## Entry and Exit in Treasury Auctions

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#### Abstract

Many financial markets are populated by dealers, who commit to participate regularly in the market, and non-dealers, who do not commit. This market structure introduces a trade-off between competition and volatility, which we study using data on Canadian treasury auctions. We document a consistent exit trend by dealers and increasing, but irregular, participation by non-dealer hedge funds. Using a structural model, we evaluate the impact of dealer exit on hedge fund participation and its consequences for market competition and volatility. We find that hedge fund entry was partially driven by dealer exit, and that gains thanks to stronger competition associated with hedge fund entry are offset by losses due to the irregular market participation of hedge funds. We propose an issuance policy that stabilizes hedge fund participation at a sufficiently high average level and achieves revenue gains.

Topics: Debt management; Financial markets; Financial institutions; Market structure and pricing JEL codes: D44, D47, G12, G28


## Résumé

De nombreux marchés financiers comptent des courtiers, qui s'engagent à participer régulièrement au marché, et d'autres participants qui ne sont pas des courtiers et donc, qui ne s'engagent pas à le faire. Cette structure crée un compromis entre concurrence et volatilité, que nous étudions à l'aide de données sur les adjudications de titres du Trésor canadien. Nous décrivons une tendance constante de retrait de courtiers et une participation croissante, mais irrégulière, de fonds de couverture qui ne sont pas courtiers. À l'aide d'un modèle structurel, nous évaluons l'incidence du retrait des courtiers sur la participation des fonds de couverture et sur la concurrence et la volatilité du marché. Nous constatons que I'entrée de fonds de couverture était en partie attribuable au retrait de courtiers, et que les gains réalisés grâce à la plus forte concurrence associée à l'entrée des fonds de couverture sont compensés par les pertes causées par la participation irrégulière des fonds de couverture au marché. Nous proposons une politique d'émission qui stabilise la participation des fonds de couvertures à un niveau moyen suffisamment élevé et qui entraîne des hausses de revenu.

Sujets : Gestion de la dette; Marchés financiers; Institutions financières; Structure de marché et fixation des prix
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## 1 Introduction

Governments worldwide have traditionally relied on regulated banks, known as primary dealers, to consistently purchase government debt and regularly facilitate trade between investors, such as firms, public entities, and individuals. More recently, other institutions have assumed an increasingly important role. For instance, in the U.S., hedge funds doubled the total value of their U.S. Treasury holdings between early 2018 and February 2020, reaching $\$ 2.39$ trillion (Banegas et al., 2021). Given that hedge funds and other non-dealer institutions, referred to as customers, are subject to less-stringent regulations than dealers and have no obligation to participate in market-making activities, the implications of this observed trend on the functioning of Treasury markets remain uncertain. ${ }^{1}$

Broadly, we are interested in better understanding why customers have entered Treasury markets and evaluating the consequences for market functioning. Answering these questions is challenging because there is limited data that allows us to study how customers trade Treasuries. We overcome this challenge by focusing on the Canadian primary market in which the government issues bonds via regularly held auctions. We document dealer exits as well as the increasing, yet irregular, incidents of customer participation. We then introduce and estimate a structural model to assess the role of dealers' exits in explaining customers' entries and to quantify the benefits of greater customer competition against the costs of higher market volatility.

Our data combines bidding information on all Canadian Treasury auctions, from 1999 to 2022, with price information from the secondary market and the futures, foreign exchange and repo markets. Two types of bidders participate in these auctions: dealers and customers. Only dealers can submit bids directly to an auctioneer; customers must bid via a dealer-a common feature in Treasury auctions. For all securities, we observe bidder types, unique anonymized bidder identifiers, and all submitted bids. We also know through which dealer a customer submits bids and where the market cleared.

With these data, we document a series of facts, the first set of facts being related to entry and exit. We show that dealers have been systematically exiting the primary market since these auctions began. In contrast, customers have entered, in particular hedge funds.

[^1]However, unlike dealers, who have an obligation to regularly attend auctions and buy sufficient amounts of debt, customer participation is irregular. This suggests that customers select specific auctions, depending on market conditions. In line with this idea, we show that customers participate in auctions when secondary-market spreads in the days leading up to the auction are high. The second set of facts helps us model bidding behavior, conditional on auction participation. We show that when dealers observe an aggressive customer bid, they systematically adjust their own bids in anticipation of greater competition in the auction. Sophisticated customers, such as hedge funds, should take this adjustment into account when bidding.

Motivated by the empirical evidence and institutional features of the market, we construct a model that mimics a fiscal year. At the start of each year, when the government announces its debt issuance plan, each dealer decides, at a cost, whether to commit to bidding in every auction in the upcoming year and the number of participating dealers is announced publicly. Then, prior to each auction, customers observe the market conditions and decide whether or not to enter that specific auction, at an entry cost. Conditional on participation, all customers and dealers draw private signals about how much they value the bond-representing how much profit they expect to generate post-auction, given their (private) balance sheet positions - and place their bids. The auction clears at the price at which aggregate demand meets supply and each bidder pays their offered prices for each unit won.

To solve for the equilibrium conditions of this game and to estimate the dealer and customer values and entry costs, we build on the empirical literature on multi-unit auctions, in particular Guerre et al. (2000), Hortaçsu (2002), Kastl (2011), and Hortaçsu and Kastl (2012). Unlike existing studies, we allow customers to behave strategically, in that they anticipate dealers updating their own bids, and we endogenize bidder participation. This contributes to the empirical auction literature that allows for endogenous entry but has so far focused on single-object auctions (see Hortacsu and Perrigne (2021) for an overview). Estimation approaches for entry in single-object auctions require knowledge of how the equilibrium bid function behaves. In multi-unit auctions, this is a complicated object and so these tools are not directly transferable. Instead, we exploit estimated bounds on bidderspecific surpluses together with a matching procedure to estimate the entry costs of multi-unit auctions. This approach could be used to endogenize bidder participation in other auction
settings, including those for electricity, renewable energy, or carbon allowances.
We find that both dealers and customers have high entry costs, and that customers are willing to pay more for bonds than dealers are. The latter suggests that, post-auction, participating customers expect to execute more-profitable trading strategies with the bonds than participating dealers. One reason for this may be that customers face less-stringent regulation than dealers, especially post global financial crisis. Consistent with this, we find that prior to 2007-2009, dealer values were not so different from customer values but that they differed significantly afterwards.

To disentangle whether the surge in customer engagement resulted from dealers leaving the market, potentially influenced by regulatory shifts, or from broader alterations in market conditions favoring customer bond purchases, we employ our structural model and conduct a counterfactual analysis. In this process, we calculate counterfactual bids without relying on the conventional assumption of truthful bidding, as is typically employed in multi-unit auction studies. Instead, we utilize the empirical guess-and-verify method introduced by Richert (2021). By reintroducing the dealers who withdrew from the market after 2014 and simulating customer participation in these counterfactual auctions, we can evaluate the effect of increased dealer competition on customers' participation decisions. We find that if dealers had not exited, then an average customer would have been approximately $31.56 \%$ less likely to partake in an average auction. This substantial reduction in participation indicates that dealer exit is a significant economic driver of customer entry.

We next use our model to evaluate the market consequences from the rise in customer participation. On one hand, stronger participation may increase competition (as in Bulow and Klemperer 1996), which reduces debt funding costs and price distortions due to bid shading. On the other hand, irregular bidder participation may increase the volatility in market outcomes, such as the market price. For example, two auctions may clear at different prices only because one auction attracts more customers than the other, and not because the auctions offer bonds of different fundamental values. This not only introduces unnecessary volatility that might destabilize financial markets but also increases the total cost of issuing government debt, since auction revenues fall by more when customer participation is low than they rise when it is high.

To build an intuition for these effects, we start with a simplified environment with one
type of bidder (dealers) that bids directly to the auctioneer and we ask by how much the (expected) auction outcomes vary in the number of competing bidders. We find that when we remove one dealer (from the status quo), the (expected) price drops by $0.06 \%$ because of stronger shading. When the expected number of participating customers increases by one, the expected price increases by $0.0004 \%$ and bid shading decreases by $51.51 \%$ due to stronger competition.

To compare the competition and volatility effect from the increasing yet irregular customer participation, we contrast the revenue gain from attracting one extra customer in expectation with the expected revenue loss coming from the across-auction variation in the customer participation probabilities. Irregular participation results in revenue losses relative to consistent average participation because the expected auction revenues are concave in the customer participation probabilities. The concavity implies that the revenue improvements when participation is high can be more than offset by the losses under low participation. With our model estimates, the revenue gains from attracting an additional customer versus losses from volatility are not too dissimilar: $\mathrm{C} \$ 0.2$ million versus $\mathrm{C} \$ 0.04$ million.

In light of the competition-volatility trade-off, we propose a simple issuance policy that aims to increase competition while simultaneously decreasing volatility. Our idea is to strategically shift the supply from auctions in which we predict strong customer participation to auctions with low predicted customer participation. The hope is that by doing so, we can stabilize customer participation and attract a sufficient number of market participants to guarantee a high level of competition. Indeed, we find that the median revenue increases by about $\mathrm{C} \$ 0.38$ million per auction when implementing our proposed rule.

The competition-volatility trade-off we highlight might be present in other settings. It is common in auctions for financial products to have a set of regular and irregular bidders; for example, see Hendricks et al. (2023) for mortgage securities and Richert (2022) for credit event auctions. Our framework can be easily adjusted to fit these applications. Furthermore, the economic insights generalize to non-auction markets that are populated by regular and irregular participants. Examples include market makers versus opportunistic traders in financial markets, global versus local firms in production markets, loyal versus non-loyal customers in consumption markets, and irregular versus stable energy generation in electricity markets (Petersen et al., 2022).

## 2 Institutional details and data

In most countries, government bonds are issued in primary auctions to a small set of regulated banks, often called primary dealers, and to customers (see Appendix Figure A2). While specific auction rules may vary slightly, the main market features are common across developed economies.

Canadian primary market. In November 1998, Canada adopted a typical primary dealer market structure to distribute its debt. Since then, Treasury auctions have been held according to an annual schedule. For instance, between 1999 and 2022 there have been about twenty-eight government bond auctions per year, with an average nominal issuance size of $\mathrm{C} \$ 3.24$ billion, given the bonds' face value is $\mathrm{C} \$ 100$. The auction schedule specifies the timing of auctions and the total debt to be issued. One week prior to the auction, the precise issuance size is announced.

Anyone can participate in Treasury auctions, but only dealers can bid directly. ${ }^{2}$ Other bidders, called customers, can only participate indirectly by placing their bids via a dealer, who can observe the customers' bids (see Appendix Figure A3). Indirect customer bidding is a common feature of Treasury auctions across countries. It is also present, for example, in Japan and the U.S. (Boyarchenko et al., 2021).

Similar to Treasury auctions in other countries, customers include different types of institutions, such as pension, mutual and exchange traded funds, insurance companies, sovereigns, or bank treasuries. For over a decade, the biggest customer category has consisted of alternative investments companies, which included hedge funds. For simplicity, we use the term hedge fund to describe these investment companies throughout the paper.

A bidder may submit and update two types of bids from the time the tender call opens until the auction closes. This is also a common feature of most Treasury auctions worldwide. The first type of bid is a competitive bid. This is a step function, with at most 7 steps, which specifies how much a bidder offers to pay for specific amounts of the asset for sale. ${ }^{3}$

[^2]During normal times, primary dealers can demand up to $25 \%$ of the supply of Treasuries for their own account and $25 \%$ for their customers (with an aggregated cap of $40 \%$ ), in the form of competitive bids.

The second type of bidding is a non-competitive bid. This is a quantity order that the bidder wins for sure at the average price of all the accepted competitive bid prices. The Bank of Canada actively utilizes non-competitive bids to reduce the announced auction supply (before observing the submitted bids). For example, the Bank buys Treasuries (assets) to match the value of its bank notes (liabilities). For dealers and customers, non-competitive bids are trivial since they cannot be larger than $\mathrm{C} \$ 10$ million for dealers and between $\mathrm{C} \$ 3$ million and $\mathrm{C} \$ 5$ million for customers.

All submitted bids are aggregated and the market clears at the price at which aggregate demand equals auction supply. In case of excess demand, bidders are rationed pro-rata on the margin, which means that the auctioneer proportionally adjusts demand at the clearing price until supply equals demand (see Kastl 2012, for a formal definition). Every bidder wins the units they demanded at bids weakly above the clearing price, and pays the bids for each unit won. ${ }^{4}$

Regulation. In Canada, as elsewhere, dealers have an obligation to actively buy bonds in the primary market. Concretely, in normal times, primary dealers face minimal bidding requirements of roughly $10 \%$ of the auction supply. The minimum level of bidding must be at reasonable prices, and accepted bids should be approximately equal to a dealer's secondary market share over a specified time period. Primary dealers are also expected to act as market makers in the secondary cash and repo markets, where they provide liquidity to investors who seek to exchange government bonds for cash. ${ }^{5}$ In exchange, primary dealers enjoy benefits. For instance, they have privileged access to liquidity facilities, overnight and term repurchase operations, and extract auction rents from observing customer bids. There are also positive reputation effects that potentially spills over to many market segments, coming from the fact that primary dealers are financial institutions the central bank trusts, and this can help

[^3]attract customers.
Given the important role dealers play in the market (in Canada and elsewhere), they are heavily regulated. In particular, in the aftermath of the 2007-2009 financial crisis, regulation tightened for dealers and large banks more broadly. For instance, banks in developed countries faced heightened capital requirements. A notable illustration of this is the Basel III leverage ratio, which was enforced at the close of 2014 and represents a significant limitation for bond trading (CGFS 2016; Wittwer and Allen 2022; Favara et al. 2022). In addition, in Canada, starting in 2016, dealers must report all trades to the Investment Industry Regulatory Organization of Canada, while customers are exempt. A similar trend happened in other countries; for example, in the U.S., dealers started reporting trades in 2017.

Traditionally, customers, such as hedge funds, played a negligible role in Treasury markets. ${ }^{6}$ We know that this has changed in recent years, but we have a limited understanding of what customers are doing, given that there are only a handful of empirical studies. For example, Sandhu and Vala (2023) argue that hedge funds can act as market makers, engaging in trades that counter the positions of other investors. In this way, they are not very different from dealers. During times of distress, such as March 2020, however, hedge funds can contribute to market imbalances, reduced liquidity, and increased price volatility (e.g., Barth and Kahn 2020; Vissing-Jorgensen 2021). Increased hedge fund trading may also have implications for systemic risk in the market, given that hedge funds are more likely to employ riskier trading strategies (Dixon et al., 2012) -an effect that we do not consider in our analysis.

Data. Understanding trading activity and customers' impacts on market functioning poses challenges due to limited data availability. For one, comprehensive long panels of tradelevel data with unique identifiers for all traders are not readily accessible. For instance, the U.S. started collecting trade-level data through TRACE in mid-2017, but customer reporting with unique IDs is not mandatory. Similarly, while the Bank of England and Bank of Canada provide firm IDs, identifying all customers remains difficult; see, for example, Kondor and Pintér (2022); Pintér and Semih (2022); Coen and Coen (2022); and Allen and Wittwer (2023). Additionally, customers, unlike dealers, are not obliged to report their

[^4]Table 1: Data summary of bond auctions

|  | Mean | SD | Min | Max |
| :--- | ---: | ---: | ---: | ---: |
| (Nominal) amount issued (in C\$B) | 3.24 | 1.04 | 1.00 | 7.00 |
| Revenue (in C\$B) | 3.25 | 1.04 | 0.88 | 7.00 |
| Number of dealers | 14.46 | 2.61 | 11 | 23 |
| Number of customers | 6.74 | 2.56 | 1 | 15 |
| Comp demand of a dealer (in \%) | 14.80 | 7.51 | 0.00 | 40 |
| Comp demand of a customer (in \%) | 5.83 | 4.70 | 0.01 | 25 |
| Non-comp demand of a dealer (in \%) | 0.09 | 0.04 | 0.00 | 0.30 |
| Non-comp demand of a customer (in \%) | 0.19 | 0.16 | 0.00 | 0.76 |
| Non-comp demand of the Bank of Canada (in \%) | 13.84 | 4.00 | 0.01 | 20 |
| Number of submitted steps of a dealer | 4.34 | 1.71 | 1 | 7 |
| Number of submitted steps of a customer | 1.86 | 1.02 | 1 | 7 |
| Amount won by a dealer (in \%) | 4.79 | 5.85 | 0 | 35 |
| Amount won by a customer (in \%) | 4.02 | 5.90 | 0 | 25 |

Table 1 displays summary statistics of our sample, which covers the period February 10, 1999 to January 27, 2022. There were 645 auctions, none of which failed due to insufficient demand. The typical auction issues $\$ 3.2$ billion worth of debt. The total number of competitive bidding functions (including updates) is 62,813 . Competitive bids are step functions with at most 7 steps. The total number of non-competitive bids is 10,552 . Demand and amount won are in percentage of supply.
trades. Consequently, studies analyzing hedge fund trading behavior face limitations in that they miss trades between hedge funds and non-dealers, which could represent a biased sample of hedge fund trades. ${ }^{7}$

We overcome the lack of data on hedge fund secondary market activity by focusing on the primary market - where we can trace market participants over a long time horizon, thanks to unique identifiers. Bidder identifiers are created by Bank of Canada staff, who observe bidder names and are, therefore, able to account for mergers, acquisitions, and name changes. We observe all winning and losing bids in regular government bond auctions from the beginning of 1999 to the end of January 2022. This represents the entire auction history with the exception of three auctions held in December 1998. Table 1 provides summary statistics for our auction sample.

We augment the auction data with the daily average prices of each security from the secondary market and the futures, foreign exchange and repo markets, from the Canadian Depository for Securities. These data range from the beginning of 2014 to the end of 2021.

[^5]
## 3 Empirical evidence: Exit, entry and bid updating

We document a series of stylized facts to motivate our structural model.
Exit and entry. We observe two striking time trends regarding entry and exit in the Canadian primary market. Appendix Figure A4 visualizes similar trends using public data of U.S. Treasury auctions; moreover, policy reports and newspaper articles indicate similar patterns for other countries. ${ }^{8}$ This highlights that these trends are not Canadian-specific.

On one hand, dealers have (voluntarily) been exiting the market since the first Treasury auctions took place. While our data shows this fact only for the auctions, we know from exit interviews that primary dealers exited the Canadian Treasury market entirely. The total number of dealers declined from 24, in 1999, to 15, in 2021 (see Figure 1A). After an early round of exits, the number of dealers remained stable until 2014, when global playerssuch as Deutsche Bank and Morgan Stanley - exited the primary market for Canadian debt. Smaller broker-dealers such as PI Financial Corporation and Ocean Securities followed in 2015 (see Appendix Table A1). The exit of global banks from the Canadian bond market around 2013-2015, during a period of tighter banking regulations and monitoring, suggests that stricter regulations may have pushed some banks out of the market. ${ }^{9}$ More recently, two dealers sought buyers, which would have meant exiting via acquisition: RBC (which is also a dealer) purchased HSBC in December 2023 and Laurentian Bank failed to find a buyer.

