BANK OF CANADA
BANQUE DU CANADA

Staff Discussion Paper/Document d'analyse du personnel—2024-9

Last updated: July 8, 2024

Deriving Longer-Term Inflation Expectations and Inflation Risk Premium Measures for Canada

by Bruno Feunou and Zabi Tarshi

Financial Markets Department
Bank of Canada
BFeunou@bankofcanada.ca, ZTarshi@bankofcanada.ca

DOI: https://doi.org/10.34989/sdp-2024-9 | **ISSN** 1914-0568

Acknowledgements

We are grateful to Virginie Traclet and Gitanjali Kumar Garriott for their helpful comments and suggestions since the very beginning of this work. We also thank colleagues from the Financial Markets Department for insightful discussions.

Abstract

We present two models for long-term inflation expectations and inflation risk premiums for Canada. First, we estimate inflation expectations using a vector autoregressive model based on the relationship of inflation with both the unemployment gap and the term structure of the Government of Canada nominal bond yields. Then we estimate the inflation risk premium by regressing the nominal term premium on a set of inflation risk factors. We find that our model-implied measure of inflation expectations generally follows a trend similar to that of break-even inflation rates. We also find that the estimated inflation risk premium is negative or near zero through most of the sample period because most of this period was dominated by low inflation and low growth, with investors concerned about deflation. However, the model-implied inflation risk premium becomes positive in 2021. Because real return bonds will eventually disappear in Canada, a market-derived indicator for long-term inflation expectations is particularly relevant for central bankers. Similarly, capturing the individual components of the nominal term premium can be highly useful from a policy perspective.

Topics: Econometric and statistical methods
JEL codes: C58, E43, E47, G12

Résumé

Nous présentons deux modèles servant à mesurer les attentes d'inflation à long terme et les primes de risque d'inflation pour le Canada. Tout d'abord, nous estimons les attentes d'inflation à l'aide d'un modèle vectoriel autorégressif basé sur la relation entre l'inflation et deux facteurs, soit l'écart du taux de chômage et la structure par terme des taux d'intérêt des obligations du gouvernement du Canada. Ensuite, nous estimons la prime de risque d'inflation en effectuant une régression de la prime de terme nominale sur un ensemble de facteurs de risque d'inflation. Nous constatons que la mesure des attentes d'inflation implicite dans notre modèle suit, en général, une tendance semblable à celle du taux d'inflation neutre. Nous constatons également que la prime de risque d'inflation estimée est négative ou près de zéro sur presque toute la période considérée du fait que celle-ci était marquée par un taux d'inflation bas, une faible croissance et, du côté des investisseurs, la crainte d'une déflation. Cependant, la prime de risque d'inflation implicite dans le modèle devient positive en 2021. Étant donné que les obligations à rendement réel finiront par disparaître au Canada, un indicateur des attentes d'inflation à long terme basé sur le marché est d'un intérêt particulier pour les banques centrales. De même, il peut être très utile de tenir compte des composantes individuelles de la prime de terme nominale du point de vue des politiques.

Sujet : Méthodes économétriques et statistiques

Codes JEL : C58, E43, E47, G12

1. Introduction

In Canada and other economies, break-even inflation rates (BEIRs) are typically used to proxy for inflation expectations, even though it is well known that BEIRs are affected by a certain degree of illiquidity and reflect the presence of an inflation risk premium—which suggests that BEIRs are, at best, a proxy for inflation compensation.

Inflation expectations matter because they can cause actual inflation. Thus, policy-makers need to measure inflation expectations and inflation risk premiums as well as their drivers because it helps inform monetary policy.

A term structure of inflation expectations can help break down the yield of a Government of Canada (GoC) bond into its two drivers:

- the nominal expectations (reflecting inflation expectations and the real short-term rate)
- the nominal term premium (reflecting the real term premium and inflation risk premium)

Since the risk premium components can reflect uncertainty about growth or inflation, capturing these two key components separately is desirable from a central bank perspective. To that end, we develop an empirical approach to estimate market-implied long-term (10-year) inflation expectations and the inflation risk premium. We focus on the long-term inflation expectations because having this measure well anchored is important for policy-makers. Depending on economic developments, the degree to which long-term inflation expectations are anchored can change, which can lead to inflation expectations becoming unmoored. Moreover, having such a market-based indicator of inflation expectations will be particularly useful because BEIRs will eventually disappear in Canada, given the federal government's announcement in the 2022 Fall Economic Statement that it would immediately stop issuing real return bonds.

This paper is organized as follows: Section 2 describes the methodology, data and results for modelling market-implied inflation expectations. Section 3 does the same for the inflation risk premium, and section 4 concludes.

2. The inflation expectations model

Our goal is to forecast long-term inflation expectations using a simple approach. To that end, we rely on a vector autoregressive (VAR) framework. We chose this modelling approach because it is one of the most common for analyzing the dynamic relationship between key macroeconomic variables. It is also useful for describing the dynamic behaviour of economic and financial time series (Killian and Murphy 2012).

Our different multivariate time-series models use information on:

- the nominal GoC yield curve
- the unemployment rate
- consumer price index (CPI) inflation

We use the term structure of nominal interest rates because it embeds inflation expectations at different horizons. In principle, an increase in inflation expectations would result in an increase of the nominal yield.

Conversely, an increase in the nominal yield could signal high borrowing costs and thus a reduction in demand and a decrease in inflation expectations. To reduce dimensionality and to summarize the informational content of the nominal curve, we extract the first three principal components, which are routinely interpreted as the level, slope and curvature factors. These factors have macroeconomic interpretations: the level is associated with long-term inflation expectations, the slope captures temporary business cycle conditions, and curvature is interpreted as an independent monetary policy factor (Dewachter and Lyrio 2006).

Duffee (2013) argues that no variables beyond current interest rates are necessary for forecasting future rates if bond markets are efficient because any information useful for predicting interest rates would rapidly be incorporated into current bond yields. However, a few recent studies find that some macroeconomic variables seem to have additional predictive power for bond returns that is not contained in yields.[1] Therefore, we also include two macroeconomic variables to account for real activity and inflation, which is in line with findings in the literature. More specifically, we use the unemployment gap as our measure of activity because the Phillips curve relationship implies that the unemployment gap contains information necessary for forecasting inflation expectations.

In our set-up, we stack the five variables of interest in the following state vector Z_t:

$$Z_t = (\pi_{t-1}, y_{t-1}, L_t, S_t, C_t)',$$

where π_{t-1} and y_{t-1} represent inflation and the unemployment gap, respectively. Inflation and the unemployment gap enter the state vector with *t-1* subscript because those variables reflect information from the previous month, even though the data are released at time *t* and this information is incorporated in the term structure of interest rates at time *t*. L_t reflects the level, S_t the slope and C_t the curvature of the GoC nominal interest rate term structure.

We define a VAR(*p*) process for the state vector such that:

$$Z_t = \omega + \varphi_1 Z_{t-1} + \ldots + \varphi_p Z_{t-p} + \varepsilon_t, \quad (1)$$

conditional on the *p* initial values of Z_0 , Z_{-1} ,..., $Z_{-(p-1)}$.

We make the standard assumption that $\varepsilon_t \mid Z_{t-1}, Z_{t-2}, \ldots, Z_{t-k} \sim N(0, \Omega)$, with ε_t serially uncorrelated and Ω a symmetric and positive definite matrix. Therefore, for each given lag (*p*), our model has $m + pm^2$ parameters in the conditional mean (where *m* reflects the dimension of Z_t vector and is equal to 5) and $m(m+1)/2$ parameters in the symmetric covariance matrix, Ω. As we increase the number of coefficients that need to be estimated, the forecast's estimation error will also increase.

