variable appears on the right-hand side of the equation as part of the liquidation revenue. In the next paragraphs, we draw on aggregate-level data to provide descriptive evidence for the quantity channel.

Supplementary Evidence To complement our transaction-level analysis, we examine the total quantity of ETH liquidated as a share of all ETH pledged for each protocol. To calculate this share, we obtain the total quantity of ETH liquidated during each month in our sample as the numerator. Then we gather data on the end-of-day aggregate balances of ETH deposited at Aave and Maker, respectively, and take the monthly average to form the denominator.[29] We choose monthly frequency since the occurrences of liquidations are relatively infrequent. Figure 4 plots the share of ETH liquidated for the two protocols.

During earlier episodes of liquidations, Aave has a larger spike than Maker in the share of ETH liquidated. In later episodes with smaller spikes from Aave, Maker's shares barely spike. Overall, the average share of ETH liquidated is 0.5% for Aave and 0.16% for Maker. This evidence suggests that Maker contributes to a smaller aggregate supply from liquidation

[29]Our calculation also includes Lido Staked Ether (stETH), which is derivative from ETH accepted by Aave as collateral.

Table 3: Liquidation discount and price change

	(1) Liquidation Discount (%)	(2) Price Change Δp (%)
Auction	-3.187***	0.00392
	(-57.94)	(0.35)
Liquidation Revenue ($K)	-0.0000782	-0.000133**
	(-1.97)	(-4.23)
Pre-Liquidation Price ($)	0.000143	-0.000262**
	(1.34)	(-4.04)
Wave Length (min)	0.0876**	-0.0232***
	(4.20)	(-12.75)
Wave Size	-0.00907**	0.00209***
	(-4.54)	(8.20)
Position in Wave	0.00263***	0.000116
	(4.87)	(0.18)
Liquidation Discount (%)		-0.0162*
		(-2.60)
Date FE	Y	Y
DEX FE	Y	Y
Debt Token FE	Y	Y
Observations	7060	7060
R^2	0.633	0.045

Column (1) shows the regression of liquidation discount as the dependent variable. Column (2) shows the regression of price change as the dependent variable. Definitions of independent variables are the same as in Table 2. Standard errors are clustered by DEX with t- statistics in parentheses. One, two, and three stars indicate significance at the 5%, 1%, and 0.1% level, respectively.

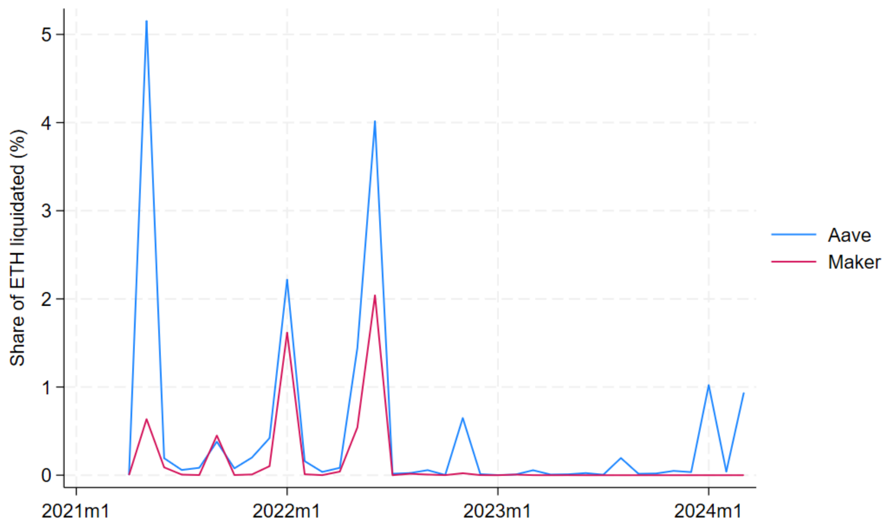


Figure 4: Share of ETH liquidated for Aave and Maker

This figure shows the share of ETH liquidated in each month from 2021/04/26 to 2024/03/30. This share is the total quantity of ETH liquidated divided by the month average balance of all ETH deposited, expressed in %.

per unit of collateral deposited, consistent with the earlier finding of a smaller price impact.