On the other hand, customers - in particular, hedge funds-have become more active (see Figures 1B and 2A). The total number of hedge funds increased from zero to ten in 2021. Furthermore, they have been buying an increasingly larger share of the auction allotments relative to dealers and other customer groups (see Appendix Figures A5 and A6). How-

[^6]Figure 1: Dealer exit and hedge fund entry from 1999 to 2022


Figure 1A shows how many dealers have participated in primary auctions, on average, in a year since the first auction in 1999 to 2022. Figure 1B shows how many hedge funds participate, on average, each year.
ever, since customers have no obligation to participate regularly, like dealers, their auction participation is highly volatile, as shown in Figure 2.

Predicting customer participation. In Appendix B.1, we analyze potential predictors of customer auction participation, without claiming causality. ${ }^{10}$ These predictions are used in our policy counterfactuals to demonstrate how an auctioneer could shift the supply of Treasuries across auctions to boost competition and reduce volatility. Our selection of explanatory variables is inspired by emerging research into the trading behavior of hedge funds, particularly during 2020 .

Across regression specifications, we find that the spread between the highest and lowest price at which a bond trades shortly prior to being auctioned is positive and significant. Since this spread correlates closely with the bid-ask spread and other liquidity measures (see Duffie et al. 2023), this finding suggests that customers buy bonds at auction when they can quickly sell them at high prices in the secondary market. Additionally, some coefficients, detailed in the appendix, indicate that fewer customers participate when there is increased uncertainty about interest rates. The issuance amount (supply) is also a significant positive

[^7]Figure 2: Irregular hedge fund participation

(A) Number of hedge funds per auction

(B) Variance in participation

Figure 2A shows the number of participating hedge funds in all auctions. The first box plot of Figure 2B, called dealers, shows the distribution of the variance in the percentage of dealers bidding across auctions in a year out of all active dealers in that year. The second box plot, called hedge funds, shows the analogue for hedge funds. The distribution is similar when including all customers.
factor, although the estimate is noisy. The $R^{2}$ ranges between $37 \%$ and $41 \%$, highlighting the influence of unobservable factors on customer participation - a feature that our model will incorporate.

Dealer updating. Our auction model, presented in Section 4, builds on Hortaçsu and Kastl (2012), who model the bidding process of Canadian Treasury auctions, in which customers must bid via dealers. To provide evidence that dealer updating plays a role in our data, we would like to regress the "change in a dealer's bid conditional on observing a customer's bid" on "the customer's bid." Given that bids are step functions, this is non-trivial.

We compute the change in the dealer's bid as the difference between the average quantityweighted bid of the dealer's update, having observed a customer's bid, minus the average of the dealer's bid, excluding the update. If no update is placed, we use the last bid before observing the customer bid, since by refusing to update, the dealer decided that that bid was still optimal. We regress this variable on three moments of the customer's bidding function to get a sense of what aspects of the step function affect updating the most: the quantityweighted average bid (that is, the price a bidder is willing to pay per unit of the bond), the number of steps, or the highest amount demanded. We normalize the customer's quantityweighted average bid by the average of all customer bids within an auction to capture the idea that dealers update their bid when the customer's bid they observe differs from what

Table 2: Dealer updating

| Change in qw-bid of dealer |  |  |
| :--- | :---: | :---: |
| Customer's qw-bid - average customer qw-bid | $+0.047^{* * *}$ | $(0.009)$ |
| Customer's number of steps | 0.032 | $(0.029)$ |
| Customer's total demand | +0.001 | $(0.001)$ |
| Auction fixed effects | Yes |  |
| Observations | 8193 |  |
| Adjusted $R^{2}$ | 0.21 |  |

Table 2 shows the results from regressing the change in a dealer's quantity-weighted average (qw) bid, calculated as the difference between the qw bid of their next submitted bid and the average qw bid of all other steps that they submit in the auction on the quantity-weighted average bid of the customer less the average customer qw-bid in that auction (which reflects how aggressive a particular customer's bid is relative to the average customer at that auction), the number of steps, and the customer's total demand. The regression also includes auction fixed effects. We use bidding data from all bond auctions from 1999 to 2022. Bids are in bps, quantities are in C $\$$ millions. Standard errors are in parentheses.
they expected. In rare cases in which the dealer's update follows multiple customer bids, we average over them.

We report in Table 2 that a dealer bids more aggressively when observing a more aggressive customer bid. Of the moments in the customer's step function that we consider, only the quantity-weighted average is statistically significant - a detail that we exploit in our estimation presented in Section 5.

## 4 Model of the Canadian primary market

Motivated by the empirical evidence, we construct a model with two main features, which are both novel relative to the existing literature. First, we endogenize the bidders' participation decision. Here, we distinguish between the dealer's decision to exit the market at an annual frequency and the customer's decision to enter specific auctions. This allows us to highlight the benefit of greater competition versus the cost of higher market volatility when an increasing share of bidders participates irregularly. Second, we allow customers to be sophisticated in that they can anticipate that dealers might update their bids when observing a customer bid. We show in Figure 4 as well as Appendix B. 3 that this feature is empirically relevant in that the model estimates are biased if we assume that customers do not anticipate dealer updating.

### 4.1 Players, timing, and preferences

Our model describes bidding decisions over the course of a fiscal year. We highlight four assumptions in particular because they directly relate to the model primitives that we later estimate. We denote random variables in bold.

There are two groups ( $g$ ) of market participants: dealers $(d)$ and customers ( $h$ ). The number of dealers who consider remaining as dealers, $\bar{N}^{d}>1$, and the number customers who are interested in bidding, $\bar{N}^{h} \geq 0$, are commonly known. ${ }^{11}$ Similarly, all distributions and functional forms are commonly known.

At the beginning of the year, the debt issuance plan is announced, stating $T$ auctions will be held. At this point, dealers decide whether they wish to continue being a dealer, that is, whether they commit to participating in all auctions in the upcoming year. Whether this is profitable depends on the private annual cost each dealer faces, $\gamma_{i}^{d}$. One may think of this as an opportunity cost that an institution that acts as a dealer suffers because it cannot do other things during the time it fulfills its dealer activities, such as bidding at auction and making markets.

Assumption 1. At the beginning of the year, dealers' private entry costs for all $T$ auctions of the year, $\gamma_{\boldsymbol{i}}^{\boldsymbol{d}}$, are drawn independently from a common atomless distribution, $G^{d}$.

After each dealer makes their decision, the market is informed about the number of bidders who will act as dealers in the upcoming year, $N^{d}$. In reality, this information is posted on the website of the auctioneer. The game ends if no dealer enters.

Before each auction, $t$, customers observe how costly it is for them to enter the auction. Similar to dealers, one may think of these entry costs as opportunity costs, since it takes time to monitor the market and compute competitive bids. ${ }^{12}$

Assumption 2. Before each auction t, customers' entry costs for auction $t$, $\boldsymbol{\gamma}_{\boldsymbol{t i}}^{\boldsymbol{h}}$, are drawn independently from a common atomless distribution, $G^{h}$.

[^8]In addition, customers observe the distribution from which they will draw their (multidimensional) private signals, $s_{t i}^{h}$,-which affects their willingness to pay-if they choose to bid at auction. The signal captures institution-specific knowledge, including information about the balance sheet or outstanding client orders, on auction day. The signal distribution is specified in Assumption 3. It captures current market conditions, which can be unobserved by the econometrician. For example, the expected willingness to pay may be high when secondary spreads are high (as suggested by the evidence in Appendix Table A2), or when interest rates are expected to fall.

Observing the signal distribution, each customer decides whether to enter an auction before learning their private signal. This timing assumption is standard in the literature on endogenous bidder participation in single-object auctions and reflects the idea that most customers are part of large institutions that tend to allocate tasks (such as bidding at auction) some time in advance, before the bidding process starts. An alternative would be to assume that customers first observe private signals and then enter; that is, allow for selective entry. In this case, customers with lower signals would enter an auction after a dealer exits, as fewer bidders make the auction less competitive, lowering the entry threshold. Consequently, the average customer value in auctions after a dealer exits should be lower than before. While we cannot perfectly test this implication, we can provide some supporting evidence by comparing the average customer bid (relative to dealer bids) around the dealer exits. We find no evidence of a change in relative bids; the same is true when comparing value estimates.

Assumption 3. Dealers' and customers' private signals $\boldsymbol{s}_{\boldsymbol{t i}}^{\boldsymbol{d}}$ and $\boldsymbol{s}_{\boldsymbol{t i}}^{\boldsymbol{h}}$ are, for all bidders, $i$, independently drawn from common atomless distribution functions $F_{t}^{d}$ and $F_{t}^{h}$ with support $[0,1]$ and strictly positive densities $f_{t}^{d}$ and $f_{t}^{h}$.

Within an auction, a bidder's signal must be private and independent from all other signals, conditional on everything that bidders know when bidding, which includes a reference price range provided by the auctioneer. To support the assumption that signals are private, we follow Hortaçsu and Kastl (2012) and test whether dealers who observe customer's bids only learn about the degree of competition in the auction (and not about the fundamental value of the bond). Our findings, reported in Appendix Table A5, support the assumption of private values.

Assumptions 1-3 rule out that bidders have an incentive to adopt strategies that connect multiple auctions. Thus, technically, our game consists of dealers' exit decisions plus $T$ exante identical and separate auction sub-games. We think this is reasonable in our setting, given that the typical dealer sells most of their auction-purchased bonds before the next auction takes place and there is no evidence that dealers are willing to pay more in subsequent auctions when they were allocated fewer bonds than expected. Further, customer participation in two subsequent auctions is uncorrelated (once we control for the upward time trend in customer participation, in Appendix Table A3). In other settings, it can be important to take inter-temporal strategic considerations into account (e.g., Rüdiger et al. 2023). ${ }^{13}$

How bidders bid in a specific auction, $t$, depends on how much they value the bond at that time. This, in turn, is driven by their signals.

Assumption 4. A bidder, $i$, of group $g \in\{d, h\}$ with signal $s_{t i}^{g}$ values amount $q$ of the bond by $v_{t}^{g}\left(q, s_{t i}^{g}\right)$. This value function is non-negative, measurable, bounded strictly increasing in $s_{t i}^{g}$ for all $q$, and weakly decreasing in $q$ for all $s_{t i}^{g}$.

Given these values, bidders place bids. Each bid is a step function that characterizes the price the bidder would like to pay for each amount offered at auction. Specifically, bidder $i$ has the following action set to place a bid in auction $t$ :

$$
A_{t i}=\left\{\begin{array}{l}
(b, q, K): \operatorname{dim}(b)=\operatorname{dim}(q)=K \in\{1, \ldots, \bar{K}\}  \tag{1}\\
b_{k} \in[0, \infty) \text { and } q_{k} \in\left[0, \bar{Q}_{t}\right] \\
b_{k}>b_{k+1} \text { and } q_{k}<q_{k+1} \forall k<K
\end{array}\right.
$$

where $\bar{Q}_{t}$ is the (maximal) auction supply, which is explained in more detail below.
Following Hortaçsu and Kastl (2012), bidding evolves in three rounds. First, dealers can place early bids directly with the auctioneer. Second, each participating customer is

[^9]randomly matched to a dealer and places their bid with this dealer. ${ }^{14}$ Third, each dealer observes these customer bids (if any) and may update their own bids.

To rationalize early bidding (which we observe in the data), we let one dimension of each dealer's signal, $s_{t i}^{d}$, be a random variable, $\Psi_{t i} \in[0,1]$, which is the mean of another Bernoulli random variable, $\boldsymbol{\Phi}_{\boldsymbol{t} \boldsymbol{i}}$, that determines whether the dealer's later bid will be made before the auction closes. The idea is that, when observing a customer's bid shortly before the auction, the dealer might not have sufficient time to recompute their bid and enter it into the bidding interface. Whether there is sufficient time or not is revealed only in the last stage. Formally, in the last stage, the dealer observes the realization $\omega_{t i} \in\{0,1\}$ of $\boldsymbol{\Phi}_{\boldsymbol{t} \boldsymbol{i}}$, where $\omega_{t i}=1$ means that the late bid will make it on time, in addition to their signal, $s_{t i}^{d}$, and information that includes the bid(s) of the customer(s) that was (were) matched to this dealer or the fact that no customer bid has arrived.

A pure bidding strategy is a mapping from the information set of a bidder to the action space at each stage of the game. To capture everything that a bidder knows, we introduce a bidder type, labeled $\theta_{\tau i}^{g}$. This is the private signal of the bidder in the first and second stages of the game, and it includes information about the observed customer bid at the final stage. To highlight the symmetry across bidder groups when defining strategies, we use subscript $\tau$, which summarizes the auction and bidding stages within a bidder group. For a dealer, $\tau=t 1$ in the first stage and $\tau=t 2$ in the third stage in auction $t$, while for a customer, who only moves in stage 2 of the game, $\tau=t$. With this, bidding strategies can be represented by bidding functions, labeled $b_{\tau i}^{g}\left(\cdot, \theta_{\tau i}^{g}\right)$, for bidder $i$ of group $g$ with type $\theta_{\tau i}^{g}$ at time $\tau$.

When choosing the bidding function, a customer anticipates that their dealer can update their own bid. This differs from Hortaçsu and Kastl (2012), who focus on dealers. Since the customer does not know the dealer's type, $\theta_{\tau i}^{d}$, they do not know whether and how the dealer will update their bid. As a result, a customer cannot be sure if the market-clearing price will increase or decrease when they marginally increase their own demand at step $k$-anything

[^10]can happen. ${ }^{15}$ This makes it complex for the customer to determine their optimal bid.
To render the customers' optimization problem solvable, we assume that dealers only pay attention to finite sets of $L_{t}$ moments of the customers' bidding function when updating their own bid-motivated by the empirical evidence presented in Table 2. ${ }^{16}$ Formally, a moment is a mapping $\mu_{t}^{l}$ that transforms the bidding function, $b_{\tau i}^{h}\left(\cdot, \theta_{\tau i}^{h}\right)$ for type $\theta_{\tau i}^{h}$, into a real number $\mathbb{R}$. We restrict our attention to moments that are differentiable with respect to the quantity at each price. This includes, for example, the intercept with the price or the quantity axis, some smooth approximation of the slope, or the quantity-weighted bid, which is defined as follows:
\[

$$
\begin{equation*}
\mu_{t}^{l}\left(b_{\tau i}^{h}\left(\cdot, \theta_{\tau i}^{h}\right)\right)=\frac{b_{1} q_{1}+\sum_{k=2}^{K} b_{k}\left(q_{k}-q_{k-1}\right)}{q_{K}} \tag{2}
\end{equation*}
$$

\]

where $\left\{b_{k}, q_{k}\right\}_{k=1}^{K}$ constitutes bidding function $b_{\tau i}^{h}\left(\cdot, \theta_{\tau i}^{h}\right)$.
Once all bidders submit their step function, the market clears at the lowest price, $P_{t}^{*}$, at which the aggregated submitted demand satisfies the total supply. The supply, $\boldsymbol{Q}_{\boldsymbol{t}}$, is unknown to the bidders when they place their bids because a significant fraction of the total allotment goes to the Bank of Canada, which is the largest non-competitive bidder. $\boldsymbol{Q}_{\boldsymbol{t}}$ is distributed according to an auction-specific distribution on $\left[0, \bar{Q}_{t}\right]$ with strictly positive marginal density conditional on $s_{t i}^{g} \forall i, g=h, d$.

Given all bidding functions, $b_{\tau i}^{g}\left(\cdot, \theta_{\tau i}^{g}\right)$, bidder $i$ of group $g$ wins amount $q_{t i}^{g *}$ at market clearing. They pay the amount they offered to win for each unit won. In case there is excess demand at the market-clearing price, each bidder is rationed pro-rata on the margin.

### 4.2 Equilibrium conditions

We first characterize the equilibrium in auction $t$, conditional on customer and dealer participation. Then, we determine the entry and exit decisions of customers and dealers, respectively.

[^11]Bidding. To find the optimal bidding strategy, a dealer maximizes their expected total surplus, taking the behavior of other bidders as given. For bidder $i$ in group $g$ of type $\theta_{\tau i}^{g}$ the expected total surplus at time $\tau$ is

$$
\begin{equation*}
T S_{\tau i}^{g}=\mathbb{E}\left[\int_{0}^{q_{t i}^{g *}}\left[v_{t}^{g}\left(x, s_{t i}^{g}\right)-b_{\tau i}^{g}\left(x, \theta_{\tau i}^{g}\right)\right] d x\right] \tag{3}
\end{equation*}
$$

Note that for customers who bid a single time, $T S_{\tau i}^{h}$ is equivalent to $T S_{t i}^{h}$. For dealers, we have two surpluses: $T S_{t 1 i}^{g}$ and $T S_{t 2 i}^{g}$, one for each bidding round. In all cases, the expectation is taken over the amount the bidder will win at market clearing, $\boldsymbol{q}_{\boldsymbol{t i}}^{\boldsymbol{g} \boldsymbol{*}}$, which depends on all bidders' strategies and types as well as the unknown supply.

We focus on the group-symmetric Bayesian Nash equilibrium (BNE) in which all dealers and customers play the same bidding strategy if they are the same type. Formally, a pure strategy BNE of auction $t$ is a collection of bidding functions, $b_{\tau i}^{g}\left(\cdot, \theta_{\tau i}^{g}\right)$, such that each bidder, $i$, and almost every type, $\theta_{\tau i}^{g}$, maximizes their expected total surplus (3) each time, $\tau .{ }^{17}$ Going forward, we use the $\tau$ subscript for dealers and $t$ for customers, to stress that customers only bid one time while dealers may update their bids.