We estimate the VAR for p = 3, 6, 9 and 12 to reduce model and estimation uncertainty.

Next, we generate s-step-ahead forecasts such that:

$$Z_{T+s|T} \equiv E(Z_{T+s} | Z_T, Z_{T-1,\ldots}) = \omega + \varphi_1 Z_{T+s-1|T} + \ldots + \varphi_p Z_{T+s-p|T} \quad (2)$$

[1] See, for example, Bauer and Hamilton (2018); Cieslak and Povala (2015); Greenwood and Vayanos (2014); Ludvigson and Ng (2009, 2010); Joslin, Priebsch and Singleton (2014); and De Pooter, Ravazzolo and Van Dijk (2010).

where $Z_{T+j|T} = Z_{T+j}$ for $j \leq 0$ is the optimal minimum mean square error s-step-ahead prediction given Z_t, $t \leq T$.

By iterating forward the equation above, the predictions are computed for s = 1, 2, etc.

2.1 Estimation methodology

We apply a direct regression technique focusing only on the inflation variable and aiming to minimize the forecast error at horizons of 1, 3, 5 and 10 years. Generally, the root mean squared error (RMSE) minimization can yield different results based on the choice of forecast horizon (short or long). However, if we assume we have the correct model, the forecast error of the RMSE minimization should be the same whether at 1 month or at 10 years. We focus on the CPI equation and how its own lag and the lags of other variables affect it.

We find the parameter of the VAR model that minimizes the sum of the mean squared error (MSE). The MSE for forecasting horizon s is calculated as follows:

$$MSE_{\hat{\pi},t+s} = \frac{\sum_{t=T0}^{T_1-s}(\pi_{t+s}-\pi_{t+s|t})^2}{T_1-s-T_0+1}, \qquad (3)$$

where T_0 denotes one month before the start of the evaluation period, T_1 is the end of the evaluation period, π_{t+s} is the realized inflation at horizon $t+s$ (where π_t reflects the inflation equation, i.e., the first element of the state vector Z_t), and $\pi_{t+s|t}$ is the conditional forecast of inflation at horizon $t+s$. RMSE is simply the square root of equation 4. This allows us to aim to minimize s-period-ahead forecast errors.

A final step involves introducing survey forecasts by minimizing the sum of the MSE in equation 3 and the gap between the model long-term forecast and the corresponding survey forecasts. Disciplining the forecasts from our model with a survey component is consistent with research that shows that long-term forecasts from surveys of professional forecasters proxy the underlying trends or shifts in the economy reasonably well (e.g., Faust and Wright 2013; Kozicki and Tinsley 1998; Wright 2013). Moreover, using survey data on inflation helps prevent unrealistic jumps in model-implied inflation expectations (Ormeño and Molnár 2015). Feunou and Fontaine (2012) also show that surveys provide an effective counterweight to the loss of parsimony associated with conditional mean models. Finally, using surveys of yield forecasts also reduces sampling uncertainty and lessens the bias in estimates of persistence parameters (Kim and Orphanides 2012).

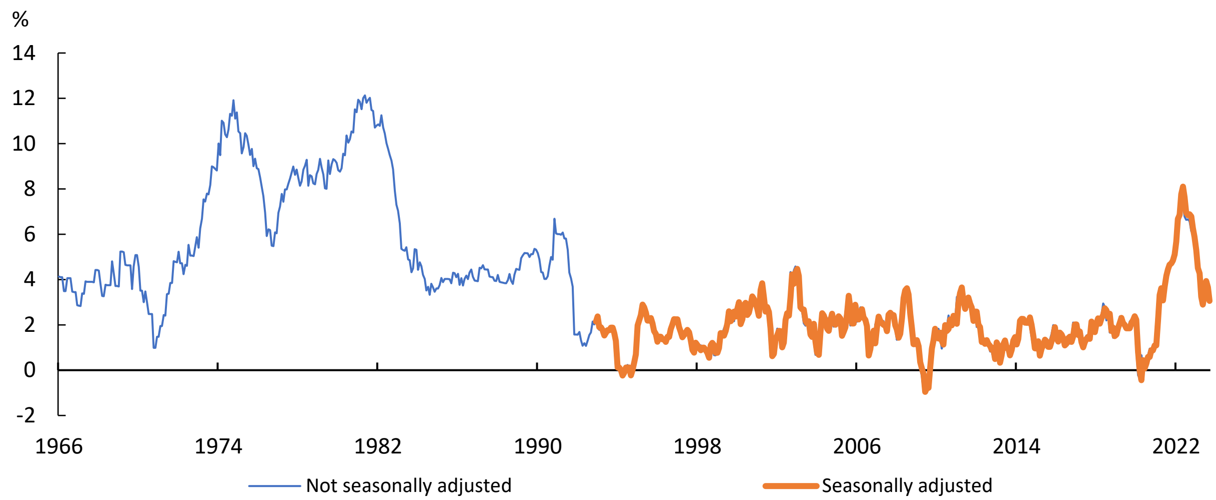
2.2 Data

Our sample period is from March 1966 to November 2023; thus, it covers the periods of high and volatile inflation that occurred before the implementation of the inflation-targeting framework in 1991 and the surge in inflation after the onset of the COVID-19 pandemic. **Table 1** lists the data series, sources and treatments applied before the estimation; **Chart 1**, **Chart 2**, **Chart 3** and **Chart 4** depict the series over the sample period.

Table 1: Data description

Series	Sources	Data transformation
Consumer price index inflation (**Chart 1**)	Statistics Canada Table 18-10-0006-01	Apply seasonal adjustment to pre-1992 data (see **Appendix**)
Employment rate (**Chart 2**)	Statistics Canada Table 14-10-0287-01	Construct unemployment gap as the difference between the unemployment rate and its long-term mean
Government of Canada bond yields with maturities averaged over 1–3 years, 3–5 years, 5–10 years and more than 10 years (**Chart 3**)	Statistics Canada Table 10-10-012222-01 and Bank of Canada	Apply principal component analysis to extract the first three components: level, slope and curvature (**Chart 4**)
Average 6-year-ahead CPI inflation expectations	Department of Finance Canada private sector survey	N/A

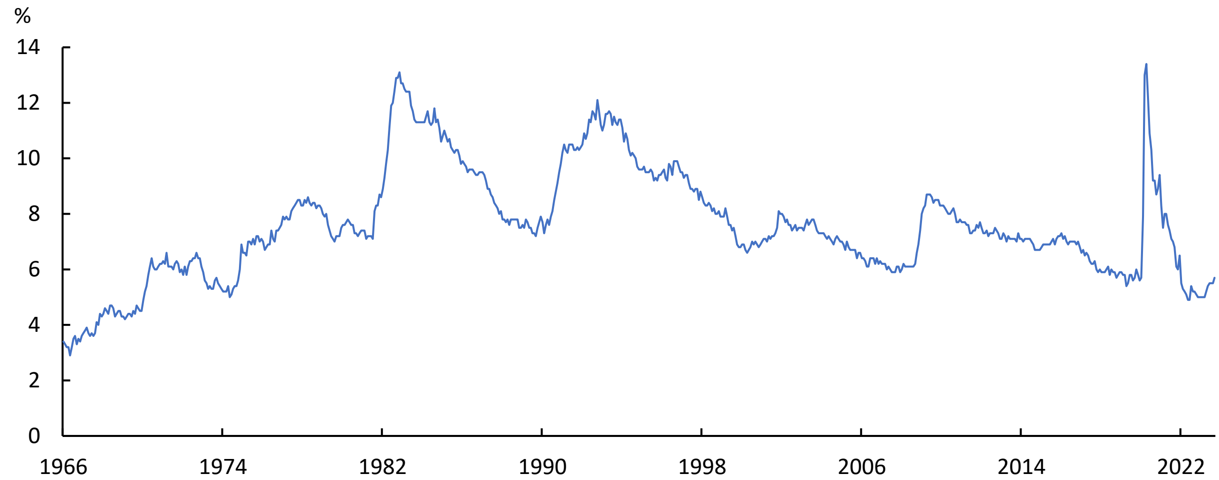
Chart 1: Consumer price index, year-over-year percentage change