6 Conclusion

This paper studies how different liquidation mechanisms of DeFi loans impact cryptocurrency price volatility. Using a static model, we compare the changes in equilibrium prices before and after a demand shock under two alternative liquidation mechanisms: the fixed-spread mechanism and the auction mechanism. These represent the two primary mechanisms used by DeFi protocols. We show that under the fixed-spread mechanism, the liquidation price is independent of the level of competition among liquidators, whereas it is directly influenced by competition under the auction mechanism.

When the level of competition is high, the auction mechanism leads to a higher liquidation price and a lower quantity of liquidated collateral compared to the fixed-spread mechanism. This implies that the auction mechanism has a smaller amplification effect on the negative demand shock. Conversely, when the level of competition is low, which can occur if the market price becomes sufficiently low due to a large negative demand shock, the fixed-spread liquidation is less attractive than the auction due to the inflexibility of the liquidation price. This discourages liquidator participation in fixed-spread liquidations, leading to more failed liquidations, a reduced supply of liquidated cryptocurrencies, and a smaller amplification effect compared to the auction mechanism.

Our empirical analysis shows that liquidation transactions have a negative price impact within the block. For Aave (fixed-spread) liquidations, the magnitude of the price impact is on average 0.05%, while for Maker (auction) liquidations, it is 0.01%. We show that the difference in price impact is significant, even conditional on the liquidation size, market condition, the common shock, and characteristics of liquidation waves. We further show that this difference is driven by the lower liquidation discounts observed in Maker liquidation, which can lead to a lower quantity of liquidated collateral. On the aggregate level, we document that Maker liquidates a smaller share of its ETH compared to Aave, consistent with the finding of a smaller price impact. These empirical findings align with the model prediction in the case of relatively low entry cost.

References

Azar, P. D., Casillas, A., and Farboodi, M. (2024). Information and market power in defi intermediation. *FRB of New York Staff Report*, (1102).

Baird, D. G. and Rasmussen, R. K. (2003). Chapter 11 at twilight. *Stanford Law Review*, 56(3):673–699.

Bhattacharyya, S. and Singh, R. (1999). The resolution of bankruptcy by auction: allocating the residual right of design. *Journal of Financial Economics*, 54(3):269–294.

Burdett, K. and Judd, K. (1983). Equilibrium price dispersion. *Econometrica*, 51(4):955–69.

Campbell, J. Y., Giglio, S., and Pathak, P. (2011). Forced sales and house prices. *American Economic Review*, 101(5):2108–2131.

Chiu, J., Ozdenoren, E., Yuan, K., and Zhang, S. (2023). On the fragility of defi lending. Technical report, Bank of Canada.

Coval, J. and Stafford, E. (2007). Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics*, 86(2):479–512.

Dinc, S., Erel, I., and Liao, R. (2017). Fire sale discount: Evidence from the sale of minority equity stakes. *Journal of Financial Economics*, 125(3):475–490.

Donner, H. (2020). Determinants of a foreclosure discount. *Journal of Housing and the Built Environment*, 35(4):1079–1097.

Dos Santos, S., Singh, J., Thulasiram, R. K., Kamali, S., Sirico, L., and Loud, L. (2022). A new era of blockchain-powered decentralized finance (DeFi) - a review. In *2022 IEEE 46th Annual Computers, Software, and Applications Conference (COMPSAC)*, pages 1286–1292.

Eckbo, B. and Thorburn, S. (2008). Automatic bankruptcy auctions and fire-sales. *Journal of Financial Economics*, 89(3):404–422.

Eckbo, B. E. and Thorburn, K. S. (2009). Bankruptcy as an auction process: Lessons from Sweden. *Journal of Applied Corporate Finance*, 21(3):38–52.

Falato, A., Hortaçsu, A., Li, D., and Shin, C. (2021). Fire-sale spillovers in debt markets. *The Journal of Finance*, 76(6):3055–3102.

Guren, A. M. and McQuade, T. J. (2020). How do foreclosures exacerbate housing downturns? *The Review of Economic Studies*, 87(3):1331–1364.

Hansen, R. G. and Thomas, R. S. (1998). Auctions in bankruptcy: theoretical analysis and practical guidance. *International Review of Law and Economics*, 18(2):159–185.

Heimbach, L. and Huang, W. (2023). Defi leverage. *Available at SSRN 4459384*.

Lehar, A. and Parlour, C. A. (2022). Systemic fragility in decentralized markets. *SSRN Electronic Journal*.