Proposition 1. Fix a set of $L_{t}$ moment functions that map a customer's bid function into a real number, $\mu_{t}^{l}: b_{\tau i}^{h}\left(\cdot, \theta_{\tau i}^{h}\right) \rightarrow \mathbb{R}$, and consider a group-symmetric BNE. Let $N^{d}>1, N^{h}>1$.
( $i$ ) Every step $k$ but the last step in the dealer's bid function, $b_{\tau i}^{d}\left(\cdot, \theta_{\tau i}^{d}\right)$, has to satisfy

$$
\begin{equation*}
\operatorname{Pr}\left(b_{k}>\boldsymbol{P}_{\boldsymbol{t}}^{*}>b_{k+1} \mid \theta_{\tau i}^{d}\right)\left[v_{t}^{d}\left(q_{k}, s_{t i}^{d}\right)-b_{k}\right]=\operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}_{\boldsymbol{t}}^{*} \mid \theta_{\tau i}^{d}\right)\left(b_{k}-b_{k+1}\right) . \tag{4}
\end{equation*}
$$

At the last step, $b_{K}=v_{t}^{d}\left(\bar{q}\left(\theta_{\tau i}^{d}\right), s_{t i}^{d}\right)$, where $\bar{q}\left(\theta_{\tau i}^{d}\right)$ is the maximal amount the dealer may be allocated in the auction equilibrium.
(ii) Every step $k$ in a customer's bid function, $b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)$, that generates moments $m_{t i}^{l}=$ $\mu_{t}^{l}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)$ for all l has to satisfy:

[^12]\[

$$
\begin{align*}
& \operatorname{Pr}\left(b_{k}>P_{\boldsymbol{t}}^{*}>b_{k+1} \mid \theta_{t i}^{h}\right)\left[v_{t}^{h}\left(q_{k}, s_{t i}^{h}\right)-b_{k}\right]= \\
& \operatorname{Pr}\left(b_{k+1} \geq P_{\boldsymbol{t}}^{*} \mid \theta_{t i}^{h}\right)\left(b_{k}-b_{k+1}\right)-\sum_{l=1}^{L_{t}} \lambda_{t i}^{l} \frac{\partial \mu_{t}^{l}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)}{\partial q_{k}}+\operatorname{Ties}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right), \tag{5}
\end{align*}
$$
\]

with $\operatorname{Ties}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)=\operatorname{Pr}\left(b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.v_{t}^{h}\left(\boldsymbol{q}_{\boldsymbol{t i}}^{\boldsymbol{h} \boldsymbol{*}}, s_{t i}^{h}\right) \frac{\left.\partial \boldsymbol{q}_{t q_{k}}^{\boldsymbol{h}}\right)}{q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]-\operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.v_{t}^{h}\left(\boldsymbol{q}_{t i}^{\boldsymbol{h} *}, s_{t i}^{h}\right) \frac{\partial \boldsymbol{q}_{t i t}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k+1} \geq\right.$ $\left.\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{t i}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{t i *}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k+1}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k+1}<\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{t i}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k+1}<\right.$ $\left.\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]$. Here we omitted the dependence on $\theta_{\tau i}^{h}$. In addition,

$$
\begin{equation*}
\lambda_{t i}^{l}\left[m_{t i}^{l}-\mu_{t}^{l}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)\right]=0 \text { for all } l, \tag{6}
\end{equation*}
$$

with Lagrange multipliers $\lambda_{t i}^{l} \in \mathbb{R}$ for all $l$.
(iii) The moments, $\left\{m_{t i}^{l}\right\}_{l=1}^{L}$, are such that the expected total surplus (3) is maximized and $m_{t i}^{l}=\mu^{l}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)$ for all $l$ and all customers.

If $N^{h}=0$, conditions (ii) and (iii) do not apply.
From the existing literature, we know how the dealer determines their equilibrium bid. Essentially, in each stage they choose their bid to maximize the total expected surplus, $T S_{\tau i}^{d}$, defined in (3), subject to market clearing:

$$
\begin{equation*}
\max _{\left\{q_{k}, b_{k}\right\}_{k=1}^{K}} T S_{\tau i}^{d} \text { subject to market clearing. } \tag{7}
\end{equation*}
$$

They trade off the expected surplus on the marginal infinitesimal unit versus the probability of winning it, summarized in Proposition 1 (i) (see Kastl 2017, p. 237, for more details). Given that it is never optimal for a dealer to submit a bid above their true value, dealer demand is never rationed in equilibrium, except for the last step. At the last step, the dealer submits their true value because it is not possible to increase the winning probability of (non-existing) subsequent steps by shading the bid.

Our innovation is to characterize the equilibrium bidding of a customer. The key difference between the dealer and the customer comes from the fact that the customers take into account the dealer's response to observing their bid. For illustration, assume for now that no dealer observes two customer bids. Since dealers only change their own bids in response to changes in the $L_{t}$ moments of the observed customer bidding function, $\left\{m_{t i}^{l}\right\}_{l=1}^{L}$, the customer's optimality conditions can be decomposed into two parts. This helps to draw the connection to the dealer's equilibrium condition. First, for each fixed set of moments,
$\left\{m_{t i}^{l}\right\}_{l=1}^{L}$, the customer's equilibrium bidding function must achieve the highest expected surplus among all of the functions that induce the same dealer updating; that is, $\left\{m_{t i}^{l}\right\}_{l=1}^{L}$ must be such that:

$$
\begin{equation*}
\max _{\left\{q_{k}, b_{k}\right\}_{k=1}^{K}} T S_{t i}^{h} \text { subject to market clearing and } m_{t i}^{l}=\mu_{t}^{l}\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right) \text { for all } l \in L_{t} . \tag{8}
\end{equation*}
$$

The corresponding optimality conditions are summarized in Proposition 1 (ii). Second, among those partially optimal functions, the customer chooses the optimal one by choosing moments $\left\{m_{t i}^{l}\right\}_{l=1}^{L}$ so that the expected total surplus, $T S_{t i}^{h}$, is maximized—Proposition 1 (iii). Both conditions generalize to the case where dealers observe more than one customer. The only difference is that the customer forms a different expectation over where the market will clear, taking into account the possibility that the dealer might observe some other customer's bid.

Dealer updating implies that it may be optimal for a customer to place a bid above their true value. As a consequence, we cannot rule out that ties may occur at any step, with positive probability. Formally, in Proposition 1 (ii) the term Ties $\left(b_{t i}^{h}\left(\cdot, \theta_{t i}^{h}\right)\right)$ includes all of the cases in which the customer ties with some other bid and the bid must be rationed.

To illustrate why it might be optimal for customers to bid above their value and tie at non-final steps with positive probability, assume customer $i$ places their bids via dealer $j$. They could submit a step function with $b_{k} \leq v_{k}$ for all $k$. Alternatively, they could deviate and bid above their value, for example, at the fourth step, $b_{4}>v_{4}$, as depicted in the solid and dotted lines, respectively in Figure 3A. This would increase their quantity-weighted bid relative to bidding below their value at all steps; that is, it would modify one moment, $m_{t}^{1}$. Observing a higher quantity-weighted average of the customer's bid, dealer $j$ updates their own bid towards a more aggressive bid- going from the solid line, when seeing the original customer bid, to the dotted line, Figure 3B. The dealer's new bid is more aggressive on most but not all units. Since they have a small number of steps to rearrange, the more aggressive bid ends up crossing their (on average) less aggressive bid for a small set of quantities. The dealer might have been willing to bid higher on those units but they cannot introduce another step to do so.

How the dealer updates their bid depends on their private beliefs about where the market will clear, which is something the customer does not know. Therefore, the customer cannot
perfectly predict how the dealer will update. However, they can predict the dealer's update probabilistically and construct the distribution of the residual supply curves against which they play (which includes the dealer's update), to choose the bidding function that maximizes their expected auction surplus.

Figures 3C-3D display one realization from the distribution of the residual supply curves and the implied point of market clearing given the customer's bid for both scenarios. Given this realization, it is profitable for the customer to bid above their value: Rather than clearing at the second step of the customer's bidding function, the auction now clears at the third step, which implies that the customer wins more units at prices below their values when deviating to $b_{4}>v_{4}$. Given another realization of the residual supply curve, the customer could tie at the new, more aggressive fourth step. In this case, the market-clearing price would increase relative to their original bid. At this step, they would be happy to be rationed and win fewer units at prices above value.

Entry and exit decisions. A customer, $i$, enters an auction, $t$, if their entry cost, $\gamma_{t i}^{h}$, is smaller than the total surplus they expect to earn from participating in the auction before they observe their private signal but they know the current market conditions, which are captured by the signal distributions, $F_{t}^{g}$, and the shape of the value functions, $v_{t}^{g}(\cdot, \cdot)$, for $g=\{h, d\}$.

Proposition 2. Customer $i$ with entry cost $\gamma_{t i}^{h}$ enters auction $t$ if

$$
\begin{equation*}
\gamma_{t i}^{h} \leq \mathbb{E}\left[\boldsymbol{T} \boldsymbol{S}_{\boldsymbol{t i}}^{h} \mid N^{d}\right] \text { with } T S_{t i}^{h} \text { given by (3). } \tag{9}
\end{equation*}
$$

Note that, relative to expression (3), we relabeled the time subscript $\tau$ to $t$ to highlight that $T S_{t i}^{h}$ is the surplus that customer $i$ expects from participating in auction $t$. The expectation is taken over the customer's private signal, $s_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}}$, and is conditional on $N^{d}$ dealers bidding in the auction.

Anticipating all auctions, $t=1, \ldots, T$, of the upcoming year, dealer $i$ exits the market if their entry cost is higher than the surplus they expect to earn from bidding in all $T$ auctions of the upcoming year.

Proposition 3. At the beginning of the year, dealer $i$ with entry cost $\gamma_{i}^{d}$ exits the market if

Figure 3: Bidding example

(A) Initial and new customer bid

(C) Initial market clearing

(B) Initial and updated dealer bid

(D) New market clearing

Figure 3 provides an example for why it can be optimal for a customer to deviate from a bidding function with $b_{k} \leq v_{k}$ (shown in solid lines) and bid above their value, here, at step $b_{4}>v_{4}$ (the dashed lines in panel A). This is true even though this triggers a more aggressive dealer bid, as shown in the dashed line of panel B. Panels C and D display one realization of market clearing for both customer bids; the downward-sloping step function is the customer's bid and the upwardsloping function is one realization of the residual supply curve that the customer faces.

$$
\begin{equation*}
\gamma_{i}^{d} \geq \sum_{N^{d}=1}^{\bar{N}^{d}}\left(\sum_{t=1}^{T} \mathbb{E}\left[\boldsymbol{T} \boldsymbol{S}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{d}} \mid N^{d}\right]\right) \operatorname{Pr}\left(\boldsymbol{N}^{\boldsymbol{d}}=N^{d}\right) \tag{10}
\end{equation*}
$$

where $T S_{t i}^{d}=\left[1-\Psi_{t i}+\Psi_{t i} \operatorname{Pr}(\right.$ no customer $\left.)\right] T S_{t 1 i}^{d}+\Psi_{t i} \operatorname{Pr}($ at least 1 customer $) T S_{t 2 i}^{d}$ with $T S_{t 1 i}^{d}$ and $T S_{t 2 i}^{d}$ given by (3) for $\tau=t 1, t 2$.

The dealer cares about the aggregate surplus they expect to earn over the entire year and, when making their decision, they do not know how many other dealers will compete in the auctions, $N^{d}$. Therefore, the dealer considers all possible realizations of $N^{d}$ and weights each by how likely it is to occur, $\operatorname{Pr}\left(N^{d}=N^{d}\right)$. Furthermore, for each auction, $t$, the dealer must take an expectation over bidding rounds, $\tau$, to determine how much surplus they expect to earn in the auction, $T S_{t i}^{d}$. This is because ex-ante the dealer does not know whether they
will place a final bid and earn a surplus of $T S_{t 2 i}$, or just an early bid which leaves them with a surplus of $T S_{t 1 i}$. This depends on the probability that the final bid will make it on time $\left(\Psi_{t i}\right)$ and the probability that at least one customer is matched to the dealer.

## 5 Identification and estimation

The goal is to learn about the unobserved bidder values, $v_{t}^{g}\left(\cdot, s_{t i}^{g}\right)$, and the entry and exit cost distributions, $G^{h}$ and $G^{d}$.

Identifying and estimating values. The idea behind identifying and estimating bidder values from bidding data is to infer how much each bidder is truly willing to pay, from the equilibrium conditions, under the assumption that everyone plays this equilibrium, here specified in Proposition 1. Other than our object of interest-the bidder values-we either observe all of the elements in Proposition 1 or can estimate them.

For dealers, it suffices to estimate the probabilities of where the market will clear to point identify the dealer values at all of the submitted steps, $q_{k}$, (Kastl, 2012). We estimate these probabilities by extending the resampling procedure in Hortaçsu and Kastl (2012) to account for the fact that in the data bidders sometimes update their bids more often than predicted by our model (see details in Appendix C). We account for differences across auctions (including differences that arise from different market conditions, such as secondary spreads), by resampling within, rather than across, auctions. ${ }^{18}$ We then leverage the monotonicity of the marginal value function to construct an upper and a lower bound for the values at the intermediate quantities, $q \in\left(q_{k}, q_{k+1}\right)$, where steps are not submitted.

It is more difficult to learn about the customer values because the customer's equilibrium condition (5) involves ties, which implies that this depends on all of the quantity points on the bidding curve and not only those at the submitted steps. This leaves us with $K$ equations but infinitely many unknowns, so that the customer values cannot be point identified.

We can, however, construct sets of informative bounds on the customer values that are consistent with the observed bids (see Proposition 4 in Appendix A). To recover $K$ upper and $K$ lower values for $v_{t}^{h}\left(q_{k}, s_{t i}^{h}\right)$ for customer $i$ in auction $t$, we assume that dealers only

[^13]pay attention to the quantity-weighted bid when updating their own bid-motivated by the empirical evidence in Table 2. Condition (5) then simplifies to an equation with a single moment, the quantity-weighted bid (2). For simplicity, we drop the superscript $l=1$.

Before describing the algorithm we use to identify the customer bounds, let us take a step back and ask what variation in the data identifies these bounds. As in the existing literature, the first-order conditions for a quantity change at each step submitted provide information about the marginal value of the bidder at that step. However, in our setting the customers' optimality condition (5) contains two additional terms, $\lambda \frac{\partial \mu}{\partial q}$, and a term that comes from ties. While $\frac{\partial \mu}{\partial q}$ is directly observed in the bidding data, $\lambda$ is an additional free parameter. To pin it down, we use the additional optimality restriction of Proposition 1, according to which the chosen moment must be optimal, together with data on how the dealer changes their own bid when observing a higher or lower quantity-weighted customer bid. This additional information is not leveraged to estimate values in the standard approach (without dealer updating); it is similar to the variation used to test the model in Hortaçsu and Kastl (2012). For details on identification, consult the proof of Proposition 4.

We proceed in three steps to back out customer bounds. First, we guess a Lagrange multiplier, $\lambda_{t i} \in \mathbb{R}$, and replace the system of equations from Proposition 1 (ii) with a system that eliminates the infinitely many unknown values due to rationing. To do this, we utilize boundedness and monotonicity to replace the values at quantities where a step is not submitted, with a bound. For example, the upper bound on the value at quantity $q \in\left(q_{k}, q_{k+1}\right)$ is $\bar{v}_{t}^{h}\left(q_{k}, s_{t i}^{h}\right)$ and the lower bound is $\underline{v}_{t}\left(q_{k+1}, s_{t i}^{h}\right)$. This results in a system of $2 K$ equalities, which are linear in the unobserved values, with $2 K$ unknown.

Second, we simplify this system of equations by showing that, at a subset of steps, rationing never occurs in equilibrium, which implies that we can cancel out all of the terms involving rationing at these steps. We do this by constructing profitable deviations at these steps in Lemma 1 in Appendix A. With these simplifications, it is straightforward to express the system of equations in matrix format and show that the matrix has full rank, which proves that the system is identified.

To provide an intuition for how Lemma 1 works, consider a customer with $\lambda_{t i}>0$. If the dealer does not update their bid $\left(\lambda_{t i}=0\right)$, this customer will submit a lower quantityweighted bid. With dealer updating, the customer cannot reach this optimum but they
can move as closely as possible to it. To do this, the customer starts at the unconstrained optimum and inflates the quantity-weighted bid in the cheapest way possible, meaning that the costs coming from dealer updating are minimized. At steps other than the last step, this may involve rationing, that is, demanding larger quantities than in the unconstrained bid so as to inflate the quantity-weighted bid without winning all of the extra units. However, at the last step, demanding a larger amount causes the quantity-weighted bid to fall and, therefore, the customer does not find it optimal to tie at the last step. Terms involving rationing at the last step are therefore zero. A similar logic applies when $\lambda_{t i}<0$. Now, if the dealer does not update, the customer will want to submit a higher quantity-weighted bid. Ties may now only occur at the last step, so that the terms involving rationing drop out at all earlier steps.

Third, we check that the guessed Lagrange multiplier, $\lambda_{t i}$, is valid in equilibrium, by using Proposition 1 (iii). To do this we rely on the fact that we observe the distribution of dealer-bid updates following any customer bid. This allows us to calculate what residual supply curves a customer would have faced had they chosen any other quantity-weighted bid. With this, and the implied marginal values from $\lambda_{t i}$, we can compute bounds on the expected surplus that the customer could have achieved at any alternative quantity-weighted bid, given $\lambda_{t i}$. We then check whether the guessed $\lambda_{t i}$ is consistent with the changes in the surplus between the submitted quantity-weighted bid and the alternative quantity-weighted bids. For example, if the guessed $\lambda_{t i}$ suggests that the customer is forgoing profitable changes in their bid in order to reduce their quantity-weighted bid, then the residual supply curves that they expect to face after submitting a larger quantity-weighted bid must imply large enough losses in their surplus. However, these losses should not be so big that the customer would rather choose a bid function that resulted in an even lower quantity-weighted bid.

Identifying and estimating cost distributions. To learn about the cost distributions, $G^{h}$ and $G^{d}$, we rely on Propositions 2 and 3 , respectively. For this, we fix the maximal number of dealers to what we observe in our sample, that is, $\bar{N}^{d}=24$. We define the number of potential customers in a year, $\bar{N}^{h}$, as the maximal number of customers we observe bidding in any auction that year.

We first compute bounds on how much the customers and dealers expect to gain from
participating in the game, that is, the RHS of (10) and (9), respectively. For the customers, this is straightforward. We can compute each customer's auction surplus (3) at the upper and lower bounds of the customer's values, $\overline{T S}_{t i}^{h}$, and $\underline{T S_{t i}^{h}}$, conditional on auction participation. For example, $\overline{T S}_{t i}^{h}$ is the area between the upper bound on the customer's value function and the submitted bid function, where each quantity is weighted by the probability that it is won at market clearance. We then average $\overline{T S}_{t i}^{h}$ across all participating customers in an auction to obtain the expected surplus prior to auction entry, $\mathbb{E}\left[\overline{T S}{ }_{t i}^{h} \mid N^{d}\right]$, and similarly for the lower bound. Here, we rely on the assumption that the customers' signals are drawn iid from the same distribution.

For the dealers, it is more difficult to construct bounds on the expected annual auction surplus, given in (9), since dealers make their entry decisions before knowing how many other dealers will be participating in the upcoming year. This implies that we need to compute two additional objects, besides the expected total surplus, given the observed number of dealers (which is constructed as for customers). First, when there is a different number of dealers than the observed participants in the market we need to compute the expected total surpluses for all of the potential numbers. Second, we need to compute how likely it is that $N^{d}$ dealers will participate from the empirical probability that a dealer participates in a fixed year. ${ }^{19}$

We could exactly compute the counterfactual surpluses at the non-observed numbers of participating dealers, following a similar approach presented in our counterfactual exercises. However, this is computationally intensive. Therefore, we approximate these surpluses by finding auctions that are similar (in that all bidders expect similar per-unit auction surpluses), but with different numbers of participating dealers. For example, to obtain the counterfactual surplus for an auction in which $N^{d}$ dealers participate, we find a similar auction in which $N_{1}^{d}$ dealers participate. For that similar auction, we compute the expected surplus for dealers: $\mathbb{E}\left[\overline{T S}_{\boldsymbol{t i}}^{\boldsymbol{d}} \mid N_{1}^{d}\right]$. Repeating this exercise for other similar auctions with different numbers of participating dealers provides an estimate for each counterfactual surplus.