Source: Statistics Canada

Last observation: November 2023

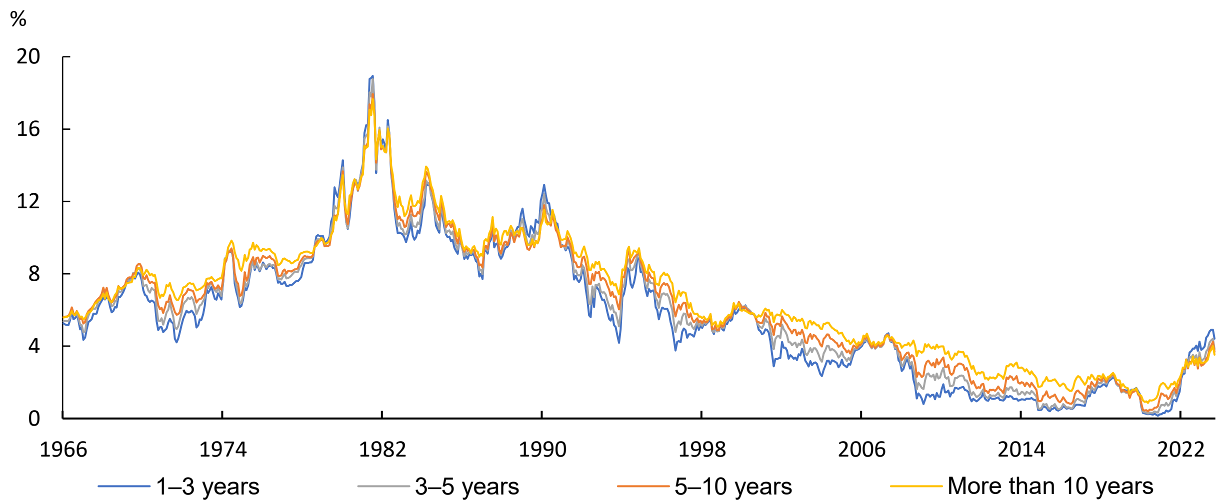
Chart 2: Unemployment rate in Canada, year-over-year percentage change



Source: Statistics Canada

Last observation: November 2023

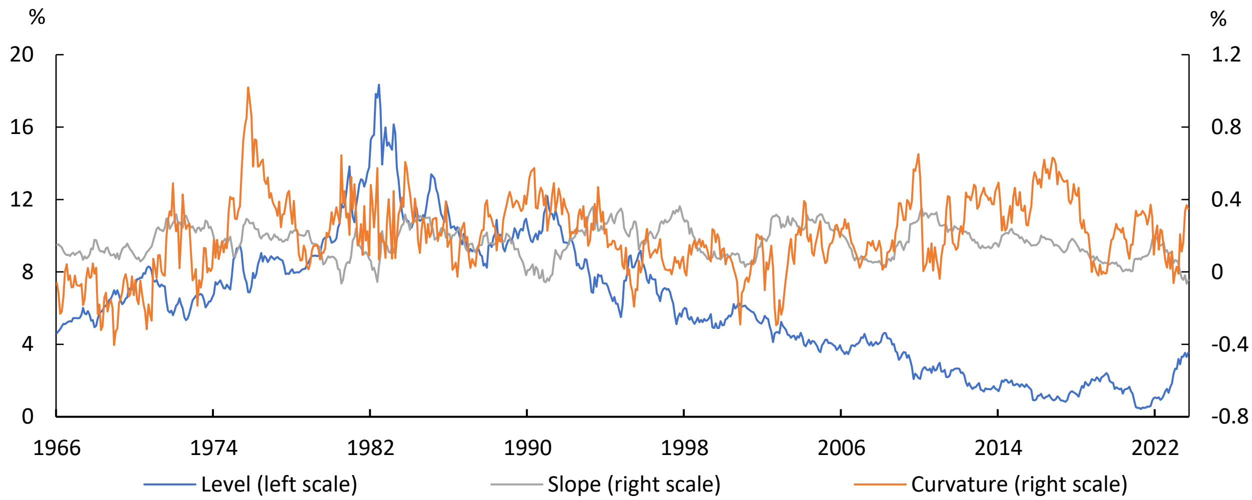
Chart 3: Government of Canada marketable bonds, average yield



Sources: Statistics Canada and Bank of Canada

Last observation: November 2023

Chart 4: Govenment of Canada yield factors



Sources: Statistics Canada and Bank of Canada calculations

Last observation: November 2023

2.3 Empirical results

We look at several measures to assess the performance of our models. We find that increases in the length of the lag are generally associated with notable improvements in the standard errors and RMSE.

Table 2 and **Table 3** show the RMSE and standard errors for the different models (i.e., different number of lags) across several forecast horizons. **Chart 5** shows the RMSE across models for the 10-year forecast horizon.

To illustrate the benefits of augmenting the model with survey forecasts, we show the 10-year forecast across the models without survey data (**Chart 6**) and with survey data (**Chart 7**). We find that inflation expectations are well behaved and monotone between different lags in both cases but that they vary more in the model without the survey component. **Chart 8** presents the average across models of survey-augmented, long-term inflation expectations. From the beginning of the inflation-targeting regime until 2020, long-term inflation expectations averaged 2%. However, inflation expectations fluctuated markedly over the past three to four years.[2]

Table 2: VAR(*p*) root mean squared error

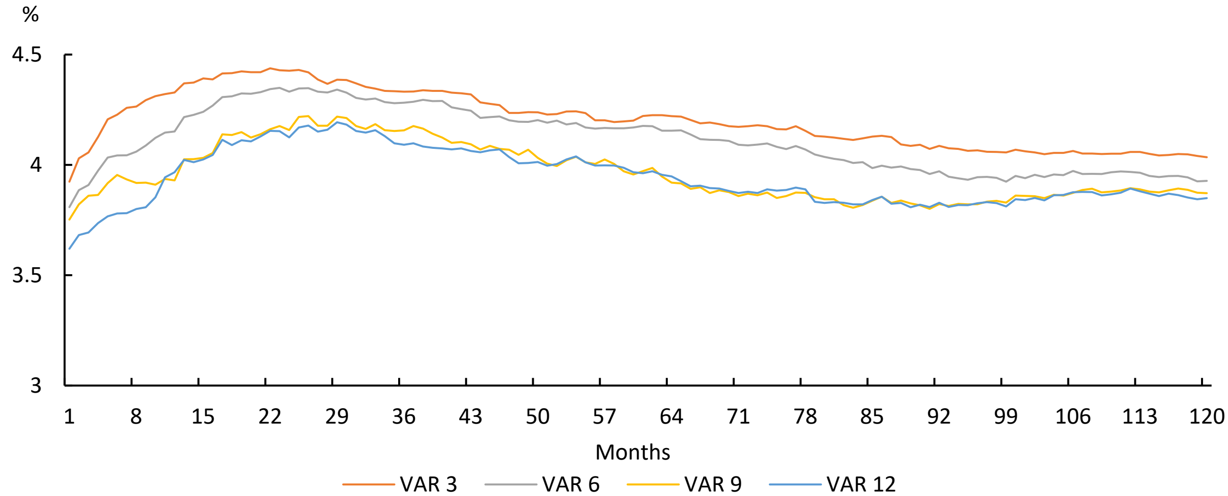
p	3	6	9	12
RMSE (1-year forecast horizon)	4.17	4.04	4.03	3.93
RMSE (3-year forecast horizon)	4.42	4.24	4.11	4.1
RMSE (5-year forecast horizon)	4.44	4.35	4.17	4.09
RMSE (10-year forecast horizon)	4.4	4.28	4.13	4.1

[2] Similar fluctuations were observed in the Canadian Survey of Consumer Expectations and the Business Outlook Survey, although these surveys gauge short- and medium-term expectations.