Lehar, A. and Parlour, C. A. (2023). Battle of the bots: Flash loans, miner extractable value and efficient settlement. *SSRN Electronic Journal*.

Myerson, R. B. (1998). Population uncertainty and Poisson games. *International Journal of Game Theory*, 27(3):375–392.

Park, Y. W. and Bang, D. W. (2014). Loss given default of residential mortgages in a low LTV regime: Role of foreclosure auction process and housing market cycles. *Journal of Banking & Finance*, 39:192–210.

Qin, K., Zhou, L., Gamito, P., Jovanovic, P., and Gervais, A. (2021). An empirical study of defi liquidations: Incentives, risks, and instabilities. In *Proceedings of the 21st ACM Internet Measurement Conference*, pages 336–350.

Sasi-Brodesky, A. and Nassr, I. K. (2023). Defi liquidations: Volatility and liquidity. *OECD Working Papers on Finance, Insurance and Private Pensions*, (48).

Wang, D., Wu, S., Lin, Z., Wu, L., Yuan, X., Zhou, Y., Wang, H., and Ren, K. (2021). Towards a first step to understand flash loan and its applications in defi ecosystem. In *Proceedings of the Ninth International Workshop on Security in Blockchain and Cloud Computing*, pages 23–28.

A Proofs

Proof of Lemma 1 . We start by showing there is no pure-strategy equilibrium. If everyone's strategy is to bid $t < \pi_f(\gamma p, l)$, a liquidator can bid $t+\epsilon$ and win every liquidation regardless of the number of competitors, which leads to a discrete increase in winning probability but only an arbitrarily small decrease in the profit conditional on winning. This results in a discrete increase in the expected profit. Therefore, the only possible pure-strategy equilibrium is $t = \pi_f(\gamma p, l)$, in which case the expected profit is 0. But then a liquidator can get a strictly positive profit by bidding $t = 0$, because there is a positive probability that they are the only one matched to this liquidation. Therefore, there is no pure strategy equilibrium.

We next show that $H(t)$ does not have any mass point and has connected support. If H has a mass point at t, bidding $t + \epsilon$ always leads to a higher expected profit than bidding t because it leads to a jump in winning probability and only ϵ decrease in profit conditional on winning. If the support is not connected, that is, there exists an interval (t_1, t_2) on which the density of G is zero, then anyone who bids to t_2 can deviate to t_1 to reduce the payment without affecting the winning probability. Therefore, bidding t_2 cannot be optimal.

By equating (6) with (7), we have

$$\mu(0;\eta_l)\pi_f(\gamma p, l) = \sum_m \mu(m;\eta_l)\left(\pi_f(\gamma p, l) - t\right) H(t)^m, \text{ for } \forall t \in [0, \bar{t}]$$

Plugging in the expression for Poisson distribution $\mu(m;\eta_l) = \frac{\eta_l^m e^{-\eta_l}}{m!}$, we have

$$\begin{aligned} \pi_f(\gamma p, l)e^{-\eta_l} &= \sum_m \frac{\eta_l^m e^{-\eta_l}}{m!}\left(\pi_f(\gamma p, l) - t\right) H(t)^m \\ \pi_f(\gamma p, l) &= \sum_m \frac{\left(\eta_l H(t)\right)^m e^{-\eta_l H(t)}}{m!}\left(\pi_f(\gamma p, l) - t\right) e^{\eta_l H(t)} \end{aligned}$$

Note that $\sum_m \frac{(\eta_l H(t))^m e^{-\eta_l H(t)}}{m!} = 1$, thus

$$\pi_f(\gamma p, l) = (\pi_f(\gamma p, l) - t)\, e^{\eta_l H(t)}$$
$$H(t) = \frac{1}{\eta_l}\left[\log \pi_f(\gamma p, l) - \log\left(\pi_f(\gamma p, l) - t\right)\right]$$

To find $\bar{t}$, we need to solve $H(t) = 1$ because $\bar{t}$ is the upper bound:

$$H(\bar{t}) = \frac{1}{\eta_l}\left[\log \pi_f(\gamma p, l) - \log\left(\pi_f(\gamma p, l) - \bar{t}\right)\right] = 1$$

which implies $\bar{t} = \pi_f(\gamma p, l)\,(1 - e^{-\eta_l})$. □

Proof of Proposition 1. In equilibrium, the expected profit of participation $\Pi_l(p; \eta_l)$ must equal the fixed cost C; otherwise, a positive measure of liquidator would find it profitable to participate. From Lemma 1, a liquidator's expected profit can be simplified using the indifference condition, as in Equation (7). Therefore,

$$e^{-\eta_f^*(p,l)} \pi_f(\gamma p, l) = C \tag{24}$$

Take the log of both sides and rearrange to obtain (9). The rest of the proposition follows by plugging the expression of $\eta_f^*(p, l)$ into the expressions in Lemma 1. □