With the bounds on how much dealers and customers expect to gain from auction participation, we identify the bounds on their cost distributions by matching the predicted participation probability of a customer and a dealer (according to Propositions 2 and 3) to

[^14]what we observe in the data. For example, the predicted expected number of customers entering an auction, $\bar{N}^{h} \operatorname{Pr}\left(\gamma_{\boldsymbol{t i}}^{\boldsymbol{h}} \leq \mathbb{E}\left[\boldsymbol{T} \boldsymbol{S}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]\right)$, must equal the observed expected number. Relying on the fact that
\[

$$
\begin{equation*}
\bar{N}^{h} \operatorname{Pr}\left(\gamma_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \leq \mathbb{E}\left[\underline{\boldsymbol{T}}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]\right) \leq \mathbb{E}\left[\boldsymbol{N}_{\boldsymbol{t}}^{\boldsymbol{h}}\right] \leq \bar{N}^{h} \operatorname{Pr}\left(\gamma_{\boldsymbol{t i}}^{\boldsymbol{h}} \leq \mathbb{E}\left[\overline{\boldsymbol{T S}}_{\boldsymbol{t i}}^{\boldsymbol{h}} \mid N^{d}\right]\right) \tag{11}
\end{equation*}
$$

\]

we could identify a lower and upper bound for the customer's cost distribution nonparametrically as long as we can construct sufficiently many different surpluses, $\mathbb{E}\left[\overline{T S}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]$ ) and $\mathbb{E}\left[\underline{\boldsymbol{T}}_{\boldsymbol{t i}}^{\boldsymbol{h}} \mid N^{d}\right]$ ), to cover the full support of the cost distribution. With our data, we impose an exponential distribution with parameter $\beta^{h}$ for customers and parameter $\beta^{d}$ for dealers. ${ }^{20}$

## 6 Estimated values and costs

For each auction, $t$, we estimate the dealer values, $\hat{v}_{t}^{d}\left(q_{k}, s_{t i}^{d}\right)$, which we call $\hat{v}_{t i k}$, and the bounds for the customer values, $\underline{v}_{t i k}$ and $\bar{v}_{t i k}$, at each submitted quantity step, $k$, of final bids. In addition, we obtain the upper and lower bounds for the parameters of the exponential cost distributions of both bidder groups, $G^{h}$ and $G^{d}$.

Customer and dealer values. Before interpreting the economic magnitudes of our value estimates, we show in Figure 4 that it is quantitatively important to account for the fact that sophisticated customers, such as hedge funds, anticipate that a dealer might update their own bid after observing the customer's bid. An alternative assumption would be to neglect the customer bidding incentives that are triggered by dealer updating. In that case, we could simply follow the estimation procedure of the existing literature and back out the customer values in the same way we back out the dealer values. However, our results highlight that the customer values would be significantly biased if we followed this approach.

Independent of the assumption we make about the customer's degree of sophistication, customers are typically willing to pay more than dealers (see Figure 5, Appendix Figure 4).

[^15]In our benchmark where customers are fully sophisticated, we estimate that, in the median, customers are willing to pay between 4.6 and 7.3 bps more per unit of the bond than dealers. This difference is economically meaningful compared to the median market yield-to-maturity of the bonds, which is 161 bps , and is sizable compared to the median difference between the quantity-weighted average winning bid and the quantity-weighted average secondary market price on auction day (one day after the auction), which is 0.4 bps ( 2 bps ). Further, the difference is statistically significant at the $5 \%$ level (as shown in Appendix Table A5) and cannot be driven by customer selection into more-profitable auctions, given that we are considering value differences conditional on auction participation. This is true when using the lower and upper bounds of the customer values.

Over time, customer values have increased relative to dealer values, as shown in Figure 5B. ${ }^{21}$ Since these values reflect how much an auction participant expects to earn from trading bonds in the secondary market, this finding suggests that customers anticipate increasingly larger per-unit returns from buying bonds, relative to dealers. This is in line with anecdotal evidence according to which more-stringent regulations for dealers, since the financial crisis in 2007-2009, have decreased profit margins for dealers relative to customers. This finding is also in agreement with evidence documented by Sandhu and Vala (2023), who argue that hedge funds that entered the market after 2009 are able to obtain higher than average returns in the secondary market for Canadian government bonds because they have more flexibility to employ complex and risky trading strategies (Ontario Securities Commission, 2007).

Finally, we test whether our estimated distribution of customer values correlates with the observed market conditions that predict customer participation (shown in Appendix Table A2). This provides a validity check for our value estimates since we have not used any information of these market conditions in our estimation. Specifically, we regress the average quantity-weighted customer value per auction, in addition to other moments of the customer value distribution, such as the standard deviation, on the explanatory variables that we used to predict customer participation shown in Appendix Table (A2). Our findings, reported in Appendix Tables A6 and A7, confirm our prior that customer values are higher (and more disperse) when secondary market spreads are wide.
${ }^{21}$ This trend could come from the same customers becoming more profitable over time. Alternatively, if different customer types (e.g., hedge funds vs. pension funds) have systematically different value distributions, this could come from a change in the composition of customer types over time.

Figure 4: Customer bid shading under different bidding assumptions


Figure 4 shows the distribution of the customer's average shading factors in $\mathrm{C} \$$, defined as the customer's quantity-weighted average value minus the quantity-weighted average bid, under different assumptions regarding the customer's degree of sophistication. The box plot called "Sophisticated" shows the distribution of the lower bound quantity-weighted customer value estimates according to our model. 'Ties only' estimates the values under the assumption that customers bid according to Proposition 1 (ii) but set $\lambda_{t i}=0$. The 'Naive' box uses the dealers' optimality condition, i.e., forcing the customer to bid without accounting for the impact of the information their bid might have on the dealers' behavior.

Entry costs. The entry costs capture the opportunity cost of the profits that customers and dealers could generate outside the Treasury market. These costs differ from values in that they are independent of how much a bidder wins at auction; they must be paid when an institution spends time bidding at auction (and in the case of dealers, conducting other market- making activities), even if the institution does not buy any bonds.

On average, we estimate an annual cost of being a dealer of between $\mathrm{C} \$ 3.263$ and $\mathrm{C} \$ 4.117$ million, with a (bootstrapped) confidence interval of [C\$3.111M, C $\$ 4.296 \mathrm{M}]$. This is sizable compared to the average annual profit a bank generates from its market-making activities in all financial markets combined, which amounts to roughly $\mathrm{C} \$ 413$ million (Allen and Wittwer, 2023). To get a sense of whether our cost estimate is sensible, we collect information on the fees that a typical trading desk must pay to act as a primary dealer for a year, for example, to access data feeds and electronic platforms, such as Bloomberg. These fees sum to C $\$ 3.1$ million, which is surprisingly close to our estimate, even though we do not use the information on fees in our estimation.

The average entry cost of a customer is between $\mathrm{C} \$ 471,505$ and $\mathrm{C} \$ 492,605$ per auction,

Figure 5: Difference between customers and dealers' values


Figure 5A shows the distribution of the difference in the quantity-weighted average values between participating customers' and dealers, using customer lower bound values on the LHS and upper bound values on the RHS. We remove outliers, defined as those with a value that is more than three scaled median absolute deviations from the median. Figure 5B plots the point estimates and the $95 \%$ confidence intervals from regressing the difference between the lower bound of the average customer value and the lower bound of the average dealer value in each auction on a set of time indicator variables. The first indicator is 1 for 1999-2000, the second indicator is 1 for 2001-2002, and so forth, until 2021-2022. The confidence intervals are computed using the point estimates of the dealer and customer values at the lower bounds and, therefore, do not account for noise in the estimation of these values. Prices are in $\mathrm{C} \$$ with a face value of $\$ 100$.
with a (bootstrapped) confidence interval of [C\$401,204, C $\$ 562,905]$. Given that there are about 28 auctions per year, the customer's cost is larger than the dealer's cost. This is in line with the idea that customers are better at executing profitable trading strategies - here, outside of the Treasury market, driving up the opportunity entry cost.

Summarizing, our findings highlight systematic differences between dealers and customers, both in terms of their values and their entry costs.

## 7 Drivers and consequences of customer participation

With the estimated model, we conduct counterfactuals to understand why hedge funds entered the market and we evaluate the consequences for market functioning. For this, we assume that all model primitives, such as the distributions of values and costs, remain fixed when we change the market rules. We use final bids only and the lower bound estimates of the values and costs. Ideally, we would compute counterfactual outcomes for all auctions in
our sample; however, this is computationally intense. We therefore only consider every third auction since 2014, that is, the period when hedge funds became the dominating customer group.

Computing counterfactuals Conducting counterfactuals for multi-unit auctions in which bidders have multi-unit demand is challenging because it is analytically impossible to solve for an equilibrium. We proceed using a numerical solution approach (explained in more details in Appendix D.1).

We extend the empirical guess-and-verify approach of Richert (2021) to allow bidders to expand their auction-specific demands in response to changes in the auction environment. We limit the amount of expansion to the quantity that would result if the bidder submitted the greater of their largest ever bid and a bid for $10 \%$ of the supply. The bidder-specific limit accounts for the unobservable constraints that restrict the amount a bidder is able to buy, such as balance sheet constraints; the $10 \%$ bound approximates the minimal bidding requirement of dealers in normal times. ${ }^{22}$

Concretely, we solve for the counterfactual bid distributions such that two conditions are satisfied. First, the distribution of the values implied by these bids and optimal bidding must be indistinguishable from the distribution of the estimated values. Second, the counterfactual distribution of the maximal amount each bidder demands in an auction must be first-order stochastically dominated by the observed distribution of maximal demands. We obtain this distribution by first fixing each bidder and computing the maximum between the largest fraction of the bond supply this bidder bids on in any auction and $10 \%$ of this supply. We then take the empirical distribution of these maximal quantities across bidders. For robustness, we alternatively impose constraints such that when each dealer is removed, the coverage ratio (which is the ratio between the sum of all bids over total supply) falls by the same amount as the average decrease in the coverage when a dealer exits, as estimated using the three exits in 2014-2015.

Our approach might not fully capture the size of the demand in counterfactual auctions that are far from any observed auction. Moreover, it does not account for endogenous changes

[^16]in the value or cost distributions, given that we need to fix the estimated model primitives to compute counterfactual bids and entry decisions. For example, since bidders at least partially derive value from the money they make from reselling bonds, we might be concerned that their values change when their competitors win more at auction-since this impacts the degree of competition the bidder faces in the secondary market. Sufficiently small changes in the auction allocation, however, should result in similar degrees of competition in the secondary market. We therefore focus our discussion on local changes where our approach provides more-reliable predictions. To avoid focusing on any particular draw from the cost or value distribution, we take the ex ante perspective and compare the expected market outcomes. For instance, we analyze the expected price at which an auction clears under the current market rules, relative to counterfactual market rules.

Why did customers enter? We first aim to understand whether customers entered the market because of dealer exit or because of changing market conditions. For this we compare the status quo, in which two dealers left in 2014, with a counterfactual with two additional dealers (fixing dealer participation but allowing customers to select into auctions).

Our findings suggest that customer participation was partially driven by dealer exit and partially due to the changes in the market that made it more profitable for customers to buy bonds (as the time trend in Figure 5B suggests). Adding two dealers reduces a customer's participation probability by roughly 17.06 percentage points, or $31.56 \%$, on average. Furthermore, Figure 6 highlights that the probability of participating in some auctions drops to almost zero. ${ }^{23}$

Competition-volatility trade-off. Next, we illustrate the competition-volatility tradeoff that arises when adding market participants who do not participate with regularity.

For this, we first consider a hypothetical auction environment with only dealers. This eliminates the effects coming from changes in the bidder composition or from dealer bid updating. In this simplified auction environment, we ask by how much the expected auction price and auction coverage vary in the number of competing dealers.

[^17]Figure 6: Why did customers enter?


Figure 6 shows the customer's participation probability in percentage points for every 3rd auction from 2014 onward in the status quo (on the x-axis) and the counterfactual in which we add back the two dealers who left (on the $y$-axis).

Consider a typical auction, shown in Figure 7: If 14 dealers compete, the market clears at a competitive price, which is similar to the observed one. If the number of dealers competing drops to 13 or 12 , the expected price drops by close to $0.06 \%$ and $0.41 \%$, respectively. ${ }^{24}$

Next, we analyze the effects of varying the probability with which customers participate in a typical auction, fixing the number of dealers-see Figures 8. The expected price decreases by $0.015 \%$, the expected revenue decreases by $0.004 \%$ (or C $\$ 0.2$ million), and bid shading increases by $12 \%$. With fewer dealers, the effects are much larger.

To compare the competition effect to the volatility effect from increasing customer participation, we compute the expected revenue loss from a reduction in the expected customer participation of one competitor on average and the expected revenue loss from across-auction variation in customer entry rates. ${ }^{25}$ The loss arises from the feature that both the expected

[^18]Figure 7: Typical auction: Varying number of dealers


Figure 7 shows how the range of the expected prices (in $\mathrm{C} \$$ ) at which an auction clears varies as the number of dealers increases in an auction that issues the average supply with medium customer participation, i.e., 3 customers. In theory, there is one expected price for each fixed number of dealers. In practice, we determine a range of prices (marked in black), given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert 2021). The blue horizontal line shows the average observed bid, which is close to the observed market-clearing price.
price and the expected revenue are concave functions of the entry probability of customers (as visible in Figure 8 ). For the median auction, the competition effect ( $\mathrm{C} \$ 0.2 \mathrm{M}$ ) is somewhat larger than the loss from irregular participation ( $\mathrm{C} \$ 0.04 \mathrm{M}$ ). However, when the distribution of customers' entry probabilities gives more weight to auctions with fewer customers than in our data the volatility loss can outweigh the competition effect. This is because the low auction revenues on bad days, on which few customers enter, dominate the smaller increases in the auction revenues on good days.

Taken together, these findings highlight the potential risk of introducing volatility in auction coverage and clearing prices when bidder participation is irregular. Our results also indicate that losing additional dealers could have detrimental effects unless an adequate number of new customers enters the market. This is not only a hypothetical concern, as one dealer (RBC) recently acquired another dealer (HSBC).

Alternative policies. In the final part of the paper, we aim to determine a simple policy that both reduces volatility and increases competition relative to the status quo.

Figure 8: Typical auction: varying customer participation probability

(A) Customer participation

(B) Customer participation less dealers

Figure 8 shows how the range of the expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the participation probability of (all) customers varies between 0 and 1 , computed at an evenly spaced grid point in an auction that issues the average supply with medium customer participation, i.e., 3 customers. The right panel, shows the same auction but with 8 dealers. In theory, there is one expected price for each counterfactual. In practice, we determine a range of prices (marked in black), given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert 2021). The blue horizontal line shows the average observed bid, which is close to the observed market-clearing price.

As a starting point, we consider modifications to the commitment requirements. We first eliminate dealer commitment, meaning that we allow dealers to freely decide whether to enter each auction, in the hope that this increases competition without harming volatility. Then, we require customers to commit to participating in the same way as dealers, in the hope that this decreases volatility without harming competition. Due to endogenous bidder participation, in both cases, it is theoretically possible to increase competition and decrease volatility relative to the status quo. Empirically, we find that neither of these two alternative policy regimes achieves that goal (see Appendix D. 2 for details).

Instead, we propose to strategically reshuffle the supply across auctions to incentivize and stabilize customer participation. The idea is that we can predict (with some noise) how many customers each auction would attract, under the current supply schedule, based on the observable market conditions, and then shift some of the supply from the attractive auctions to the unattractive ones.

Concretely, we rely on the OLS regression from Appendix Table A2 to predict how many customers will want to participate under the current supply schedule, given the observable market conditions. We exclude the lagged number of participating customers as an explanatory variable, because this variable changes endogenously in the counterfactual. We think that it is reasonable to assume that all other predictors of customer participation, such as the secondary market spread, are not materially affected by the moderate supply changes we propose. We also exclude 2020 onward, when debt issuance spiked in response to the unprecedented level of fiscal intervention during the COVID pandemic. Within each year, we rank the predicted number of participating customers, $\hat{N}_{t}^{c}$, from smallest to highest, and obtain a quantile ranking, ranging from 0 (smallest $\hat{N}_{t}^{c}$ ) to 1 (highest $\hat{N}_{t}^{c}$ ) from the empirical distribution of $\hat{N}_{t}^{c}$. The quantile ranking is a single dimensional index, $s \in[0,1]$, which captures the intensity of the predicted customer participation.

Based on this participation index, we increase the supply by $10 \%$ in the auction with the lowest participation $(s=0)$ and decrease the supply by $10 \%$ in the auction with the highest participation $(s=1)$. We adjust the supply linearly for all auctions in between. Formally, the new supply in an auction with index $s$ is $Q_{t}+0.2(0.5-s)$, given that the observed supply is $Q_{t}$. This rule implies that the supply in the auction with the median participation ( $s=0.5$ ) is left unchanged. All adjustments are made within a year, so the total volume issued in a year is unchanged. To avoid issuing amounts of debt that are too extreme in any counterfactual auction relative to the status quo, we normalize all initial supplies to one, that is, $Q_{t}=1$ for all $t .{ }^{26}$

We find that implementing such supply adjustments successfully increases competition while decreasing volatility (see Figure 9). Both the median expected price and the median number of participating customers increase, suggesting enhanced competition relative to the status quo. In addition, the customer participation probability per auction stabilizes around a median of $17 \%$ and the price volatility diminishes. As a result, the median revenue per auction increases by about $\mathrm{C} \$ 0.38$ million (or, about 0.01 bps ).

This simple rule does not account for all of the factors that influence the complicated

[^19]Figure 9: Reshuffling supply to incentivize customer participation


Figure 9A shows the distribution of customer participation probabilities across auctions in the status quo and the counterfactual in which we strategically reshuffle supply to incentivize stable customer participation (in pp). Figure 9B displays the corresponding distributions of expected auction prices (in C\$).
decision on how to issue government debt. For example, it abstracts from the term structure of bonds and, therefore, ignores the complications that arise from rolling over debt in the future.