Table 3: VAR(*p*) standard error

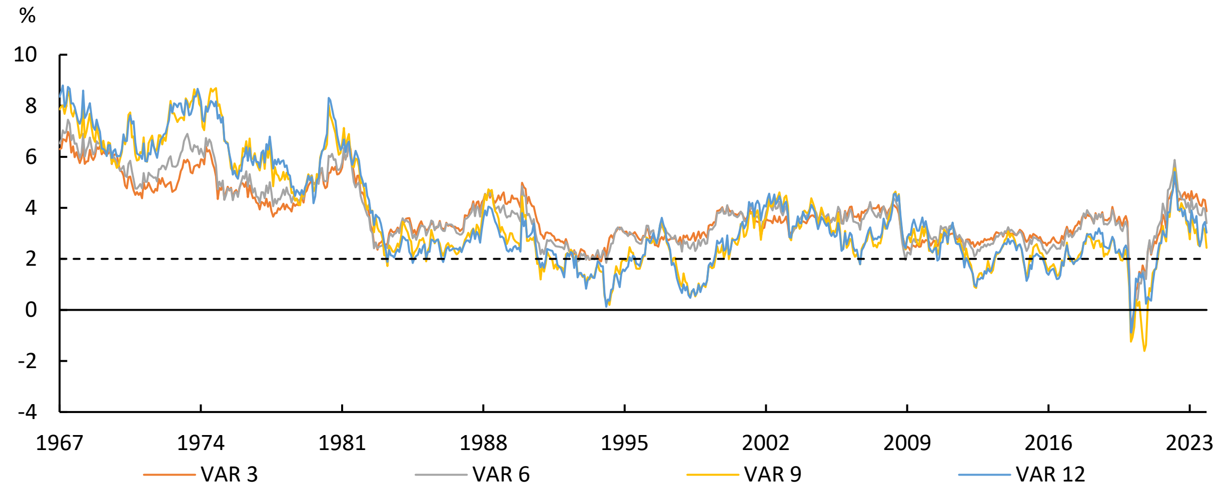
p	3	6	9	12
RMSE (1-year forecast horizon)	4.12	4.03	4.03	3.93
RMSE (3-year forecast horizon)	4.41	4.24	4.1	4.1
RMSE (5-year forecast horizon)	4.34	4.26	4.11	4.03
RMSE (10-year forecast horizon)	4.33	4.22	4.09	4.08

Chart 5: Root mean squared error at the 10-year horizon



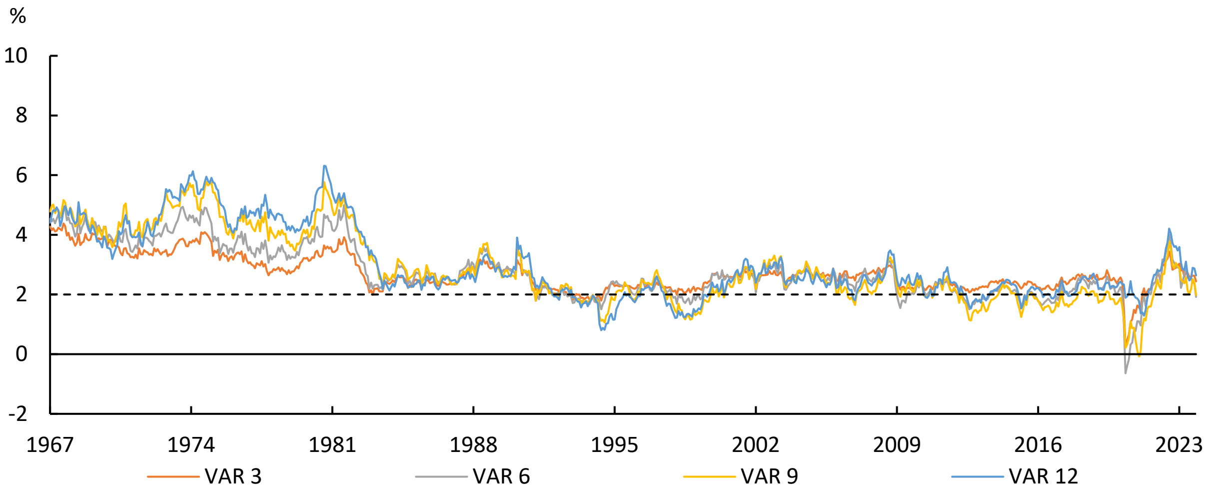
Source: Bank of Canada calculations Last observation: November 2023

Chart 6: Model-implied 10-year inflation expectations, without survey



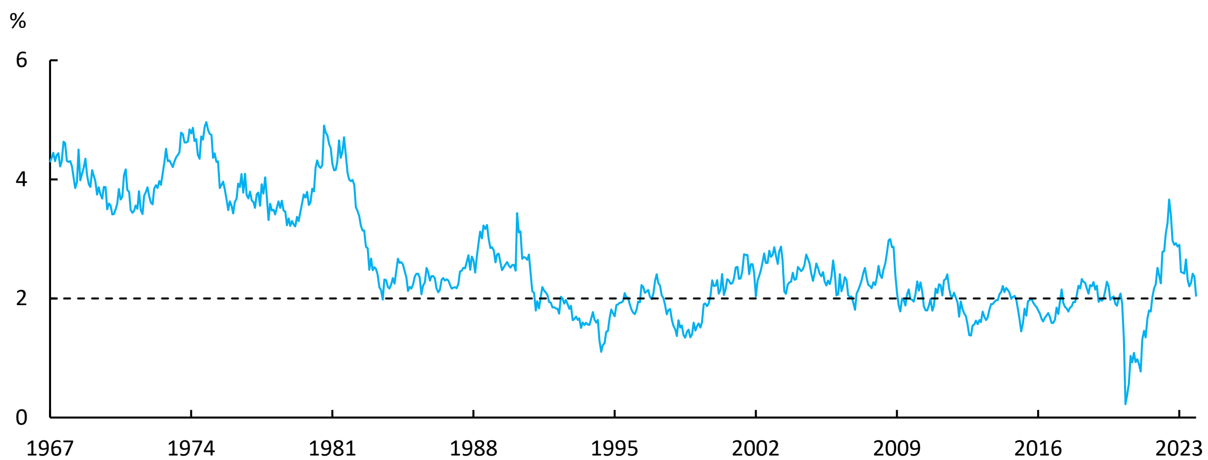
Source: Bank of Canada calculations Last observation: November 2023

Chart 7: Augmenting the model-implied 10-year inflation expectations with survey data



Sources: Department of Finance Canada and Bank of Canada calculations Last observation: November 2023