Proof Lemma 2. We solve for the symmetric mixed-strategy equilibrium G with support $[\delta p, \bar{b}]$. Because liquidators are indifferent among all b used in equilibrium, $G(b)$ must satisfy the equal profit condition, (16). Then $\bar{b}$ satisfies

$$G(\bar{b}) = \frac{1}{\eta_l}[\log \pi_a(\delta p, l) - \log \pi_a(\bar{b}, l)] = 1,$$

which can be rewritten as $\pi_a(\bar{b}, l) = e^{-\eta_l} \pi_a(\delta p, l)$. □

Proof of Proposition 2. Combining the free-entry condition with (16), we obtain

$$\pi_a(\delta p, l) e^{-\eta_a^*(p,l)} = C \tag{25}$$

Take the log of both sides and rearrange to obtain (18). The rest of the proposition follows from plugging $\eta_a^*(p, l)$ into the expressions in Lemma 2. □

Proof of Lemma 3. To ease presentation, we include C as an argument in q_a and q_f. We first calculate the expected quantity of liquidated collateral for a fixed spread liquidation.

If $\min\{l/\gamma p, 1\}(1-\gamma R)p \leq C$, the liquidator's net benefit from participating is 0. Therefore, no liquidation occurs and $q_f(p,l,C) = 0$. If $\min\{l/\gamma p, 1\}(1-\gamma R)p > C$, liquidators participate and the liquidated quantity is $\min\{l/\gamma p, 1\}$ if the liquidation is successful. The probability of a success is $1 - e^{-\eta_f^*(p,l)}$, where

$$e^{-\eta_f^*(p,l)} \min\{l/\gamma p, 1\}(1-\gamma R)p = C.$$

As a result, the expected liquidated quantity is

$$q_f(p,l,C) = \left(1 - e^{-\eta_f^*(p,l)}\right)\min\{l/\gamma p, 1\} = \min\{l/\gamma p, 1\} - \frac{C}{(1-\gamma R)p}.$$

Notice that $\min\{l/\gamma p, 1\}(1-\gamma R)p > C$ holds if $l > \gamma p$ and $p > C/(1-\gamma R)$ or if $l < \gamma p$ and $l/\gamma - Rl > C$. Therefore, we can distinguish two cases. If $l > \gamma p$,

$$q_f(p,l,C) = \begin{cases} 0 & \text{if } p \leq \frac{C}{1-\gamma R} \\ 1 - \frac{C}{(1-\gamma R)p} & \text{if } p > \frac{C}{1-\gamma R} \end{cases}.$$

If $l < \gamma p$,

$$q_f(p,l,C) = \begin{cases} 0 & \text{if } l/\gamma - Rl < C \\ \frac{l}{\gamma p} - \frac{C}{(1-\gamma R)p} & \text{if } l/\gamma - Rl > C \end{cases}.$$

Next, we move to $q_a(p,l,C)$. Again, we distinguish two cases. If $l > \delta p$

$$q_a(p,l,C) = \begin{cases} 0 & \text{if } C > (1-R\delta)p \\ 1 - \frac{C}{p-R\delta p} & \text{if } p - R\delta p > C > p - Rl \\ \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{p-Rl}{C}\right) - \frac{C}{p-R\delta p} & \text{if } C < p - Rl \end{cases}.$$

If $l < \delta p$, then

$$q_a(p,l,C) = \begin{cases} 0 & \text{if } C > l/\delta - Rl \\ \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{l/\delta - Rl}{C}\right) - \frac{C}{p-R\delta p} & \text{if } C < l/\delta - Rl \end{cases}.$$

We next calculate $\Delta(p,l,C) = q_a(p,l,C) - q_f(p,l,C)$.

First, we focus on the case with $p - Rl > 0$. According to the above discussion, we can distinguish three cases.