However, this rule has at least three attractive features. First, it is easy to implement and politically feasible, given that the central bank already changes the supply issued to bidders by placing sizable non-competitive bids. Second, the rule is supply neutral, in relative terms, in that we add the same percentage supply in one auction that we subtract from another. Since the observed supplied quantity (in dollars) is uncorrelated with the quantity changes we make (in dollars), the rule is also supply neutral in absolute terms as long as we repeat our exercise with sufficiently many auctions. Third, the rule is essentially revenue neutral. This is because there is no statistically significant correlation between the average (quantityweighted) value for the auctioned bond of participating bidders and the predicted number of participating customers. ${ }^{27}$ Therefore, our rule does not systematically shift the supply from

[^20]low-value auctions that clear at low prices, to high-value auctions that clear at high prices, or vice versa.

## 8 Conclusion

We study dealer exit and customer entry in the primary market for Canadian government debt and analyze some consequences for the functioning of the market. We show that customer participation has increased, but remains highly irregular. We introduce and estimate a structural model to trade off the benefits of higher competition from customer entry with the costs of higher market volatility. Our framework could be used in other settings with regular and irregular market participants.
confidence interval is $[-0.14,+0.07]$.

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## ONLINE APPENDIX

## Entry and Exit in Treasury Auctions

By Jason Allen, Ali Hortaçsu, Eric Richert, and Milena Wittwer

Appendix A presents mathematical details, including formal proofs.
Appendix B presents additional empirical tests supporting our model.
Appendix $\mathbf{C}$ explains how we estimate dealer and customer values.
Appendix D provides details about counterfactuals, and presents additional findings.

## A Mathematical appendix

In this appendix we prove Proposition 1. The proof relies on Lemma 1, stated and proved below. Finally, we prove that customer value bounds are identified, which is summarized in Proposition 4.

Proof of Proposition 1. The proof of statement $(i)$ is analogous to the proof of Proposition 1 in Kastl (2011).

To show statement (ii), take the perspective of customer $i$ and assume that all other bidders play an equilibrium. For ease of notation, we drop the auction and time subscripts, $t$ and $\tau$, and fix $L$ moment functions, $\mu^{l}$. Given that dealers only update their own function when the customer submits a function with at least one different moment $\left\{m_{i}^{l}\right\}_{l=1}^{L}$, we can decompose the conditions that characterize the customer's best reply (and, with that, an equilibrium, given that the other bidders play, by assumption, an equilibrium) into two parts.

First, we fix some set of moments, $\left\{m_{i}^{l}\right\}_{l=1}^{L}$, and find $b_{i}^{h}\left(\cdot, \theta_{i}^{h}\right)$ that maximizes the customer's expected total surplus,

$$
\begin{equation*}
T S_{i}^{h}=\mathbb{E}\left[\int_{0}^{\boldsymbol{q}_{i}^{h *}}\left[v^{h}\left(x, s_{i}^{h}\right)-b_{i}^{h}\left(x, \theta_{i}^{h}\right)\right] d x\right] \text { such that } \mu^{l}\left(b_{i}^{h}\left(\cdot, \theta_{i}^{h}\right)\right)=m_{i}^{l} \text { for all } l . \tag{3}
\end{equation*}
$$

Denoting Lagrange multipliers of the constraints by $\lambda_{i}^{l}$, the objective function is $T S_{i}^{h}-$ $\sum_{l=1}^{L} \lambda_{i}^{l}\left(m_{i}^{l}-\mu^{l}\left(b_{i}^{h}\left(\cdot, \theta_{i}^{h}\right)\right)\right.$. Given this objective function, we can follow the perturbation argument in the original proof in Appendix A. 2 of Kastl (2011), step by step, with one
difference. There is an additional term that comes from the constraints that all moments, $l$, of the chosen function, $\mu^{l}\left(b_{i}^{h}\left(\cdot, \theta_{i}^{h}\right)\right.$, must equal the fixed moments, $m_{i}^{l}$. This term does not create issues when taking derivatives since the moment functions are differentiable w.r.t. quantity. Further, since dealers only update their own bids if a moment of the customer's bidding function changes, and we are keeping these moments fixed, no complications arise when predicting which states of the world the market will clear relative to the original proof. Following the steps in Kastl (2011), we obtain equations A. 2 and A. 3 in Kastl (2011) plus the term $\sum_{l}^{L} \lambda_{i}^{l} \frac{\partial \mu^{l}\left(b_{i}^{h} \cdot, \cdot \theta_{i}^{h}\right)}{\partial q_{k}}$. Combining these equations gives condition (5) in Proposition 1. Unlike for dealers, these expressions do not simplify further because it may be optimal for customers to tie in steps other than the last one. This is explained in Lemma 1 below.

Statement (iii) specifies that, in equilibrium, a customer must choose a bidding function that gives rise to moments, $\left\{m^{l}\right\}_{l=1}^{L}$, so that the total expected surplus is maximized globally. Note that solving this maximization is challenging (even when we restrict our attention to moments that are real numbers) because the objective function-that is, the expected auction surplus - is not differentiable w.r.t. these moments. To see this, consider a change in moment $m_{i}^{l}$. The dealer who observes the corresponding bid function updates their own bid because a change in $m_{i}^{l}$ (weakly) changes the dealer's information set and, with that, its type, $\theta_{i}^{d}$. Therefore, the dealer submits a different bid function, that is, a step function with steps at different points. This changes the customer's beliefs about the price at which the auction will clear. Formally, the distribution of the clearing price changes when the customer fixes their own bid function, that is, since bidding functions are step functions, a change in such a function easily leads to non-continuous jumps that render the objective function, $T S_{i}^{h}$, non-differentiable.

Lemma 1. (i) For a customer, ties occur with zero probability for a.e. $s_{i}^{h}$ in any equilibrium for either all steps except the last step (if $\lambda_{i} \leq 0$ ), or at the last step (if $\lambda_{i}>0$ ). (ii) For $a$ dealer, Kastl (2011) Lemma 1 applies.

Proof of Lemma $1(i)$. For ease of notation we eliminate the auction and time subscripts, $t$ and $\tau$, as well as the customer superscript, $h$.

Case 1: $\lambda_{i}>0$. Consider the last step, $k=K$. Suppose bidder $i$ ties on step $k=K$. Take some $q_{m}=\sup \left\{q \mid v\left(q, s_{i}\right)>b_{k}\right\}$ and let $\bar{q}=\max \left\{q_{k}-\delta, q_{m}\right\}$ with $\delta$ some strictly positive
number bounded above by $q_{k}-q_{k-1}$. By moving from $q_{k}$ to $\bar{q}$, the bidder either moves to where they get a positive surplus from the amount purchased or they buy fewer units at a negative surplus. Take the deviation to bid $b_{i k}^{\prime}=b_{i k}+\epsilon$, where $\epsilon>0$ is sufficiently small at $\bar{q}$, the associated step, either bidder $i$ gets fewer units than they would in a tie and no longer pays for those units or they get more units but on each of these units earns an additional positive surplus.

Given dealer updating, this deviation is strictly profitable. To see this, let us abbreviate the moment, that is, the quantity-weighted bid of the deviated bidding function by $\mu\left(\bar{q}, b^{\prime}\right)$ and, similarly, the original bidding function. Deviating to $b_{i k}^{\prime}$ comes at a constraint penalty of $-\lambda_{i}\left(m_{i}-\mu\left(\bar{q}, b^{\prime}\right)\right)$, since the targeted moment, $m_{i}=\mu\left(q_{k}, b\right)$, is no longer met. Because $\mu\left(\bar{q}, b^{\prime}\right)>\mu\left(q_{k}, b\right)=m_{i}$ and $\lambda>0$, by assumption, $-\lambda_{i}\left(m_{i}-\mu\left(\bar{q}, b^{\prime}\right)\right)>0$, representing a strictly positive profit from deviating.

Case 2: $\lambda_{i}<0$. Suppose bidder $i$ ties on a step $k<K$. Take some $q_{m}=\sup \left\{q \mid v\left(q, s_{i}\right)>\right.$ $\left.b_{k}\right\}$ and let $\bar{q}=\max \left\{q_{k}-\delta, q_{m}\right\}$, with some strictly positive step size, $\delta$, that is bounded above by $q_{k}-q_{k-1}$, that is, the bidder steps either toward where they get a positive surplus from the amount purchased, if possible, or, if not, at least they buy fewer units at prices above their values (reducing losses). Take the deviation to bid $b_{i k}^{\prime}=b_{i k}+\epsilon$, where $\epsilon>0$ is sufficiently small, at $\bar{q}$ (the associated step), either bidder $i$ gets fewer units than they would in a tie and no longer pays for those units, or they get more units but on each of these units earns an additional positive surplus. Similar to the first case, there is a positive gain that arises due to dealer updating. The deviation comes at a constraint penalty of $-\lambda_{i}\left(m_{i}-\mu\left(\bar{q}, b^{\prime}\right)\right)$. Because $\mu\left(\bar{q}, b^{\prime}\right)<\mu\left(q_{k}, b\right)=m_{i}$, the bracketed term is positive. By assumption, $\lambda_{i}$ is negative, so that $-\lambda_{i}\left(m_{i}-\mu\left(\bar{q}, b^{\prime}\right)\right)>0$.

Case 3: $\lambda_{i}=0$. The original proof from Kastl (2011) applies.

Proposition 4. Given customer $s_{t i}^{h}$ behaves according to Proposition 1 (ii) and (iii), and the dealer only pays attention to one moment of the customer's bidding function, that is, $L=1$, the upper and lower bounds on the customer's values, $\bar{v}_{t}^{h}\left(\cdot, s_{t i}^{h}\right)$ and $\underline{v}_{t}^{h}\left(\cdot, s_{t i}^{h}\right)$, are identified.

Proof of Proposition 4. We fix an auction, $t$, and a customer, $i$, and drop the auction, $t$, time, $\tau$, customer, $h$, and bidder, $i$, subscripts and superscripts, for simplicity. Further, we assume that only one moment, $m$, matters, for instance, the quantity-weighted bid,
consistent with our estimation, and we drop the $l$-subscript.
The identification argument is complicated by the fact that condition (5)—here, slightly rearranged-not only contains values at the submitted steps, $v\left(q_{k}, s\right)$, but also the values at some intermediate quantities between the submitted steps, $v\left(\boldsymbol{q}^{*}, s\right)$ :

$$
\begin{align*}
0= & \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right) v\left(q_{k}, s\right)+ \\
& \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.v\left(\boldsymbol{q}^{*}, s\right) \frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}^{*}, \theta, m\right]+ \\
& \operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.v\left(\boldsymbol{q}^{*}, s\right) \frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1} \geq \boldsymbol{P}^{*}, \theta, m\right]- \\
& \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right) b_{k}- \\
& \operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}^{*} \mid \theta, m\right)\left(b_{k}-b_{k+1}\right)- \\
& \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}^{*}, \theta, m\right] b_{k}- \\
& \left.\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}=\boldsymbol{P}^{*}, \theta, m\right]\right] b_{k+1}+ \\
& \lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{k}} . \tag{12}
\end{align*}
$$

Here, and for all other expressions in this section, we include the fixed moment, $m$, as a condition alongside the bidder's type. Dependence on intermediate quantities implies that the customer values cannot be point identified.

To identify the customer's value bounds, we first guess a Lagrange multiplier, $\lambda \in \mathbb{R}$, and simplify the system of equations (12) to obtain a set of conditions for $\lambda>0$ and one for $\lambda<0$.

We start by relying on monotonicity and boundedness of the value function: for all $q_{k-1} \leq q \leq q_{k}$, we know that $v\left(q_{k-1}, s\right) \geq v(q, s) \geq v\left(q_{k}, s\right)$. In addition, we sign the derivatives of the rationed quantity as follows: increasing the bid at step $k$ increases the quantity rationed in the event of a tie at step $k$ and decreases the quantity rationed in the event of a tie at step $k+1$.

Next, we eliminate terms in condition (12) to obtain a system of $2 K$ linear equations with $2 K$ unknowns. To do this, we obtain an upper bound on the value at step $q_{k}$ by making the terms involving the value and the derivative of the rationed quantity in the event of a tie
as small as possible (and making them as big as possible for the lower bound). To do this, we plug in $\bar{v}\left(q_{k}, s\right)$, the maximum possible value at intermediate quantities along the next step, for the terms involving rationed quantities at the next step (e.g., line 3 of equation (12)), since the (conditional) expectation of the derivative of the random quantity won at the next step, $\mathbb{E}\left[\left.\frac{\partial q^{*}}{\partial q_{k}} \right\rvert\, b_{k+1} \geq P^{*}, \theta, m\right]$, is negative. This is because increasing $q_{k}$ decreases $\left(q_{k+1}-q_{k}\right)$, which, due to pro-rata-rationing, decreases the amount the bidder wins in case of a tie at step $k+1$. In addition, we plug in $\underline{v}\left(q_{k}, s\right)$, the smallest possible value at intermediate quantities along the current step, for terms involving ties at the current step (e.g., line 2 of equation (12)), since the (conditional) expectation of the derivative of the random quantity won at the step, $\mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=P^{*}, \theta, m\right]$, is positive. This is because increasing $q_{k}$ increases $\left(q_{k}-q_{k-1}\right)$, which, due to pro-rata-rationing, increases the amount the bidder wins in case of a tie at step $k$.

To obtain a lower bound, we instead substitute the maximum value at the current step, $\bar{v}\left(q_{k-1}, s\right)$ (in line 2 of equation (12)) and the minimum value at the next step, $\underline{v}\left(q_{k+1}, s\right)$ (in line 3 of equation (12)).

To further simplify the system of equations, we rely on Lemma 1 according to which ties never occur in equilibrium at a subset of steps. This allows us to cancel terms involving ties at these steps. There are two cases, depending on the sign of $\lambda$. When the current $\lambda$ is negative, at the steps before the last step ties cannot be optimal and the terms involving the derivative of the rationed quantity drop out except for at the second-to-last step. When the current $\lambda$ is positive, the only simplification occurs at the last step, where the rationing terms all drop out. Furthermore, at the last step, all terms involving $b_{K}$ drop out.

With these simplifications, we obtain the following system of equations for two cases, $\lambda>0$, and $\lambda<0$, where the expected payment is on the LHS and the expected benefit is on the RHS. It can help to transform this system of equations into a single matrix, one for each case, to see that the system is indeed identified (conditional on knowing $\lambda$ ).

Appendix Figure A1: Identification graphically


Appendix Figure A1 shows an example of a step function with three steps (in dashed lines) and the corresponding lower bound (in black) and upper bound (in gray) values at each step. In the example, shading is positive at all steps, which isn't crucial for identification. Instead, the main assumption we rely on to create the bounds is that the value curve is monotonically decreasing in quantity.

Define

$$
\begin{align*}
A_{k} & =\operatorname{Pr}\left(b_{k}>P^{*}>b_{k+1} \mid \theta, m\right) b_{k}+\operatorname{Pr}\left(b_{k+1} \geq P^{*} \mid \theta, m\right)\left(b_{k}-b_{k+1}\right) \\
& \left.+\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=P^{*}, \theta, m\right] b_{k}+\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}=P^{*}, \theta, m\right]\right] b_{k+1} \\
& -\lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{k}} \text { for all } k=1, \ldots, K-1,  \tag{13}\\
A_{K} & =\operatorname{Pr}\left(\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right) b_{K}+\operatorname{Pr}\left(b_{K}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=P^{*}, \theta, m\right] b_{k} \\
& -\lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{K}} \tag{14}
\end{align*}
$$

(i) When $\lambda<0$, the system of equations equalizing the expected payment with the expected benefit at the upper and lower bound is

$$
\begin{aligned}
& A_{1}=\operatorname{Pr}\left(b_{1}>P^{*}>b_{2} \mid \theta, m\right) \bar{v}\left(q_{1}, s\right) \\
& A_{1}=\operatorname{Pr}\left(b_{1}>P^{*}>b_{2} \mid \theta, m\right) \underline{v}\left(q_{1}, s\right), \text { and analogously for } k=2, \ldots K-2 .
\end{aligned}
$$

Here, we rely on the fact that when $\lambda<0$, by Lemma 1 , ties cannot occur at non-final steps.
$A_{K-1}=\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right) \bar{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \underbrace{\mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K} \geq \boldsymbol{P}^{*}, \theta, m\right]}_{<0} \bar{v}\left(q_{K-1}, s\right)$
Here, we again rely on the fact that there is no tie at $K-1$, so that we can cancel the second line in equation (12). We make the third line as small as possible to maximize $\bar{v}\left(q_{K-1}, s\right)$. All other lines are part of the expected payment and, thus, are in $A_{K-1}$. A similar logic applies when determining the lower bound at $K-1$ :
$A_{K-1}=\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right) \underline{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \underbrace{\mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K} \geq \boldsymbol{P}^{*}, \theta, m\right]}_{<0} \underline{v}\left(q_{K}, s\right)$
For the last step, we apply the same logic from above, just that now the third line of equation (12) drops out because we are at the last step. In addition, we can replace the term $\underline{v}\left(q_{K}, s\right)$ with $\bar{v}\left(q_{K}, s\right)$ since we know the value curve takes one value at that point. We obtain

$$
\begin{aligned}
& A_{K}=\operatorname{Pr}\left(b_{K}>P^{*}>0 \mid \theta, m\right) \bar{v}\left(q_{K}, s\right)+\operatorname{Pr}\left(b_{K}=P^{*} \mid \theta, m\right) \underbrace{\mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=P^{*}, \theta, m\right]}_{>0} \bar{v}\left(q_{K}, s\right) \\
& A_{K}=\operatorname{Pr}\left(b_{K}>P^{*}>0 \mid \theta, m\right) \underline{v}\left(q_{K}, s\right)+\operatorname{Pr}\left(b_{K}=P^{*} \mid \theta, m\right) \underbrace{\mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=P^{*}, \theta, m\right]}_{>0} \bar{v}\left(q_{K-1}, s\right),
\end{aligned}
$$

where $A_{k}$ and $A_{K}$ are given by (13) and (14), respectively.
(ii) Similarly, when $\lambda>0$, we rely on Lemma 1 to eliminate ties at the last step. For earlier steps, we follow the analogous point-wise argument as above to obtain

$$
\begin{aligned}
A_{1} & =\operatorname{Pr}\left(b_{1}>\boldsymbol{P}^{*}>b_{2} \mid \theta, m\right) \bar{v}\left(q_{1}, s\right)+\operatorname{Pr}\left(b_{1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{1}=\boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{1}, s\right) \\
& +\operatorname{Pr}\left(b_{2} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{2} \geq \boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{1}, s\right) \\
A_{1} & =\operatorname{Pr}\left(b_{1}>\boldsymbol{P}^{*}>b_{2} \mid \theta, m\right) \underline{v}\left(q_{1}, s\right)+\operatorname{Pr}\left(b_{1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{1}=\boldsymbol{P}^{*}, \theta, m\right] \bar{v}(0, s) \\
& +\operatorname{Pr}\left(b_{2} \geq \boldsymbol{P} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{2} \geq \boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{2}, s\right)
\end{aligned}
$$

Here, and in all of the equations for the upper bound that follow, we replace the $\underline{v}\left(q_{k}, s\right)$ in the terms involving ties with $\bar{v}\left(q_{k}, s\right)$, since we know that the values at that point are a single number. This makes the bounds more informative.

$$
\begin{aligned}
A_{2} & =\operatorname{Pr}\left(b_{2}>\boldsymbol{P}^{*}>b_{3} \mid \theta, m\right) \bar{v}\left(q_{2}, s\right)+\operatorname{Pr}\left(b_{2}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{2}=P^{*}, \theta, m\right] \underline{v}\left(q_{2}, s\right) \\
& \operatorname{Pr}\left(b_{3} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{3} \geq \boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{2}, s\right) \\
A_{2} & =\operatorname{Pr}\left(b_{2}>\boldsymbol{P}^{*}>b_{3} \mid \theta, m\right) \underline{v}\left(q_{2}, s\right)+\operatorname{Pr}\left(b_{2}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{2}=\boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{1}, s\right)+ \\
& +\operatorname{Pr}\left(b_{3} \geq \boldsymbol{P} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{3} \geq \boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{3}, s\right), \text { and analogously for } k=3, \ldots K-2, \\
A_{K-1} & =\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K}\right) \bar{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K-1}=\boldsymbol{P}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K-1}=\boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{K-1}, s\right)
\end{aligned}
$$

In this case, the third line of equation (12) drops out because there is no tie. And similarly for the lower bound equation:

$$
\begin{aligned}
A_{K-1} & =\operatorname{Pr}\left(b_{K-1}>P^{*}>b_{K}\right) \underline{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K-1}=P^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K-1}=P^{*}, \theta, m\right] \bar{v}\left(q_{K-2}, s\right) \\
A_{K} & =\operatorname{Pr}\left(b_{K}>P^{*}>0\right) \bar{v}\left(q_{K}, s\right) \\
A_{K} & =\operatorname{Pr}\left(b_{K}>P^{*}>0\right) \underline{v}\left(q_{K}, s\right),
\end{aligned}
$$

where $A_{k}$, and $A_{K}$ are given by (13) and (14), respectively. This system of equations would be identified if $\lambda$ was known. Since this isn't the case, we rely on Proposition 1 (iii) to obtain identification of the value bounds.