Chart 8: Model-implied 10-year inflation expectations, average across models



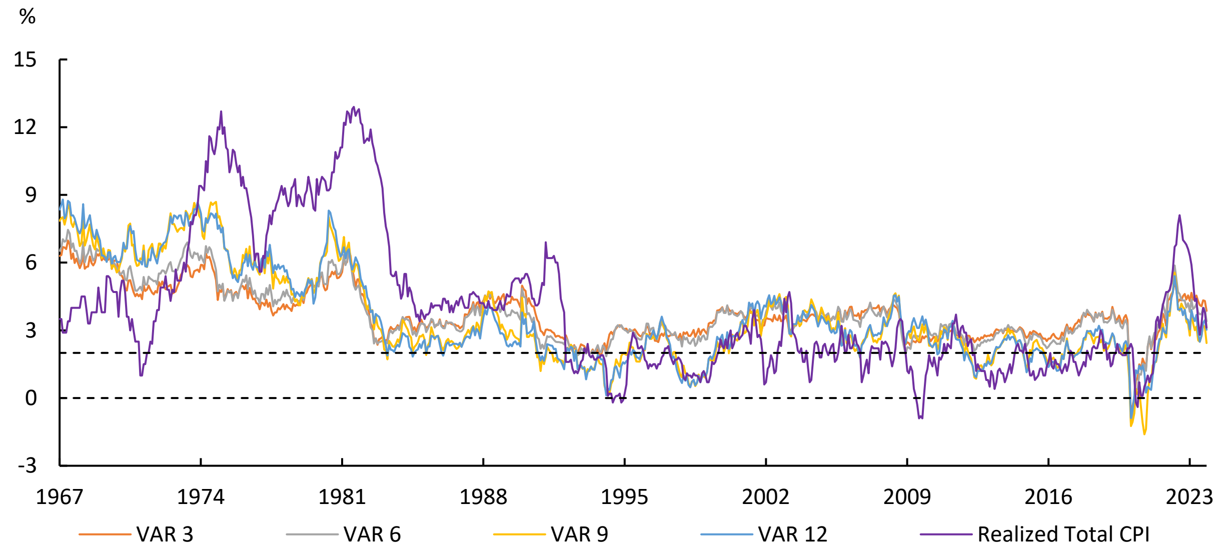
Source: Bank of Canada calculations Last observation: November 2023

2.4 Discussion of results

The historical trend in our model matches observed dynamics of inflation (somewhat by design). Canada experienced two notable episodes of inflation, triggered by food and energy price shocks: one from 1971 to 1976 and the other from 1977 to 1983 (**Chart 9**). Recently, inflation fell after the COVID-19 shock, reaching its lowest point in over a decade by May 2020. But as economies reopened and faced supply chain shocks, inflation surged to levels not seen since the 1980s, peaking in June 2022.

Our suite of models shows that these variations in realized inflation can affect long-term inflation expectations. The sharp increases in realized CPI (e.g., 12.7% in December 1974, 12.6% in April 1981, 5.3% in January 1990 and 8.1% in June 2022) tend to result in increased expectations. Conversely, as the Bank of Canada raised its policy rate, inflation expectations moved lower. Following the Bank's increase of 100 basis points (bps) in July 2022, we can observe a sharp decline in the long-term model-implied inflation expectations.[3]

Chart 9: Realized CPI and model-implied 10-year inflation expectations, without survey



Sources: Statistics Canada and Bank of Canada calculations Last observation: November 2023

3. Inflation risk premium

The inflation risk premium (IRP) is the compensation investors demand to protect themselves against inflation risk. Like other risk premiums, the IRP cannot be directly observed and must therefore be estimated from data on observable quantities such as prices, yields and macroeconomic variables.

Recent models have used inflation-linked bonds and information from surveys augmented by state-of-the-art term structure approaches to derive the IRP for the United States and the European Union. Their main findings are that nominal bonds embed a time-varying IRP, although without a consensus about the magnitude of this premium (e.g., Ang, Bekaert and Wei 2008; Grishchenko and Huang 2013). For example, Buraschi and Jiltsov (2005) find that the 10-year US inflation risk premium averaged 70 bps from 1960 to 2000. While the IRP can be linked to the uncertainty or volatility of inflation, Bekaert and Wang (2010) argue that its economic determinants are more subtle in most pricing models. Specifically, the covariance of economic growth (and therefore agents' consumption) and inflation can affect the sign and magnitude of an IRP estimate. Conventional consumption-based asset pricing theory quantifies the risk embedded in any given asset as the covariance of its returns and the growth in real activity. Piazzesi and Schneider (2007) generate a positive IRP, with a key assumption that inflation is bad news for future growth.

In contrast, recent literature since the 1990s has shown that inflation has flipped to a good news event for future growth (e.g., Burkhardt and Hasseltoft 2012; Campbell, Sunderam and Viceira 2017; Zhao 2020). This implies a negative IRP. Campbell, Shiller and Viceira (2009) also show that the positive correlation between

[3] Consensus expectations for the July 2022 meeting was at 75 bps.

inflation and stock returns (as an indicator of wealth due to money illusion) can imply a negative inflation risk premium. This correlation between an agent's wealth or consumption and inflation may well vary over time and cause substantial correlation in the conditional inflation risk premium. The above also implies that the IRP can react differently to a demand shock as opposed to a supply shock.

The IRP is also linked to the credibility of central banks. If central bank targets are credible, market participants would anticipate that higher inflation and inflation expectations (outside central bank targets) would be countered by decisive monetary policy responses, leading to negligible IRP in nominal bonds.

3.1 Modelling framework for the inflation risk premium

Our simple approach relies on breaking down the nominal term premium to an IRP and a real term premium (RTP).[4, 5] The RTP is generally viewed as reflecting growth risks, monetary policy uncertainty and other factors that affect the term premium.[6]

Since the nominal term premium is not observable, we use the measure from Feunou et al. (2015): the shadow rate model. This model is fairly similar to the Federal Reserve Bank of New York's ACM term premium model developed by Adrian, Crump and Moench (2013) (ACM), albeit with two differences:

- while the ACM model assumes a linear pricing factor, the shadow rate model assumes non-linearity
- volatility of yields is constant in the ACM model but time-varying in the shadow rate model[7]

The nominal term premium (TP) has both a cyclical and a trend component (**Chart 10**).

$$TP_t = TP_t^* + \widetilde{TP}_t \;, \quad (4)$$

where $TP_t^* = TP_{H-1} + \frac{1}{H}\sum_{s=H}^{t}(TP_s - TP_{s-H+1})$. In other words, the trend component is obtained by cumulating the rolling window (*H*) mean of daily changes.

The TP_t in Canada shows a downward trend (TP_t^*) over the past two decades, which is consistent with the secular decline in interest rates before the pandemic.

Our analysis focuses on the cyclical component ($\widetilde{TP}_t$) of the nominal term premium, which is affected by changes in risks to inflation and growth and is therefore more relevant to policy-makers.

---

[4] Recall that nominal yields can be decomposed into the sum of expectations of the nominal short rate over the horizon of the bond and the nominal term premium. Nominal expectations reflect inflation expectations and real short-rate expectations. We focus on the nominal term premium component.

[5] The nominal term premium is generally defined as the extra return that bond investors demand to hold a long-term bond instead of investing in a series of short-term securities.

[6] Other factors include the liquidity risk premium, sovereign risk premium, geopolitical risk, effects of flight to quality and search for yield.