Case 1: If $l < \delta p < \gamma p$, then

$$\Delta(p,l,C) = \begin{cases} \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{l/\delta - Rl}{C}\right) + \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} - \frac{l}{\gamma p} & \text{if } \frac{l}{\gamma} - Rl > C \\ \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{l/\delta - Rl}{C}\right) - \frac{C}{p-R\delta p} & \text{if } \frac{l}{\delta} - Rl > C \geq \frac{l}{\gamma} - Rl \\ 0 & \text{if } C \geq \frac{l}{\delta} - Rl \end{cases}.$$

If $C \to 0$, $\Delta(p,l,C) \to (R - 1/\gamma)/p < 0$. Therefore, auctions lead to fewer liquidated collateral if C is sufficiently small. If $l/\gamma - Rl > C$,

$$\frac{\partial \Delta(p,l,C)}{\partial C} = \frac{1}{p}\log\left(\frac{l/\delta - Rl}{C}\right) + \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} > 0.$$

To see this, notice $l/\gamma - Rl > C$ implies that first term after the equality is positive and $\gamma - \delta > 0$ implies the second term is positive. If $C = l/\gamma - Rl$, then $\Delta(p,l,C) > 0$. Therefore, there exists a unique $C^*_{p,l}$ such that $\Delta(p,l,C) = 0$ on $(0, l/\gamma - Rl)$. If $C < C^*_{p,l}$, $\Delta(p,l,C) < 0$ and if $C \geq C^*_{p,l}$, $\Delta(p,l,C) \geq 0$.

Case 2: If $\delta p < l < \gamma p$, then

$$\Delta(p,l,C) = \begin{cases} \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{p-Rl}{C}\right) + \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} - \frac{l}{\gamma p} & \text{if } \frac{l}{\gamma} - Rl \geq C \\ \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{p-Rl}{C}\right) - \frac{C}{p-R\delta p} & \text{if } p - Rl \geq C > l/\gamma - Rl \\ 1 - \frac{C}{p-R\delta p} & \text{if } (1-R\delta)p \geq C > p - Rl \\ 0 & \text{if } C \geq (1-R\delta)p \end{cases}.$$

Again $\Delta(p,l,C) > 0$ if $C = l/\gamma - Rl$ and $\Delta(p,l,C) < 0$ as $C \to 0$. Moreover, if $C < l/\gamma - Rl$,

$$\frac{\partial \Delta(p,l,C)}{\partial C} = \frac{1}{p}\log\left(\frac{p-Rl}{C}\right) + \frac{R(\gamma-\delta)}{(1-R\delta)(1-R\gamma)p} > 0$$

because $(p - Rl)/C > 1$ and $\gamma > \delta$. Therefore, there exists a unique $C^*_{p,l}$ such that $\Delta(p,l,C) = 0$ on $(0, l/\gamma - Rl)$. If $C < C^*_{p,l}$, $\Delta(p,l,C) < 0$ and if $C \geq C^*_{p,l}$, $\Delta(p,l,C) \geq 0$.

Case 3: If $\delta p < \gamma p < l$, then

$$\Delta(p,l,C) = \begin{cases} \frac{C+Rl}{p} + \frac{C}{p}\log\left(\frac{p-Rl}{C}\right) + \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} - 1 & \text{if } p - Rl \geq C \\ \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} & \text{if } l/\gamma - Rl \geq C > p - Rl \\ 1 - \frac{C}{p-R\delta p} & \text{if } (1-R\delta)p \geq C > l/\gamma - Rl \\ 0 & \text{if } C \geq (1-R\delta)p \end{cases}.$$

Again $\Delta(p,l,C) > 0$ if $C = p - Rl$ and $\Delta(p,l,C) < 0$ as $C \to 0$. Moreover if $C < p - Rl$,

$$\frac{\partial \Delta(p,l,C)}{\partial C} = \frac{1}{p}\log\left(\frac{p-Rl}{C}\right) + \frac{R(\gamma-\delta)}{(1-R\delta)(1-R\gamma)p} > 0.$$

We can show that $C^*_{p,l} = (p-Rl)/x^*$ where x^* solves

$$1 - x + \log(x) + \frac{R(\gamma-\delta)}{(1-R\delta)(1-R\gamma)} = 0.$$

Notice that $x^* > 1$, which implies that $C^*_{p,l} < p - Rl$. If $C < C^*_{p,l}$, $\Delta(p,l,C) < 0$ and if $C \geq C^*_{p,l}$, $\Delta(p,l,C) \geq 0$. Notice that in the three cases, $C^*_{p,l} < \min\{p, l/\gamma\} - Rl$. This proves the first claim of the lemma.