Specifically, we know that perturbing $m$ cannot be optimal. Formally, the total expected surplus must decrease when increasing and decreasing $m$ by $\epsilon>0$ :

$$
\begin{align*}
& T S(b(\cdot, \theta), m)-T S(b(\cdot, \theta), m+\epsilon) \geq \lambda \epsilon  \tag{15}\\
& T S(b(\cdot, \theta), m-\epsilon)-T S(b(\cdot, \theta), m) \leq \lambda \epsilon \tag{16}
\end{align*}
$$

where

$$
\begin{aligned}
T S(b(\cdot, \theta), m) & =\sum_{k=1}^{K}\left[\operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right) V\left(q_{k}, s\right)-\operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*} \mid \theta, m\right) b_{k}\left(q_{k}-q_{k-1}\right)\right] \\
& +\sum_{k=1}^{K} \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[V\left(\boldsymbol{q}^{*}, s\right)-b_{k}\left(\boldsymbol{q}^{*}-q_{k-1}\right) \mid b_{k}=\boldsymbol{P}^{*}, \theta, m\right]
\end{aligned}
$$

with $q_{0}=b_{K+1}=0$. This expression is equivalent to equation (3) for $g=h . \operatorname{TS}(b(\cdot, \theta), m+\epsilon)$ and $T S(b(\cdot, \theta), m-\epsilon)$ are defined analogously.

We can use (17) and (18) to find the bounds for (15) and (16), respectively. Consider (15) first and omit conditioning on $\theta$, for simplicity, and let $1(\cdot)$ denote the indicator function to obtain

$$
\begin{align*}
& \sum_{k=1}^{K}\left[\max \left\{0, \Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1}\right)\right\}\left(\sum_{j=1}^{k} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right. \\
& \left.\quad+\min \left\{0, \Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1}\right)\right\}\left(\sum_{j=1}^{k} \underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)-\Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}\right) b_{k}\left(q_{k}-q_{k-1}\right)\right]+ \\
& \sum_{k=1}^{K}\left[\max \left\{0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right\}\left(\sum_{j=1}^{k-1} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\min \left\{0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right\}\left(\sum_{j=1}^{k-1} \underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right. \\
& \left.\quad+\left[\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right)\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)\right)\right] \bar{v}\left(q_{k-1}, s\right) \\
& \quad \times 1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)>0\right) \\
& \quad+\left[\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)\right)\right] \underline{v}\left(q_{k}, s\right) \\
& \quad \times 1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)<0\right) \\
& \left.\quad-b_{k}\left(\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]\right)\right] \\
& \quad \geq \lambda \epsilon, \tag{17}
\end{align*}
$$

where $\Delta \operatorname{Pr}(\cdot)$ indicates taking a difference between $\operatorname{Pr}(\cdot \mid . ., m)$ and $\operatorname{Pr}(\cdot \mid . ., m+\epsilon)$ and $v\left(q_{0}, s\right)=$ $\bar{v}\left(q_{1}, s\right)$.

Similarly for (16)

$$
\begin{align*}
& \sum_{k=1}^{K}\left[\max \left\{0, \Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1}\right)\right\}\left(\sum_{j=1}^{k} \underline{v}\left(q_{k}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right. \\
& \left.\quad+\min \left\{0, \Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1}\right)\right\}\left(\sum_{j=1}^{k} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)-\Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}\right) b_{k}\left(q_{k}-q_{k-1}\right)\right]+ \\
& \sum_{k=1}^{K}\left[\max \left\{0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right\}\left(\sum_{j=1}^{k-1} \underline{v}\left(q_{k}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\min \left\{0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right\}\left(\sum_{j=1}^{k-1} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right. \\
& \\
& \quad+\left[\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)\right)\right] \underline{v}\left(q_{k}\right) \\
& \quad \times 1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)>0\right) \\
& \quad+\left[\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)\right)\right] \bar{v}\left(q_{k-1}\right) \\
& \quad \times 1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)<0\right) \\
& \left.\quad-b_{k}\left(\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m\right]\right)\right],  \tag{18}\\
& \quad \leq \lambda \epsilon
\end{align*}
$$

where $\Delta \operatorname{Pr}(\cdot)$ instead now indicates taking a difference between $\operatorname{Pr}(\cdot \mid . ., m-\epsilon)$ and $\operatorname{Pr}(\cdot \mid . ., m)$.
Summarizing, we now have a system of $2 K$ linear equations, $2 K+1$ unknowns and 2 inequalities from the optimality of the $K$ steps submitted by customer $i$. Therefore, the customer value bounds are identified.

## B Empirical tests

In Appendix B.1, we analyze what factors predict customer participation. In Appendix B.2, we provide evidence in favor of independent private values. In Appendix B.3, we test whether customers take dealer updating into account and, in Appendix B.4, we test whether customer values are different from dealer values.

## B. 1 Predicting customer participation

To better understand what predicts customer participation, we regress the number of participating customers on a set of explanatory variables, using data from 2014 onward, a period
when customers are almost exclusively hedge funds. Appendix Table A3 predicts customer participation at the bidder level and the results are similar. Importantly, we do not try to estimate the causal effects of customer participation. Instead, we focus on prediction, which we use in Section 7 to propose a policy rule that stabilizes customer participation at a sufficiently high average.

The first predictor of customer participation we include indicates the auction dates for which we estimate that a basis trade, which means buying a bond and shorting the future, could be profitable, an idea inspired by Barth and Kahn (2020) and Banegas et al. (2021). ${ }^{28}$ The second indicator variable, denoted $C I P$, is the ten-year cross-currency basis swap with the U.S. dollar. A non-zero basis indicates a violation of covered interest parity and an opportunity for arbitrage (e.g., Du et al. (2018)). The third variable tells us whether the to-be-issued bond has benchmark status, which is the Canadian equivalent of being on the run (Berger-Soucy et al. (2018)). It could be, for example, that hedge funds buy more-liquid on-the-run bonds because they are easier to sell.

The fourth and fifth predictors are indicator variables that capture the importance of monetary policy committee meetings (MPC) and quantitative easing (QE). Including an indicator for MPC meetings is inspired by the findings of Lou et al. (2023), which demonstrate that hedge funds tend to purchase bonds outside of the pre-MPC window to avoid interest rate uncertainty. This is why we also include the bond coupon rate-higher coupon rates are correlated with lower interest rate risk. Including an indicator for the auction days on which the central bank conducts a bond issuance in the morning and engages in QE in the afternoon, where hedge funds have an opportunity to sell bonds back, is inspired by An and Song (2018, 2023).

The sixth variable measures the buy-sell spread at which a to-be-issued bond is traded prior to the auction. We approximate this spread by the average difference between the highest and lowest price within a day during which a bond is to be issued is traded in

[^21]the secondary market three days prior to the auction. The seventh and eight variables count the number of dealers who participate in the auction and the number of customers who participated in the previous auction. Finally, we control for supply in the auction. Anecdotally, we know that margins are thin in Treasury markets; therefore, countries with large supplies can attract more participants.

In addition, we add controls that capture the interest rate environment and the expectations about the stance of monetary policy and, therefore, future bond prices. Specifically, we construct an overnight index swap (OIS) curve that captures the market expectations of the Bank of Canada's interest rate target for the overnight lending rate over 12 months. ${ }^{29}$

Our estimation findings, reported in Appendix Table A2, indicate that customers are more likely to participate in auctions when the secondary-market spread is high. This suggests that they buy bonds at auction when they can quickly sell them at high prices. In addition, we find some support for the idea put forth in Lou et al. (2023) that hedge funds avoid buying bonds prior to monetary policy announcements. The coefficient on the coupon rate is positive and this is consistent with hedge funds disliking interest rate risk. The other explanatory variables are statistically insignificant when including year-fixed effects. As for the number of dealers, this is because there is little variation within a year-a feature we will incorporate in our model. For some of the other explanatory variables, this might be because of low statistical power. For example, between 2014 and 2021 there were only five cases in which we estimate that a basis trade could have been profitable.

Low statistical power, in addition to a moderately sized $R^{2}$, indicates that there are unobservable factors that play a significant role in driving customer participation-a feature our model will incorporate.

## B. 2 Testing independent private values

We perform a formal test for independent private values, introduced in Hortaçsu and Kastl (2012), for auctions of the bonds in our sample. The test checks for equality of the dealers' estimated values before and after observing a customer bid. In a common values environment,

[^22]a customers' bid would provide the dealer with information that changes their expected values for acquiring the bond being sold. Under independent private values, this bid reveals information about the expected level of competition but it should not affect their value. Similar to the findings of Hortaçsu and Kastl (2012) for bills, we fail to find evidence that dealer values are shifted by the information learned through the customers' bids in the bond market. We calculate a p-value of 0.48 and, therefore, do not reject the null (of no learning about fundamentals).

## B. 3 Testing bounds

We test whether customers take dealer updating into account when placing a bid. Formally, we want to know if $\lambda_{t i}=0$ in Proposition 1 (ii). To do this, we fix an auction (and therefore omit the auction $t$ subscript). We construct measures $T_{i}=\left|v^{h}\left(q, s_{i}^{h} ; \lambda_{i}=0\right)-v^{h}\left(q, s_{i}^{h}\right)\right|$ for each customer $i$. Here, $v^{h}\left(q, s_{i}^{h} ; \lambda=0\right)$ denotes the customer's value for amount $q$, assuming that $\lambda_{i}=0$ and $v^{h}\left(q, s_{i}^{h}\right)$ is the value if $\lambda_{i} \neq 0$. With this, we construct a test statistic analogous to $S S Q_{T}$ from Hortaçsu and Kastl (2012). The test rejects the null hypothesis with a p -value of 0 , where the p -value is computed via bootstrap.

## B. 4 Testing value differences

We test whether customer values are significantly above dealer values. The null hypothesis is that customer values at the lower bound are weakly smaller than dealer values at the upper bound.

Following Hortaçsu and Kastl (2012), we compute three aggregate test statistics: the first test is in the spirit of a Chi-squared test, the second is based on the 95 th percentile of the across-auction differences, and the third is based on the maximum difference in these values across auctions. Since we are interested in a one-sided null hypothesis (are customer values larger than dealer values), we drop the absolute value, which differs from Hortaçsu and Kastl (2012). In all cases we omit the subset of auctions where not a single customer participated. We compute these test statistics for the differences in the average quantity-weighted value, the average maximum value, and the average minimum value of dealers/customers. In addition, we compute the confidence intervals for a set estimate of the mean difference. This allows us to evaluate the size of the expected difference and to compare it to 0 .

The results are reported in Table A5. For all measures, the customer values appear to be above the dealer values, however, the differences in the average maximum values are less precise, with some of the test statistics being insignificant.

## C Details regarding the estimation of values

To back out the dealer values and bounds on customer values from the equilibrium conditions of Proposition 1, we need to estimate the probabilities that enter these conditions, which are determined by the distribution of the market-clearing price, $\boldsymbol{P}_{\boldsymbol{t}}^{*}$. For customers, we also need to estimate the Lagrange multiplier, $\lambda_{t i}$, and the terms that capture the ties in condition (5). For simplicity, we assume that the number of customers matched to each dealer is 1 . This is the case in the data, except for rare cases.

We estimate the market-clearing price distributions by simulating market clearing. If all bidders were ex-ante symmetric and bid directly to the auctioneer, we would fix a bidder in an auction and draw $N-1$ bid functions, with replacement, from all of the observed bids in that auction. This would simulate one possible market outcome for the fixed bidder. Repeating this many times, we would obtain the distribution of the market-clearing price, $\boldsymbol{P}_{\boldsymbol{t}}^{*}$, for this bidder. Our setting is more complicated because there are both dealers and customers and customers must bid via dealers. Hortaçsu and Kastl (2012) introduce a resampling procedure to estimate the price distribution from the dealer's perspective. We extend their method to learn about the customers. Further, we allow signals within a bidder to be correlated over the course of an auction. This is to avoid estimation bias arising from the fact that we observe some bidders updating their bids without observing a customer bid. ${ }^{30}$

Specifically, we resample as follows: We first construct the residual supply curve that bidder $i$ faces in auction $t$. For this, we start by randomly drawing a customer bid from the set of $\bar{N}^{h}$ potential customer bids. If the customer did not participate in the auction, their bid is 0 ; if they updated their bid, we randomly select one of their bids. Next, we find a dealer that observed a similar bid to that customers' bid. ${ }^{31}$ If the selected dealer made

[^23]multiple bids, we select a random bid from the set of bids submitted by that dealer after they observed a bid similar to the current customer's bid. If the dealer did not update their bid after learning the customer's bid, we choose the last bid before learning it. Once a bid is selected, we drop all other bids from that dealer. We repeat this procedure $\bar{N}^{h}$ times if $i$ is a dealer and $\bar{N}^{h}-1$ times if $i$ is a customer. Next, we resample the dealers that did not observe customer bids. Starting with a list of "uninformed" dealers, we draw one such dealer. ${ }^{32}$ If they submitted more than one bid, we randomly select one bid and drop the others. We continue drawing from the set of uninformed dealers so that there are $N^{h}+N^{d}-1$ bidding curves. This is one realization of the residual supply curve that $i$ expects to face. We repeat the process many times to estimate the distribution of the clearing price, each time starting with the full set of bids made in the auction.

As in Kastl (2011), consistency of the estimator requires that the probability of market clearing at each step is strictly bounded away from zero. However, in our finite sample this event may occur. For steps with estimated win probabilities close to or equal to zero, we mix the estimated clearing-price distribution with a uniform distribution over the range of placed bids in order to give all bids a small win probability. In addition to reducing the sensitivity of the analysis to these small probabilities, we truncate less than $10 \%$ of the estimated values by assuming that these values are below the maximum bid plus $\mathrm{C} \$ 0.1$ times the maturity length in months divided by 12 , which is roughly equivalent to 10 bps in terms of yield-to-maturity.

With the estimated price distributions, using condition (4), we can solve for the value that rationalizes a dealer's bid at each step. To obtain the customer value bounds, we implement an estimation procedure that follows our identification argument that is presented in the main text, and formally explained in the proof of Proposition 4 in Appendix A.

To begin, we search over the (one-dimensional) set of $\lambda_{t i}$. For each feasible $\lambda_{t i}$, there is a unique set of lower and upper bounds for the value at each step, $q_{k}$, where that customer submitted a step that satisfies equation (5). Using these implied values together with the definition of the total surplus allows us to obtain upper and lower bounds on the $\lambda_{t i}$ based on expressions (17 and 18). This range of $\left(\lambda_{L}, \lambda_{U}\right)$ is the set of feasible $\lambda_{t i}$ that is consistent with

[^24]the observed choices and values. To obtain the maximum and minimum, we find the value at each point that maximizes (minimizes) the change in the total surplus. Whether this is the upper or lower envelope of the set of values consistent with the observed bids depends only on the sign of the change in the clearing probabilities under the increased moment, $m$. If the initial $\lambda_{t i}$ is within the range $\left(\lambda_{L}, \lambda_{U}\right)$, then the associated value curve is part of the identified set of values that can rationalize the behavior of a given customer. When it is outside the range, the bid is not consistent with equilibrium behavior for that set of values. To trace out the identified set, we repeat this exercise along a grid of possible $\lambda_{t i}$.

## D Details regarding counterfactuals

In Appendix D. 1 we explain how we compute counterfactual equilibria. In Appendix D. 2 we present what happens when we make changes to the rules of bidder commitment.

## D. 1 Computational details

We are interested in finding a set of bid distributions that implies a value distribution that is similar to the true (in our case estimated) value distribution. We therefore construct a criterion function that compares these distributions along several dimensions. The criterion has three components.

First, we evaluate the distribution of the values at quantiles of the quantity-bid distribution, corresponding to orders for $1.1,1.7,2.3,3.4,3.6,4.5,5.6$ and $25 \%$ of the total supply. To reduce the width of the predicted set of solutions, we add to this set the marginal distribution of the values corresponding to quantities of $10 \%$ and $15 \%$. For each auction and bidder group, $g$, we construct bounds on the value distribution at each discrete level of quantity, using an evenly spaced grid running from the $5^{\text {th }}$ percentile to the $95^{\text {th }}$ percentile. At each point, we compare the bounds on the implied values from the guess of the bid distribution to the true values and add to the criterion function $\max \left(F_{L}^{I M}-F_{U}, 0\right)^{2}$ and $\min \left(F_{U}^{I M}-F_{L}, 0\right)^{2}$, where $F_{L}^{I M}$ denotes the implied value distribution at each of the quantity levels evaluated on the grid points and $F_{U}$ denotes the corresponding upper bound on the distribution known from the data. $F_{U}^{I M}$ and $F_{L}$ are defined analogously.