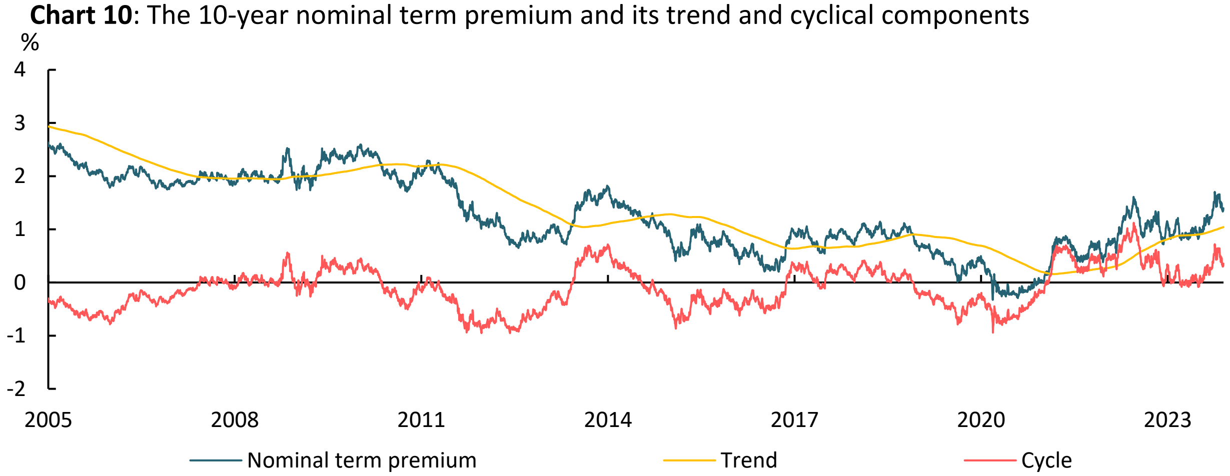
[7] Many models face a tension in capturing prices and quantities of risk in normal times, which is further complicated at the zero lower bound. Including the non-linearity assumption provides freedom in modelling the short rate that is consistent with the presence of a lower bound. Because the volatility term structure varies, assuming constant volatility remains a challenge for existing models. The shadow rate incorporates these two changes without compromising the estimation of risk premiums and offers a tractable avenue to overcome this tension.

In contrast, the trend component moves slower. The decline in the trend component of the term premium in Canada reflects a combination of domestic and US factors that spill over onto the Canadian trend term premium. Cao et al. (2023) provide evidence of interest rate spillovers from large economies to small open economies, such as Canada. The downward trend in the US term premium since the 2008–09 global financial crisis coincides with increases in the Federal Reserves and foreign officials holdings of US Treasuries and the notion that demand pressures from these sources helped push down yields.

While the Bank of Canada did not engage in quantitative easing during the global financial crisis, long-term Government of Canada bonds followed similar trends due to the spillovers from the US Treasury market. Moreover, the downward trend of the US nominal term premium has also been linked to the more appealing risk properties of bonds. Specifically, bond yields tended to fall in response to any sign of setbacks in the economic recovery because investors raised their expectations of further monetary stimulus or pushed back the expected start of policy normalization (Cohen, Hördah and Xia 2018).

As often happens after a crisis, awareness of tail risks rose, and with it the desire to insure against such risks. Hence, in the aftermath of the global financial crisis, bonds took on some insurance-like properties. As a result, investors may have been willing to hold bonds even as the term premium fell toward zero or became negative. The flight to safety during the global financial crisis raised the demand for safe assets. Tighter regulatory requirements may also have played a role, such as for banks' holdings of liquid assets or collateralization of derivatives positions. This downward trend reached a trough at the onset of the COVID-19 pandemic. As the economic recovery started to crystalize and growth prospects improved, term premiums started rising in early 2021.

Chart 10: The 10-year nominal term premium and its trend and cyclical components



Source: Bank of Canada calculations

Last observation: November 2023

3.2 Estimation methodology for the inflation risk premium

We use a rolling window regression framework to regress changes in the cyclical component of the nominal term premium on changes in inflation risk factors to capture the time-varying aspect of the IRP. Key inflation risk factors (denoted by X_t) are the factors that investors face when buying nominal bonds. Key inflation risk factors typically comprise macroeconomic and financial variables. For example, García and Werner (2010) use various measures of realized inflation, unemployment rate, output gap and yield curve slope as well as two risk factors computed from the European Central Bank's Survey of Professional Forecasters on inflation expectations: skewness (perceived asymmetry in inflation risk at different horizons) and a measure of inflation uncertainty.

We focus primarily on scenarios where investors would demand higher compensation for bearing inflation risks. For example, investors could be concerned about central banks deviating from their targets, leading to possible tail inflation outcomes, among other factors.

Similarly, high inflation could be perceived differently in good times than it is in bad times. For example, in the two decades before the COVID-19 pandemic, elevated inflation was perceived as a good outcome because it potentially implied stronger economic activity. This changed after the post-pandemic surge in inflation, which was primarily driven by supply factors.

We decompose daily changes in the cyclical component of the nominal term premium according to the following regression estimated in a two-year rolling window (this corresponds to setting *H = 500*):

$$\Delta\widetilde{TP} = \beta_{lt}'\widetilde{\Delta X}_t + \beta_{it}'\widetilde{X\Delta X}_t + \varepsilon_t\,, \qquad (5)$$

where $\Delta\ \widetilde{TP}$ is the daily change in the 10-year cyclical nominal term premium, $\widetilde{\Delta X}_t$ is the vector of daily demeaned changes in inflation risk factors ($\widetilde{\Delta X}_t = \Delta X_t - \frac{\sum_{s=t-H+1}^{t}\Delta X_s}{H}$), $\widetilde{X\Delta X}_t$ is the vector of daily demeaned interaction between the change in inflation risk factors and the level of inflation risk factors ($\widetilde{X\Delta X}_t = X_t\Delta X_t - \frac{\sum_{s=t-H+1}^{t}X_s\Delta X_s}{H}$), and β_{lt}' and β_{it}' are the regression coefficients of $\Delta\widetilde{TP}_s$ on $\widetilde{\Delta X}_s$ and $\widetilde{X\Delta X}_s$ for $s = t - H + 1, \cdots, t$.

We include the interaction term ($\widetilde{X\Delta X}_t$) to account for non-linearities. Because the price of risk reflects the sensitivity to change in the risk factor, the price of risk could depend on the level of that risk factor. Consistent with investor risk aversion, when the quantity of risk is higher, investors could be more sensitive to the additional change compared with when risks are lower. The price of risk implied by equation (5) is given by:

$$\beta_t = \beta_{lt}' + \beta_{it}'\,X_t. \qquad (6)$$

The risk factors (X_t) are:

- The risk of inflation deviating from the Bank of Canada's inflation-control target range, both on the upside and the downside. Specifically, we use 12-month-ahead model-implied probabilities of inflation being above 3% and below 1%.
- Joint probabilities of high inflation and growth because the IRP is known to co-vary with expectations of economic growth and react differently to supply and demand shocks. Specifically, we consider joint probabilities of:
  - inflation above 3% and growth above 2%, and
  - inflation above 3% and growth below 0%.
- Risks around forecasted inflation, i.e., volatility, skewness and kurtosis. If inflation is more volatile or risks of tail inflation outcomes are elevated, investors would demand a higher IRP.
- We also include the volatility of West Texas Intermediate (WTI) to gauge risks around oil prices, which are an important driver of inflation.

The inflation risk factors are obtained from the joint economic distributions model proposed by Feunou, Azizova and Kyeong (2023). Using realized mean, variance, skewness and kurtosis of Canadian inflation and growth, the authors model daily conditional moments of inflation and gross domestic product as moving averages of economic and financial conditions. The conditional moments are then translated to conditional distributions using a flexible parametric distribution known as a skewed error distribution model.