Next, we consider the case with $p - Rl < 0$. Because $R\delta < R\gamma < 1$, $p - Rl < 0$ implies that $\delta p < \gamma p < l$. Therefore,

$$\Delta(p,l,C) = \begin{cases} \frac{R(\gamma-\delta)C}{(1-R\delta)(1-R\gamma)p} & \text{if } l/\gamma - Rl \geq C > 0 \\ 1 - \frac{C}{p-R\delta p} & \text{if } (1-R\delta)p \geq C > l/\gamma - Rl \\ 0 & \text{if } C \geq (1-R\delta)p \end{cases},$$

which is non-negative. Because $\gamma < R^{-1}$, the first branch always exists. Therefore, as $C \to 0$, $\Delta(p,l,C) \to 0$. This proves the second claim of the lemma. $\square$

Proof of Proposition 4. We now prove each of the claims.

Proof of Claim 1: Recall that

$$Q_a(p,C) = \alpha \int_{\theta p}^{\theta p^*} q_a(p,l,C) f(l)\, dl$$
$$Q_f(p,C) = \alpha \int_{\theta p}^{\theta p^*} q_f(p,l,C) f(l)\, dl.$$

Because $\theta p < p$ and $R\theta < 1$, the set $(\theta p, p/R)$ is not empty.

$$\begin{aligned}\Delta_{\tilde{Q}}(p,C) &= Q_a(p,C) - Q_f(p,C) = \alpha \int_{\theta p}^{\theta p^*} \Delta(p,l,C) f(l)\, dl \\ &= \alpha \int_{\theta p}^{\min\{p/R, \theta p^*\}} \Delta(p,l,C) f(l)\, dl + \alpha \int_{\min\{p/R,\theta p^*\}}^{\theta p^*} \Delta(p,l,C) f(l)\, dl.\end{aligned}$$

By the proof of Lemma 3, the first term is positive if C is sufficiently small and the second term can be made arbitrarily small if C is sufficiently close to 0, which implies $\Delta_{\tilde{Q}}(p) < 0$ if

C is sufficiently small. Define

$$\underline{C}_p = \sup \left\{ \tilde{C} : \Delta_{\tilde{Q}}(p, C) < 0 \text{ for all } C < \tilde{C} \right\}.$$

Then $\underline{C}_p > 0$ for every $p \in [\underline{v}, p^*)$. Define $\underline{C} = \inf \left\{ \underline{C}_p : p \in [\underline{v}, p^*) \right\}$. Then by definition, $\Delta_{\tilde{Q}}(p) < 0$ for all $p \in [\underline{v}, p^*)$. We only need to show $\underline{C} > 0$.

To see this, first notice that for any small $\varepsilon > 0$, $\underline{C} = \min \left\{ \underline{C}^1(\varepsilon), \underline{C}^2(\varepsilon) \right\}$ where $\underline{C}^1(\varepsilon) = \inf \left\{ \underline{C}_p : p \in [\underline{v}, p^* - \varepsilon] \right\}$ and $\underline{C}^2(\varepsilon) = \inf \left\{ \underline{C}_p : p \in [p^* - \varepsilon, p^*) \right\}$. We next show $\underline{C}^1(\varepsilon) > 0$ and $\underline{C}^2(\varepsilon) > 0$. Suppose, toward contradiction, $\underline{C}^1(\varepsilon) = 0$. This implies there exists a sequence $\{p_n\}_{n=1}^{\infty}$ such that $p_n \in [\underline{v}, p^* - \varepsilon]$ and $\lim_{n\to\infty} \underline{C}_{p_n} = 0$. Because $[\underline{v}, p^* - \varepsilon]$ is a compact set, there exists a converging subsequence of $\{p_n\}_{n=1}^{\infty}$, denoted as $\{p_{n_k}\}_{k=1}^{\infty}$. Denote its limit as $\hat{p}$. Notice that $\hat{p} \in [\underline{v}, p^* - \varepsilon]$. Then $0 = \lim_{k\to\infty} \Delta\left(p_{n_k}, \underline{C}_{p_{n_k}}\right) = \lim_{k\to\infty} \Delta\left(\hat{p}, \underline{C}_{p_{n_k}}\right) > 0$, which leads to a contradiction. Therefore, $\underline{C}^1(\varepsilon) > 0$ for all small $\varepsilon > 0$.