In addition, for each bidder group, we want the (across-bidder) distribution of the largest
quantities demanded to be smaller than the observed distribution of the largest quantities a bidder ever purchased in all auctions. The restriction is designed to capture the fact that some bidders might be capacity constrained below the regulatory $25 \%$ maximum and, therefore, even if the counterfactual price is low, bidders may not be interested in purchasing up to $25 \%$. Therefore, we require the distribution of the largest quantity bid by bidders of group $g$ in auction $t$ to be first-order stochastically dominated by the distribution of the largest quantities ever bid by bidders of group $g$ in any auction $t$. We add to the criterion function the squared difference in the probabilities any time the implied maximum quantity distribution falls below the distribution from the data. These violations are evaluated along a set of grid points (evenly spaced by quantity, from 0 to $25 \%$ ).

In an alternative specification, we require the predicted average coverage, that is, the sum of all bids over the total supply, of the auction to match the observed one, after accounting for the change in coverage that is induced by a change in the number of bidders. Specifically, we estimate the change in coverage caused by dealer exits (pooling 10 auctions before and after each exit) and reduce the coverage by the difference in the number of dealers between the counterfactual and the factual, multiplied by the average reduction in coverage. If we allow each bidder to demand up to the bidding limit (25\%), no auction will fail. However, it is unrealistic to assume that all bidders have the capacity to buy Treasuries worth more than C $\$ 81$ million (which is $25 \%$ of the average amount issued) in each auction. This would be possible only if a bidder received an extraordinary number of client orders or had sufficient balance sheet space despite stringent regulatory constraints.

## D. 2 Evaluating the importance of commitment

In light of the competition-volatility trade-off presented in the main text, we evaluate two alternative policy regimes regarding bidder commitment. First, we make changes to assess the extent to which primary auctions run smoothly without forcing regular dealer participation. Second, we attempt to minimize volatility by requiring customers to commit to participating in the same way as dealers. In both cases, it is theoretically ambiguous whether competition and volatility increase or decrease relative to the status quo, due to endogenous bidder participation.

To analyze the importance of dealer commitment, we compare two settings-in the first, dealers commit as in the status quo but we allow customers to place bids directly with the auctioneer; in the second, both bidder groups bid directly with the auctioneer but we do not impose obligatory participation on dealers. To compute the counterfactual without dealer commitment, we assume that a dealer's cost of entering one auction equals their estimated annual cost divided by the average number of auctions in a year.

We find that most auctions attract sufficiently many bidders to guarantee full auction coverage, even without obligating dealers to regularly participate (see Appendix Figure A12). However, both dealer and customer participation is highly irregular. Moreover, three out of one hundred and one auctions risk failure without dealer commitment (where we define an auction to be at risk if its chance of not clearing is above $5 \%$ ). The expected price of these at-risk auctions drops by more than $5 \%$. Further, even in fully covered auctions, expected revenue decreases by $0.04 \%$, in the median, without dealer commitment.

To assess the impact of customer commitment, we force customers to decide at the beginning of each year whether or not to commit to participating in all auctions of the upcoming year. They enter the market if their annual entry cost (approximated by the estimated auction-specific cost scaled by the average number of auctions in a year) is larger than the total surplus they expect from participating in all auctions of that year. Dealer participation is fixed.

We find that between one and two fewer customers would have participated if they had to commit (see Appendix Table A8). Nevertheless, auctions would have remained relatively competitive since sufficiently many bidders would have remained in the market. Expected revenue would have dropped by $\mathrm{C} \$ 3.6 \mathrm{M}$, or about $0.11 \%$ on average.

Appendix Table A1: List of dealers that exited and entered auctions, and year

| Dealer name | Bill auctions | Bond auctions |
| :--- | :--- | :--- |
| CT Securities | Entry 1999, exit 2000 | Entry 1999, exit 1999 |
| Salomon Brothers | Entry 1999 exit 2000 | Entry 1999, exit 2001 |
| Goldman Sachs | Entry 1999, exit 2001 | Entry 1999, exit 2002 |
| SG Valeurs | Entry 1999, exit 2004 | Entry 1999, exit 2004 |
| JP Morgan | Entry 1999, exit 2007 | Entry 1999, exit 2007 |
| Deutsche Bank | Entry 1999, exit 2014 | Entry 1999, exit 2014 |
| Morgan Stanley | Entry 1999, exit 2001 | Entry 1999, exit 2014 |
| PI Financial Corp | Entry 2009, exit 2015 | Entry 2009, exit 2015 |
| Ocean Securities | Entry 2006, exit 2008 | Entry 1999, exit 2015 |
| Sherbrooke SSC | Entry 2020 | Entry 2020 |

Appendix Table A1 lists all entries and exits of dealers in bill and bond auctions from 1999 to 2022 . We only list years, even though we do observe the exact dates of entry and exit.

Appendix Table A2: Predictors of customer participation

| Number of customers | (OLS1) |  | (OLS2) |  | (Year-FE) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| $\beta_{1}:$ Cash-futures basis trade | 0.172 | $(0.692)$ | -0.0589 | $(0.700)$ | -0.331 |  |
| $\beta_{2}:$ CIP basis trade | $-0.088^{* * *}$ | $(0.0192)$ | $-0.109^{* * *}$ | $(0.0204)$ | -0.0117 |  |
| $\beta_{3}:$ Benchmark status | 0.158 | $(0.314)$ | 0.0129 | $(0.319)$ | -0.139 |  |
| $\beta_{4}:$ MPC | $-1.708^{* *}$ | $(0.766)$ | $-1.908^{* *}$ | $(0.372)$ | $-1.888^{* *}$ |  |
| $\beta_{5}:$ QE | -0.142 | $(0.380)$ | -0.350 | $(0.403)$ | -0.358 |  |
| $\beta_{6}:$ Spread | $0.260^{* * *}$ | $(0.0579)$ | $0.264^{* * *}$ | $(0.0588)$ | $0.226^{* * *}$ |  |
| $\beta_{7}:$ Number of dealer | $-0.182^{* *}$ | $(0.0900)$ | $-0.187^{* *}$ | $(0.0947)$ | 0.0745 |  |
| $\beta_{8}:$ Lagged number of customers | $0.158^{* * *}$ | $(0.0513)$ | $0.123^{* *}$ | $(0.0522)$ | 0.0655 |  |
| $\beta_{9}:$ Coupon | $0.940^{* * *}$ | $(0.176)$ | $0.973^{* * *}$ | $(0.186)$ | $0.985^{* * *}$ |  |
| $\beta_{10}:$ Supply | $1.043^{*}$ | $(0.550)$ | 0.838 | $(0.669)$ | 0.0547 |  |
| Extra controls | - |  | $\checkmark$ |  | $\checkmark$ |  |
| Adjusted $R^{2}$ | 0.375 |  | 0.389 |  | 0.411 |  |
| Observations | 326 |  | 326 |  | 326 |  |

Appendix Table A2 shows the estimation results of regressing the observed number of participating customers in an auction on a series of explanatory variables using data from the beginning of 2014 to the end of 2021 in column (OLS1). "Cash-futures basis trade" is an indicator variable equal to 1 if buying a bond at auction and shorting the future is profitable (calculated as in Hazelkorn et al. 2022). "CIP basis trade" captures deviations from covered interest parity using the 10 -year cross-currency swap basis with the U.S. dollar. "Benchmark status" is an indicator equal to 1 if the issued bond is on the run and 0 otherwise. "MPC" and "QE"" capture conventional and unconventional monetary policy, respectively. "Spread" is the high-minus the low-trading price for the bond being auctioned. "Number of dealers" and "Lagged number of customers" are the number of dealers who participate at auction and the number of customers who participated in the previous auction. "Coupon" is the coupon rate on the bond being issued. Supply is the residual from regressing the log-supply on the bond maturity at issuance. In column (OLS2) we add additional controls that capture the interest rate environment and expectations about the stance of monetary policy, using eight points on the OIS curve. In column (Year-FE) we include year fixed effects, in addition. Standard errors are in parenthesis.

Appendix Table A3: Predictors of customer participation-Individual level

| Participation | (OLS) |  | (Bidder-FE) |  | (Bidder-Year-FE) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\beta_{1}:$ Cash-futures basis trade | 0.0189 | $(0.0304)$ | -0.00307 | $(0.0251)$ | -0.0108 | $(0.0226)$ |
| $\beta_{2}:$ CIP | $-0.00259^{* *}$ | $(0.000861)$ | -0.000521 | $(0.00141)$ | -0.00120 | $(0.00179)$ |
| $\beta_{3}:$ Benchmark status | -0.0176 | $(0.0137)$ | -0.0197 | $(0.0135)$ | -0.0180 | $(0.0130)$ |
| $\beta_{4}:$ MPC | -0.0413 | $(0.0329)$ | -0.0619 | $(0.0314)$ | -0.0611 | $(0.0330)$ |
| $\beta_{5}:$ QE | -0.0218 | $(0.0161)$ | -0.0312 | $(0.0178)$ | $-0.0366^{*}$ | $(0.0161)$ |
| $\beta_{6}:$ Spread | $0.0170^{* * *}$ | $(0.00251)$ | $0.0127^{* *}$ | $(0.00441)$ | $0.00996^{* *}$ | $(0.00369)$ |
| $\beta_{7}:$ Number of dealers ${ }_{d}$ | $0.0096^{* *}$ | $(0.00483)$ | 0.0070 | $(0.0115)$ | $0.0136^{*}$ | $(0.00663)$ |
| $\beta_{8}:$ Lagged-participation ${ }_{i}$ | $0.532^{* * *}$ | $(0.0106)$ | $0.230^{* * *}$ | $(0.0521)$ | 0.0354 | $(0.0300)$ |
| $\beta_{9}:$ Coupon | $0.042^{* * *}$ | $(0.00791)$ | $0.0522^{* * *}$ | $(0.0140)$ | $0.0549^{* * *}$ | $(0.0144)$ |
| $\beta_{10}:$ Supply | $0.0528^{*}$ | $(0.0236)$ | -0.00198 | $(0.0367)$ | 0.00887 | $(0.0296)$ |
| Extra controls | $\checkmark$ |  | $\checkmark$ |  | $\checkmark$ |  |
| Adjusted $R^{2}$ | 0.289 |  | 0.427 |  | 0.510 |  |
| Observations | 6,567 |  | 6,551 |  | 6,548 |  |

Appendix Table A3 is analogous to Appendix Table A2, but zooms in on customer bidding participation at the individual level. We regress an indicator for whether customer $i$ participated in an auction ("Participation") on the same explanatory variables as in Appendix Table A2 only that we replace the number of customers who participated in the previous auctions by whether customer $i$ participated in the previous auction, called "Lagged-participation." To take the time variation in the set of potential customers into account, we use data from all auctions between the first and last time we observe the customer bidding at auction to construct all customer-specific participation and lagged participation indicators. In column (Bidder-FE) we include a bidderfixed effect and, in column (Bidder-Year-FE), we include a year-bidder fixed effect. Standard errors are in parenthesis. They are clustered at the bidder level in columns (Bidder-FE) and (Bidder-Year-FE). Our preferred specification includes bidder-year fixed effects, analogous to column (Year-FE) in Appendix Table A2. As in Appendix Table A2, Spread and Coupon are the only significant predictors among the market-level explanatory variables. The coefficient of Lagged-participation is positive without controlling for the upward time trend in customer participation. However, when accounting for this trend, this coefficient becomes statistically insignificant, suggesting that customers are not more likely to participate in an auction based on their participation in the previous auction. The number of dealers is weakly statistically significant, which likely arises from the fact that a fiscal year does not start in January.

Appendix Table A4: Dealers do not demand more when a dealer exits the market

|  | Demand in C\$ |  | Demand in \% of supply |  |
| :--- | :---: | :---: | :---: | :---: |
| exit | -12.22 | $(28.28)$ | -0.288 | $(0.929)$ |
| Adjusted $\mathrm{R}^{2}$ | 0.0221 | 0.0027 |  |  |
| Observations | 286 | 286 |  |  |

Appendix Table A4 provides evidence that (participating) dealers do not significantly adjust their auction demands when a dealer exits the market. Concretely, we regress the maximal amount any participating dealer demands in the closest auctions around a dealer exit (which we observe, but cannot display, in Appendix Table A1) on an indicator exit variable that is one post-exit and also exit-event fixed effects. We report the estimated coefficients and standard errors in parenthesis, with demands expressed in millions $\mathrm{C} \$$ and in percentages of supply. In both cases, the exit coefficient is statistically insignificant at $10 \%$. This is also the case when estimating separate regressions for each of the eight exit events.

Appendix Table A5: Differences in customer and dealer values

|  | P95 | Sum | Max | CI |
| :--- | :---: | :---: | :---: | :---: |
| QWA value | 0.00 | 0.00 | 0.00 | $[825,2906]$ |
| Max value | 0.06 | 0.00 | 0.93 | $[-836,2406]$ |
| Min value | 0.00 | 0.00 | 0.00 | $[615,1976]$ |

Appendix Table A5 shows the results from testing whether customer values are above dealer values. Columns P95, Sum, and Max present the p-values for the test statistics that take the 95th percentile, the sum of squared standardized differences, and the maximum difference across auctions of the average values of dealers less the lower bound of customer values. P-values are computed using the bootstrap. The confidence intervals (CI) are for the interval estimates of the mean difference. The QWA value is the average (within customers and dealers) of the individual participant's quantity-weighted average values. The Max value row compares the within group average values of the individual bidders' maximum value (at their first submitted step). The Min value row compares the within group average values of the individual bidder's minimum value (at their last submitted step).

Appendix Table A6: Predictors of customer participation, customer values and bids

|  | Values (OLS) |  | $\begin{gathered} \text { Bids } \\ \text { (OLS) } \end{gathered}$ |  | Values(Year-FE) |  | Bids(Year-FE) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta_{1}$ : Cash-futures basis trade | -1.038 | (1.340) | -0.185 | (1.352) | -1.554 | (1.301) | -0.729 | (1.308) |
| $\beta_{2}$ : CIP | 0.0426 | (0.0390) | 0.0387 | (0.0394) | -0.0101 | (0.0698) | -0.0117 | (0.0701) |
| $\beta_{3}$ : Benchmark status | 0.476 | (0.612) | 0.223 | (0.617) | 0.612 | (0.593) | 0.389 | (0.595) |
| $\beta_{4}$ : MPC | -1.942 | (1.478) | -2.017 | (1.491) | -1.889 | (1.418) | -1.984 | (1.425) |
| $\beta_{5}$ : QE | 2.896*** | (0.771) | 2.935*** | (0.778) | $2.345^{* * *}$ | (0.762) | 2.350*** | (0.766) |
| $\beta_{6}$ : Spread | 0.428*** | (0.113) | 0.457*** | (0.114) | 0.480*** | (0.112) | 0.519*** | (0.112) |
| $\beta_{7}$ : Coupon | $3.831^{* * *}$ | (0.356) | $3.679^{* * *}$ | (0.359) | $3.796^{* * *}$ | (0.351) | $3.612^{* * *}$ | (0.353) |
| $\beta_{8}$ : Number of dealers | $-0.467^{* *}$ | (0.181) | -0.470** | (0.183) | -0.218 | (0.256) | -0.265 | (0.258) |
| $\beta_{9}$ : Lagged number of customers | -0.0309 | (0.1000) | -0.0223 | (0.101) | 0.0491 | (0.0997) | 0.0595 | (0.100) |
| $\beta_{1} 0$ : Supply | 1.984 | (1.281) | 1.958 | (1.293) | 2.413* | (1.326) | 2.472* | (1.332) |
| Extra controls | $\checkmark$ |  | $\checkmark$ |  | $\checkmark$ |  | $\checkmark$ |  |
| Adjusted $R^{2}$ | 326 |  | 326 |  | 326 |  | 326 |  |
| Observations 0.481 |  | 0.468 |  | 0.491 |  | 0.477 |  |  |

Appendix Table A6 is similar to Appendix Table A2. In the "Values" and (OLS) column, we regress our estimated quantity-weighted average values of customers on all of the explanatory variables we used in Appendix Table A2 to predict customer participation. We add a year-fixed effect in the (Year-FE) column. In the "Bids" columns, we replace the value estimates by the observed quantity-weighted bids of customers. The data ranges from the beginning of 2014 to the end of 2021. Standard errors are in parenthesis.

Appendix Table A7: Predictors of customer participation and moments of the customer value distribution

|  | (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Average | Median | 5-Percentile | 95-Percentile | Std | Range |
| Cash-futures basis trade | -1.554 | -1.376 | -2.737* | -1.047 | $0.763^{* *}$ | 1.690* |
|  | (1.301) | (1.317) | (1.400) | (1.345) | (0.320) | (0.888) |
| CIP | -0.0101 | -0.0111 | 0.0343 | -0.0328 | -0.0268 | -0.0671 |
|  | (0.0698) | (0.0707) | (0.0751) | (0.0721) | (0.0172) | (0.0476) |
| Benchmark status | 0.612 | 0.465 | 1.270** | 0.542 | -0.261* | -0.728* |
|  | (0.593) | (0.600) | (0.637) | (0.612) | (0.146) | (0.404) |
| MPC | -1.889 | -1.980 | -0.820 | -2.361 | -0.531 | -1.542 |
|  | (1.418) | (1.435) | (1.525) | (1.465) | (0.348) | (0.968) |
| QE | $2.345^{* * *}$ | $2.335^{* * *}$ | $2.309^{* * *}$ | $2.406^{* * *}$ | -0.0162 | 0.0976 |
|  | (0.762) | (0.772) | (0.820) | (0.788) | (0.187) | (0.520) |
| Spread | 0.480 *** | $0.527^{* * *}$ | 0.0957 | $0.613^{* * *}$ | $0.174^{* * *}$ | $0.517^{* * *}$ |
|  | (0.112) | (0.113) | (0.120) | (0.115) | (0.0274) | (0.0762) |
| Number of dealers | -0.218 | -0.229 | -0.0920 | -0.250 | -0.0534 | -0.159 |
|  | (0.256) | (0.259) | (0.276) | (0.265) | (0.0630) | (0.175) |
| Lagged number of customers | 0.0491 | 0.0508 | 0.0443 | 0.0493 | 0.00298 | 0.00500 |
|  | (0.0997) | (0.101) | (0.107) | (0.103) | (0.0245) | (0.0680) |
| Coupon | $3.796^{* * *}$ | $3.718^{* * *}$ | 3.671*** | $4.113^{* * *}$ | 0.169* | 0.442* |
|  | (0.351) | (0.356) | (0.378) | (0.363) | (0.0864) | (0.240) |
| Supply | 2.413* | 2.298* | 2.939** | $2.280{ }^{*}$ | -0.298 | -0.658 |
|  | (1.326) | (1.342) | (1.426) | (1.370) | (0.326) | (0.905) |
| Extra controls | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Year fixed effect | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Adjusted $R^{2}$ | 0.525 | 0.521 | 0.420 | 0.557 | 0.232 | 0.247 |
| Observations | 326 | 326 | 326 | 326 | 326 | 326 |

Appendix Table A7 regresses moments of the estimated customer value distribution (at the lower bound) on all explanatory variables that we include to predict customer participation in Appendix Table A2, plus year-fixed effects. "Average" stands for the quantity-weighted average value, which approximates the quantity-weighted expected value. "Median" considers the median value, " $5-$ " and " 95 -percentile" show the 5 th and 95 th percentiles of the quantityweighted average values, "Std" is the standard deviation, and "Range" is the difference between the 95 th and 5 th percentiles. The data ranges from the beginning of 2014 to the end of 2021. Standard errors are in parenthesis.