The fitted value of the above regression provides an estimate of the change in the cyclical IRP:

$$\Delta \widetilde{IRP}_t = \beta_{lt}' \widetilde{\Delta X}_t + \beta_{it}' \widetilde{X\Delta X}_t = \beta_t' \widetilde{\Delta X}_t \,. \quad (7)$$

The estimated β_t' from this regression can be interpreted as investor risk aversion (price of risk), and the factors inside $\widetilde{\Delta X}$ as the change in the quantity of risk.

Since, ultimately, we are interested in estimating the level of the IRP, we need to add the trend and cyclical components. To do so, we estimate changes in the term premium trend that are explained by changes in the IRP trend:

$$\Delta IRP_t = \Delta IRP_t^* + \Delta \widetilde{IRP}_t \,. \qquad (8)$$

We estimate the daily change in the trend component (ΔIRP_t^*) as a fitted value of the regression of ΔTP_t^* on $\Delta X_t^* \equiv \frac{\sum_{s=t-H+1}^{t} \Delta X_s}{H}$, that is,

$$\begin{cases} \Delta TP_t^* = \gamma' \Delta X_t^* + \epsilon_t \\ \Delta IRP_t^* = \hat{\gamma}' \Delta X_t^* \end{cases}.$$

Given the estimates of ΔIRP_t^*, $\Delta \widetilde{IRP}_t$ and ΔIRP_t, we recover the level by cumulating the estimated changes as following:

$$\begin{cases} \widetilde{IRP}_t = \widetilde{IRP}_{H-1} + \sum_{s=H}^{t} \Delta \widetilde{IRP}_s \\ IRP_t^* = IRP_{H-1}^* + \sum_{s=H}^{t} \Delta IRP_t^* \\ IRP_t = IRP_t^* + \widetilde{IRP}_t \end{cases}, \qquad (9)$$

where $\widetilde{IRP}_{H-1}$ and IRP_{H-1}^* are the initial cyclical and trend components to be fixed.

3.3 Data

We estimate the model using daily data from May 7, 2002, to November 24, 2023. We obtain the 10-year nominal term premium at the daily frequency from Feunou et al. (2015). The inflation risk factors are obtained from Feunou, Azizova and Kyeong (2023). Oil price is measured by the WTI daily returns. Two distinct treatments are applied to the data before estimation. First, after differencing the term premium and the covariates, we demean the variables to remove the time trend because our analysis focuses on the cyclical component. This implies that the average daily change in the variables is zero. Since we apply a linear regression, not detrending could result in the violation of independent residuals or in spurious regressions. Second, to mitigate the effect of outliers in the sample, we limit the absolute values of changes in all the covariates at within two standard deviations of the mean.

Chart 11 shows the time series of the higher moments of the inflation distribution (volatility, skewness and kurtosis). The skewness of the inflation distribution is positive and stable for most of the sample, with a few leaps around 2008 and 2020. The kurtosis is stable for most of the sample, with a couple of significant spikes around 2008 and 2020. Data for 2008 and 2020 show that the distribution of inflation was close to a normal distribution before the pandemic started, with a skewness value close to 0 (for most of the time) and a kurtosis value close to 3. The surge in inflation after the start of the pandemic led to a spike in the skewness. Similarly, the kurtosis spiked because more of the variance is caused by these high values for inflation.

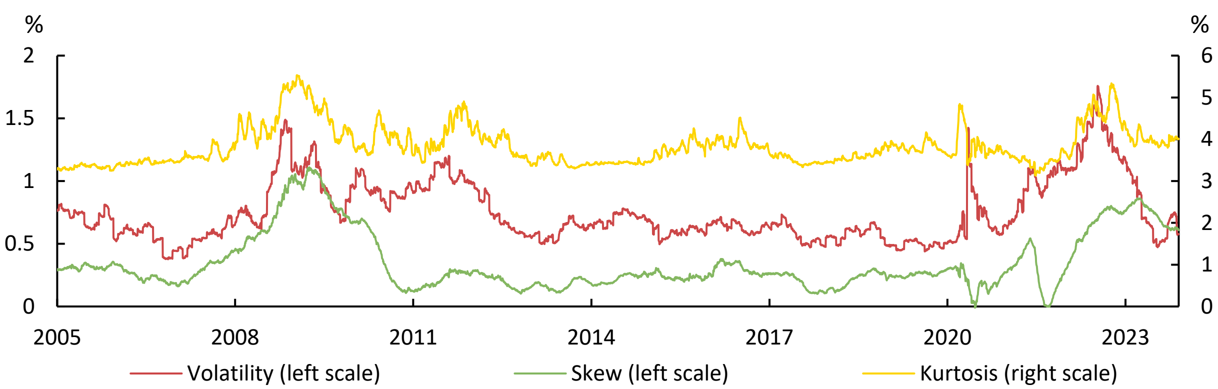
Chart 12 shows implied probabilities of inflation being outside the central bank target range. The likelihood of inflation below 1% increases significantly after the global financial crisis and briefly at the onset of the COVID-19 pandemic. However, as supply chain issues started to emerge later in 2020, the probability of above-target inflation rose significantly. **Chart 13** shows that the risk of stagflation (i.e., the joint probability of inflation above 3% and negative growth) was negligible for most of the period but rose around late 2020. The joint probability of high inflation (>3%) and high growth (>2%) was fairly stable over the sample period, with a significant leap once in September 2021. However, since the monetary policy tightening cycle took hold, the probability of this outcome has slowly been trending down.

The volatility of oil prices (**Chart 14**) is measured by the one-year rolling window of WTI daily returns as follows:

$$WTI_{vol} = \sqrt{\sum_{i=1}^{n} (ln(WTI_t / WTI_{t-1}))^2} \ . \qquad (10)$$

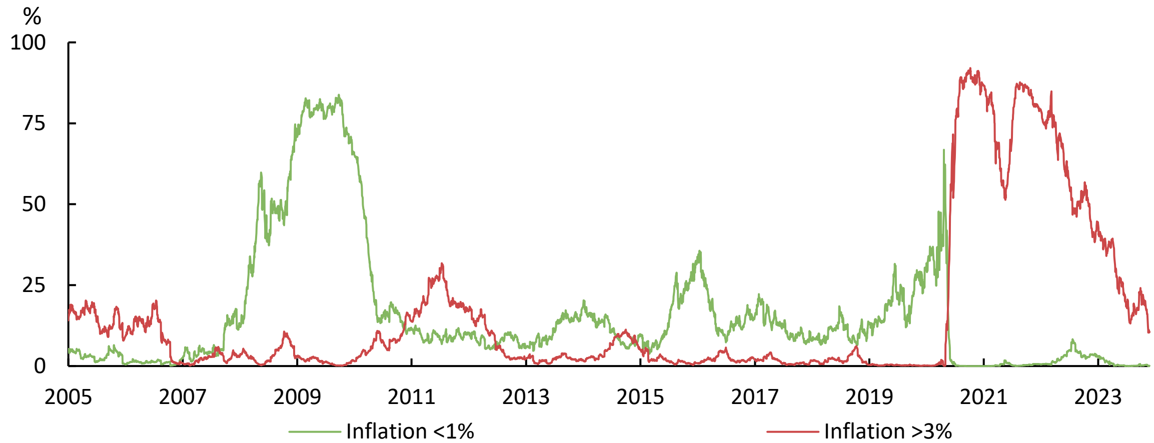
These quantities and the oil price volatility can be thought of as quantities of risk, i.e., the Q_i that we stack in the Xt vector.