Next, choose $\varepsilon > 0$ sufficiently small such that $R\theta p^* < p^* - \varepsilon$. This implies that for all $p \in [p^* - \varepsilon, p^*]$, $Rl < p$ if $l \in [\theta p, \theta p^*]$. Then for all $l \in [\theta(p^* - \varepsilon), \theta p^*]$ and $p \in [p^* - \varepsilon, p^*]$, $C^*_{p,l} > 0$. Using a similar argument as the above, we can show that

$$C^2 = \inf \left\{ C^*_{p,l} : p \in [p^* - \varepsilon, p^*], l \in [\theta(p^* - \varepsilon), \theta p^*] \right\}$$

is positive and if $C < C^2$, $q_a(p, l, C) > q_f(p, l, C)$ for all $p \in [p^* - \varepsilon, p^*]$ and $l \in [\theta(p^* - \varepsilon), \theta p^*]$. Therefore, if $p \in [p^* - \varepsilon, p^*)$ and $C < C^2$

$$\Delta_{\tilde{Q}}(p, C) = \alpha \int_{\theta p}^{\theta p^*} [q_a(p, l, C) - q_f(p, l, C)] f(l)\, dl < 0,$$

which implies that $\underline{C}_p \geq \underline{C}_1 > 0$ for all $p \in [p^* - \varepsilon, p^*)$. As a result, $\underline{C}^2(\varepsilon) \geq \underline{C}_1 > 0$.

Proof of Claim 2: Notice that if $p < Rl$, $q_a(p, l, C) \geq q_f(p, l, C)$ for all $C > 0$, where the inequality holds strictly if $C \in \left(0, \min\left\{\frac{l}{\delta p}, 1\right\}(p - R\delta p)\right)$. We can then define $C^*_{p,l} = 0$. If $p > Rl$, $q_a(p, l, C) \geq q_f(p, l, C)$ for all $C \geq C^*_{p,l}$ where the inequality holds strictly if $C \in \left(C^*_{p,l}, \min\left\{\frac{l}{\delta p}, 1\right\}(p - R\delta p)\right)$. Define $\bar{C}^* = \sup \left\{ C^*_{p,l} : p \in [\underline{v}, p^*), l \in [\theta \underline{v}, \theta p^*] \right\}$. Then $\bar{C}^* < p^*(1 - R\delta)$ and if $C > \bar{C}^*$, $q_a(p, l, C) \geq q_f(p, l, C)$ for all $p \in [\underline{v}, p^*), l \in [\theta \underline{v}, \theta p^*]$, which in turn implies that $Q_a(p, C) \geq Q_f(p, C)$ for all $p \in [\underline{v}, p^*)$.

Next, define

$$\overline{C}_p = \inf \left\{ \tilde{C} : \Delta_{\tilde{Q}}(p, C) \geq 0 \text{ for all } C > \tilde{C} \right\}$$

and

$$\overline{C} = \sup \left\{ \overline{C}_p : p \in [\underline{v}, p^*) \right\}.$$

Because $\overline{C}_p \leq \bar{C}^*$ by definition, $\overline{C} < \overline{C}^* < \infty$. Moreover, $\Delta_{\tilde{Q}}(p, C) \geq 0$ for all $C > \overline{C}$ and $p \in [\underline{v}, p^*)$, and if $C < \overline{C}$, there exists $p \in [\underline{v}, p^*)$ such that $Q_a(p) < Q_f(p)$. Notice that by definition, $\overline{C} \geq \underline{C}$.

Proof of Claim 3: Notice that if $(1 - R\delta) p > C > (1 - R\gamma) p$, $q_f(p, l, C) = 0$ for all p, l and $q_a(p, l, C) > 0$ for $l > p$. If $(1 - R\delta) p < C$, $q_f(p, l, C) = q_a(p, l, C) = 0$ for all p, l. Therefore, if $p < \min\{C/(1 - R\gamma), \theta p^*\}$, $Q_a(p) \geq Q_f(p)$. Moreover, if at the same time $p > (1 - R\delta) p$, $Q_a(p) > Q_f(p)$. This concludes the proof of the proposition. $\square$