Appendix Table A8: Customer commitment

| Year | No. of customers |  | Clearing Price |  | Revenue |  | Std of Revenue |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Com | SQ | Com | SQ | Com | SQ | Com | SQ |
| 2015 | 2 | 2.90 | 100.05 | 100.04 | 3.58 | 3.58 | 0.21 | 0.21 |
| 2016 | 2 | 2.46 | 100.28 | 100.24 | 3.60 | 3.60 | 0.18 | 0.18 |
| 2017 | 2 | 2.40 | 99.29 | 99.33 | 3.62 | 3.62 | 0.17 | 0.17 |
| 2018 | 2 | 3.06 | 98.74 | 98.85 | 3.62 | 3.62 | 0.32 | 0.32 |
| 2019 | 2 | 3.35 | 98.82 | 98.93 | 3.61 | 3.61 | 0.39 | 0.39 |
| 2020 | 6 | 4.4312 | 98.10 | 98.41 | 3.45 | 3.45 | 0.48 | 0.47 |
| 2021 | 6 | 3.98 | 98.15 | 98.45 | 3.50 | 3.51 | 0.43 | 0.42 |
| 2022 | 6 | 4.4304 | 98.01 | 98.32 | 3.38 | 3.38 | 0.40 | 0.40 |

Appendix Table A8 compares the counterfactual with customer commitment (Com) to the status quo (SQ), where customers make per-auction entry decisions. Dealer participation is fixed. Note that the equilibrium number of customer entrants depends on auction-specific profits for each of the 30 auctions per year across 8 years. To avoid computing the equilibrium in all of these auctions for each possible number of customers, we utilize a selected sample of five auctions. These auctions are strategically chosen to align the number of customers with percentiles (5th, 25 th, 50 th, 75 th, and 95 th) of the customer participation distribution since 2014, while the quantity sold approximates the average amount. When calculating profits, surpluses, and prices for each year, we re-weight the predictions from these five auctions to match the composition of auctions in that specific year. Expected revenues are in C $\$$ billions.

Appendix Figure A2: Primary auctions in different countries


Appendix Figure A2, taken from Muller (2019), shows an overview of how different countries issue debt. Towards the left are countries like Canada, which heavily relies on dealers to make markets. Towards the right are countries such as the U.S., which lets anyone participate in primary auctions.

Appendix Figure A3: What a dealer sees when bidding


Appendix Figure A3 shows a screenshot of what a dealer sees when placing its bids, either for its own account or on behalf of a customer.

Appendix Figure A4: Auction allotment by investor class for U.S. government bond auctions


Appendix Figure A4 shows the auction allotment in percentage of supply in U.S. government bond auctions from the beginning of 2010 to the end of January 2022 for broker/dealers (plus) and for investment funds (circle). Broker/dealers include primary dealers, other commercial bank dealer departments, and other non-bank dealers and brokers; investment funds include mutual funds, money market funds, hedge funds, money managers, and investment advisors. To create this graph, we use public data from TreasuryDirect.org, available at https://home.treasury.gov/ data/investor-class-auction-allotments, accessed on July 19, 2023.

Appendix Figure A5: Purchased amount by dealers, customers, and hedge funds


Appendix Figure A5 shows the distribution of how much dealers, customers, and hedge funds win (as a group) in percentage of the total amount issued across all bond auctions in our sample for each year from 1999 to 2022.

Appendix Figure A6: Purchased amount by investor groups


Appendix Figure A6 shows a binned scatter plot of how much each investor group wins in percentage of the total supply bought by non-dealers from 1999 to 2022.

Appendix Figure A7: Primary dealers are above minimal bidding requirements


Appendix Figure A7 provides evidence that primary dealers are, with rare exception, above the minimal bidding limit of $10 \%$, which is required to maintain their primary dealer status, conditional on market participation in a given year. In addition, supervisory data show that there have been extremely few violations in the past decade. This suggests that Canadian dealers do not face the dynamic trade-off, noted by Rüdiger et al. (2023), according to which dealers forgo one-shot auction surpluses in order to fulfill the minimal bidding requirements that must be met over a longer horizon. Concretely, the figure shows the distribution of the maximal amount an active primary dealer demands in an auction (as percentage of supply), where a primary dealer is active if they place at least one bid over the course of an entire year and the maximal demand is zero if the dealer does not participate in an auction. The distribution is taken over auctions and primary dealers. Outliers are excluded.

Appendix Figure A8: Random matching of customers to dealers


Appendix Figure A8A shows the distribution of how many dealers a customer places a bid through within an auction. The median is 1 . Figure A8B plots the distribution of the number of unique dealers used by a customer in all auctions in the data (in pink) and the number of unique dealers that would be predicted for each customer under random matching (in blue). The model prediction fixes the maximum number of dealers at the median number of dealers across years (12). The predicted distribution of the number of dealers used by each customer contains predictions from a single simulation, drawing independently one of the 12 dealers with equal probability for each customer each time they bid. The histogram plots the total number of unique dealers matched to each customer in the simulated sample. The distributions are broadly similar, but the modelpredicted distribution somewhat overestimates the probability that a customer sometimes uses all of the possible dealers.

Appendix Figure A9: Revenue and price effect-adding back dealers


Appendix Figure A9A shows hedge fund (HF) participation probabilities (in percentage points) in every 15th auction, from 2014 onward, in the status quo (on the x-axis) and the counterfactual in which we add back the two dealers who left (on the y-axis). Figure A9B shows the expected number of dealers and HFs that participate in each auction, in the status quo and the counterfactual. Figure A9C shows the distribution of the expected auction revenues in million C\$. Figure A9E is a time series of the percentage change in the expected price when going from the status quo to the counterfactual. Prices are in $\mathrm{C} \$$ with a face value of 100 .

Appendix Figure A10: Expected price and number of dealers for auctions p5 and p95


Appendix Figure A10 is analogous to Figure 7. It shows how the range of the expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the number of dealers increases from 7 to 14 in two representative auctions, which issue the average supply with 1 participating customer in A10A, and with 4 participating customers in A10B. Here, 1 and 4 are the 5th and 95 th percentiles of the observed distribution of the number of participating customers. In theory, there is one counterfactual equilibrium for each fixed number of dealers. In practice, we determine a range of prices (marked in black), given that our numerical procedure to determine the counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (for details, see Richert 2021). The blue horizontal line shows the average observed bid, which is close to the observed market-clearing price.

Appendix Figure A11: Varying number of dealers: Alternative distribution of maximal quantities


Appendix Figure A11 is analogous to Figure 7, but this figure uses the alternative distribution of the maximal quantities of Appendix D.1. Figure A11 shows how the range of the expected prices (in $\mathrm{C} \$$ ) at which an auction clears varies as the number of dealers increases in an auction that issues the average supply with medium customer participation, i.e., 3 customers. In theory, there is one expected price for each fixed number of dealers. In practice, we determine a range of prices (marked in black), given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert 2021). The blue horizontal line shows the average observed bid, which is close to the observed market-clearing price.

Appendix Figure A12: No dealer commitment


For every third auction starting in 2014, Figure A12A shows the probability, in percentage points, that a customer participates in the status quo (on the x-axis) and the counterfactual in which we add back the two dealers who left (on the y-axis). Figure A12B shows the expected number of dealers and customers that participate in each auction in the status quo and the counterfactual. Figure A12C shows the distribution of the percentage change in the expected auction revenue. Figure A12D is the time series of the percentage change in the expected price when going from the status quo to the counterfactual. Prices are in $\mathrm{C} \$$ with a face value of 100 .


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[^1]:    ${ }^{1}$ Unprecedented market turmoil in March 2020 triggered a policy debate on whether to reform Treasury market rules (e.g., Logan 2020; Ackerman and Hilsenrath 2022; Grossman and Goldfarb 2022).

[^2]:    ${ }^{2}$ Strictly speaking, there are two types of dealers in Canada. Most dealers are primary dealers, but some are government security distributors. These are smaller dealers, who also place bids on behalf of customers, but face fewer market-making requirements. For simplicity, we do not distinguish between groups.
    ${ }^{3}$ Prices are expressed with three decimal places, e.g., C $\$ 99.999$, and quantities must be stated in multiples of $\mathrm{C} \$ 1,000$. The minimum demand is $\mathrm{C} \$ 100,000$.

[^3]:    ${ }^{4}$ Shortly after the auction clears, the clearing price and some additional aggregate summary statistics about the auction are publicly announced. This implies that no one has the incentive to participate in the auction only to learn about the market price, without wanting to win.
    ${ }^{5}$ For the most recent terms and conditions, see https://www.bankofcanada.ca/wp-content/uploads/ 2016/08/standard-terms-securities180816.pdf.

[^4]:    ${ }^{6}$ In stock markets, hedge funds have long been active, and their role in these markets has been discussed in the academic literature (e.g., Stein 2009).

[^5]:    ${ }^{7}$ For instance, Banegas et al. (2021) acknowledge challenges in assessing the extent of hedge funds' Treasury sell-offs during March 2020, given the lack of detailed data on their cash and derivatives positions.

[^6]:    ${ }^{8}$ For example, the OECD raised the concern that primary dealers might not have the capacity to commit capital to debt markets post-global financial crisis and would exit (Blommestein et al. 2010); Bloomberg reports on dealer exit in the U.K., in particular the exit of Credit Suisse Group AG and Societe General SA in 2016. More recently, Association for Financial Markets highlights the exit of 7 primary dealers in the Eurozone between June 2021 and January 2022; The Economist and the Wall Street Journal discuss the rising role of hedge funds in the U.S.; Reuters highlights the increasingly important role of hedge funds in European Treasury markets. All websites were accessed on 07/05/2024.
    ${ }^{9}$ This would align with anecdotal evidence. For instance, according to research by Greenwich Associatesa leading financial consultancy—regulations implemented after the 2008 global financial crisis caused a general (voluntary) retreat from Canadian debt markets in 2014 (Altstedter, 2014).

[^7]:    ${ }^{10}$ Since hedge funds in the Canadian fixed income market are large international players, it is impossible to capture all of their potential uses of Canadian bonds and how these interact with their activities in global markets.

[^8]:    ${ }^{11}$ We could distinguish between different types of customers, for instance hedge funds versus other customers. Theoretically, such an extension would be straightforward. However, empirically, non-hedge fund customers play such a small role that the cost of complicating the model and increasing measurement error (due to more bidder groups in the resampling procedure described below) outweighs the benefit of separating non-hedge fund customers from other customers.
    ${ }^{12}$ In principle, these costs could be constant over time or within customers. However, both of these alternative assumptions imply entry patterns that are inconsistent with the data.

[^9]:    ${ }^{13}$ Rüdiger et al. (2023) analyze inter-temporal incentives of Argentinian primary dealers to forgo shortterm auction gains to fulfill the longer-term minimal bidding requirements necessary to maintain their dealer status. Appendix Figure A7 provides evidence that Canadian primary dealers are above minimal bidding requirements and, therefore, do not face the same inter-temporal trade-off as in Rüdiger et al. (2023). In addition, if these bidding requirements played an important role, dealers should be willing to pay more in subsequent auctions when winning less than expected, for which we find no evidence in our setting.

[^10]:    ${ }^{14}$ The assumption of random matches simplifies the equilibrium conditions and the estimation procedure. In Appendix Figure A8, we provides some evidence that random matching is a reasonable approximation of reality. Further, we provide evidence that since 2014, dealers have not been leaving the market because they systematically observe fewer customers than dealers who remain in the market, which would violate the random matching assumption. We find that on average the total number of customers that an exiting dealer observes in a year (because the customer bids via this dealer) is not statistically different from the average number of customers that the remaining dealers observe. The test's p-value is 0.28 for auctions post-2014.

[^11]:    ${ }^{15}$ As a comparison, when dealers do not update their bids, the market-clearing price will weakly increase in all states of the world if the customer increases their quantity $q_{k}$ at price $b_{k}$ by a little bit, assuming that all other participants play as in the equilibrium.
    ${ }^{16}$ Alternatively, we could assume that customers only think that this is the case, even though the dealer responds to the full curve.

[^12]:    ${ }^{17}$ Since there is no cost of submitting steps, all bidders choose the maximal number of steps in equilibrium: $K=\bar{K}$. To rationalize the variation in the number of steps in the data, we could follow Kastl (2011) and include a private cost of computing and submitting steps. We refrain from doing so since this does not add additional economic insights; instead, we treat the observed K as the bidder-specific $\bar{K}$.

[^13]:    ${ }^{18}$ Hortaçsu and Kastl (2012) also resample within auctions. Their data features a similar number of unique bidders and slightly fewer bids per auction than our sample.

[^14]:    ${ }^{19}$ Formally: $\operatorname{Pr}\left(\boldsymbol{N}^{\boldsymbol{d}}=N^{d}\right)=\binom{\bar{N}^{d}}{N^{d}}\left(N^{d} / \bar{N}^{d}\right)^{N^{d}}\left(1-\left(N^{d} / \bar{N}^{d}\right)^{\bar{N}^{d}-N^{d}}\right.$.

[^15]:    ${ }^{20}$ Concretely, for customers we estimate a set of $\beta^{h}$, using the following criterion function: $Q^{\prime}\left(\beta^{h}\right)=$ $Q\left(\beta^{h}\right)-\inf _{\beta^{\prime}} Q\left(\beta^{\prime}\right)$ with $\left.\left.Q\left(\beta^{h}\right)=\left(\frac{N_{t}^{h}}{N^{h}}-H\left(\mathbb{E}\left[\underline{\boldsymbol{T}}_{t i}^{h} \mid N^{d}\right] ; \beta^{h}\right)\right)_{+}\right)^{2}+\left(\frac{N_{t}^{h}}{N^{h}}-H\left(\mathbb{E}\left[\overline{\boldsymbol{T S}}_{t i}^{h} \mid N^{d}\right] ; \beta^{h}\right)\right)_{-}\right)^{2}$, where $H$ is the CDF of an exponential distribution with parameter $\beta^{h}$. As the sample size grows, all points in the identified set produce criterion values of zero. To account for finite sample errors, we define a contour set of level $c_{n}$, and estimate the parameter set $\left\{\beta^{h} \mid Q^{\prime}\left(\beta^{h}\right) \leq c_{n}\right\}$. We choose the cutoff $c_{n}$ proportionally to the number of auctions in our sample $c_{n}=\log (645) / 645$, inspired by Chernozhukov et al. (2007).

[^16]:    ${ }^{22}$ For small changes in the number of dealers, this approach may be conservative, given that dealers do not increase their maximum quantity demanded when a dealer exits the market, as shown in Appendix Table A4.

[^17]:    ${ }^{23}$ Appendix Figure A9 shows by how much the expected price and revenue change in the counterfactuals relative to the status quo. Given that all auctions are relatively competitive, these effects are relatively small.

[^18]:    ${ }^{24}$ These large changes rely on the assumption that the dealers are limited in their increases in demand around an exit. This is consistent with evidence when a dealer exits the market (illustrated in Appendix Table A4). Appendix Figure A10 shows the analogous results for other auctions, specially p5 and p95.
    ${ }^{25}$ Formally, let $p r$ denote the probability that a customer participates and pr_data $=\frac{1}{T} \sum_{t} N_{t}^{h} / \bar{N}^{h}$. The competition effect $=\mathbb{E}\left[\right.$ revenue $\left.\mid p r=p r_{-} d a t a\right]-\mathbb{E}\left[\right.$ revenue $\left.\left\lvert\, p r=p r_{-} d a t a-\frac{1}{\frac{1}{T} \sum_{t} N_{t}^{h}}\right.\right]$ is the difference between the expected auction revenue with the observed customer entry probabilities and the expected revenue when we remove one customer, in expectation. The volatility effect $=\mathbb{E}\left[\right.$ revenue $\left.\mid p r=p r \_d a t a\right]-$ $\mathbb{E}[\mathbb{E}[\boldsymbol{r e v e n u e} \mid p r]]$ is the expected revenue with observed entry probabilities minus the expectation of expected revenues over the distribution of customer entry probabilities.

[^19]:    ${ }^{26}$ To illustrate why normalizing the supply reduces the supply changes we propose, consider one auction that supplies $\mathrm{C} \$ 4$ billion and one auction that supplies $\mathrm{C} \$ 1$ billion in the status quo. Assume that our rule would suggest shifting $10 \%$ of the $\mathrm{C} \$ 4$ billion in bonds from the first to the second auction. This would mean increasing the supply in that auction by C $\$ 400$ million - a massive percentage increase of $40 \%$.

[^20]:    ${ }^{27}$ The point estimate from regressing the average quantity-weighted value of all bidders, in an auction that issues a bond with $\mathrm{C} \$ 100$ face value, on the predicted number of participating customers is -0.03 . The

[^21]:    ${ }^{28}$ We calculate the basis as in Hazelkorn et al. (2022). Specifically, to determine profitability of buying bonds at auction and shorting the corresponding futures contract, we approximate the bond's value as the quantity-weighted average price of winning bids (by customers) plus the accrued interest between the auction date and the futures' expiration date. If this price is below the price of the futures contract multiplied by a conversion rate that is determined by the Bank of Canada, we say that a basis trade is profitable. The conversion rates are published here: https://www.m-x.ca/en/markets/interest-rate-derivatives/ bond-futures-conversion-factor, accessed on 08/23/2023.

[^22]:    ${ }^{29}$ Furthermore, even though transactions data only starts in 2016, we did experiment with including interdealer repo rates and repo spreads to capture the cost of overnight borrowing, but the coefficients are not statistically significant. The same is true for 1- and 3-month Canadian Dollar Offered Rates, which are important interest rate benchmarks (e.g., McRae and Auger 2018), but are not statistically significant.

[^23]:    ${ }^{30}$ For simplicity, our model does not rationalize such updates, but an extended model based on Hortaçsu and Kastl (2012) could.
    ${ }^{31}$ Ideally, we would choose a dealer that observed an identical bid. Given our limited sample size, however, this event is extremely unlikely. To reflect customer uncertainty about the value of the dealer observing their bid, we set a bandwidth and define similar bids using the quantity-weighted average bids.

[^24]:    ${ }^{32}$ This includes sampling bids from dealers that later become informed and placed a later bid but that were not selected in the simulated residual supply curve in the customer resampling step.