Chart 11: Risks around forecast inflation (%)



Source: Bank of Canada calculations	Last observation: November 2023

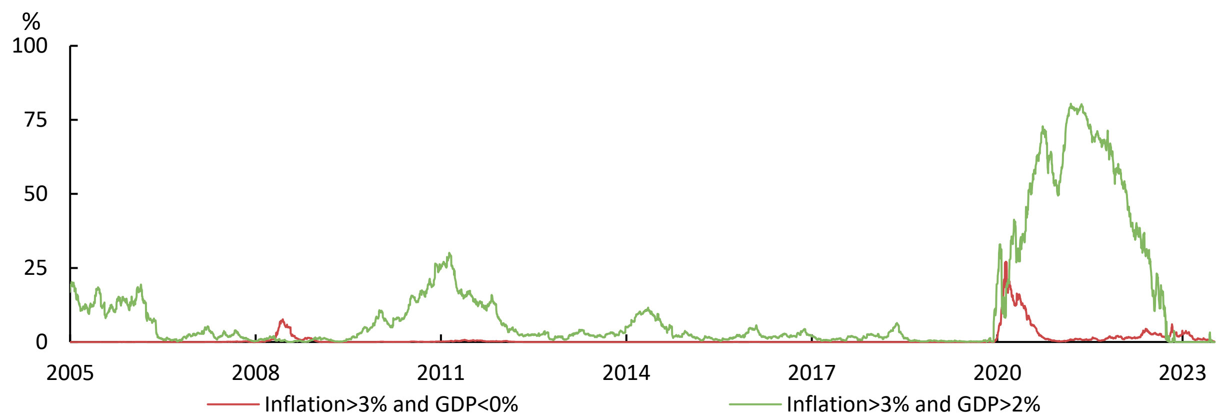
Chart 12: Probability of inflation deviating from central bank target



Source: Bank of Canada calculations

Last observation: November 2023

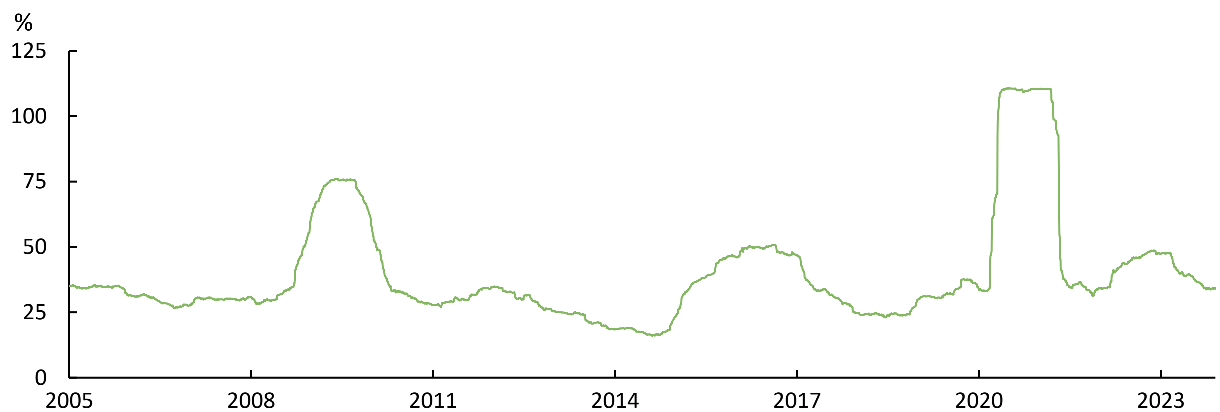
Chart 13: Joint probability of high inflation/strong growth, and high inflation/slow growth



Source: Bank of Canada calculations

Last observation: November 2023

Chart 14: West Texas Intermediate 1-year rolling volatility



Sources: Bloomberg and Bank of Canada calculations

Last observation: November 2023

3.4 Empirical results

We find that our set of inflation risk factors explain, on average, about 25% of the daily variation in the nominal term premium. The rolling window R^2 over the sample ranges from around 10% to 40% (**Chart 15**). This implies that the remaining variation in the nominal term premium is driven by the real term premium.

Our estimate of the IRP is negative or near zero through most of the sample period because most of this period was dominated by low inflation and slow growth, with investors being concerned about deflation. However, the model-implied IRP jumps into positive territory in 2021 as the situation changed drastically (**Chart 16**).

Our analysis also finds that risks around forecast inflation (volatility, skew and kurtosis) have been the key drivers of the model-implied IRP throughout the sample period. The probability of inflation above 3% became a more prominent contributor after the pandemic started. The Bank embarked on its tightening cycle in early 2022, and the policy rate reached 4.5% by January 2023. We can isolate how each risk factor and its beta affect the IRP over 2023. The probability of high inflation and slow growth can be seen to contribute positively to the IRP. This reflects the speed and magnitude of rate hikes, which increased the risk of stagflation. Finally, the probability of high inflation and risks around forecast inflation mostly contribute to a higher IRP (**Chart 17**).

3.5 Robustness check

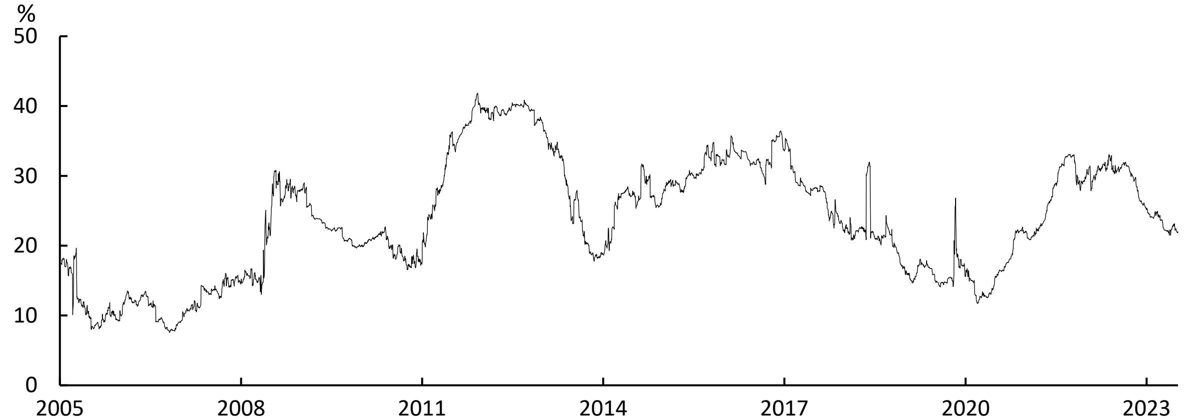
The first section of our paper provides a tool to gauge market-implied long-term inflation expectations, and the second section offers a simple approach to extract the inflation risk premium. As a robustness check, we can compare our model estimates of inflation expectations and IRP with BEIRs. Recall that BEIRs comprise two of the above-mentioned components as well as an illiquidity premium (because usually real return bonds are less liquid than nominal bonds).

Our approach does not have to account for an illiquidity premium because neither of the two models relies on real return bonds. Therefore, we can construct a synthetic BEIR by adding the two components, and we compare this synthetic BEIR with actual BEIRs. We use the average across VARs for the model-implied inflation expectations at monthly frequency and take the end-of-month values for the IRP (since its frequency is daily) to construct the model-implied BEIR.

Both the actual BEIR and the sum of our two model-implied measures share a similar trend. The BEIR is on average about 40 bps lower (**Chart 18**).[8] The model-implied BEIR varies more and has recently been diverging from the market-based one. The increase in the model-implied BEIR is primarily due to an increase in the IRP as model-implied inflation expectations have edged down. The presence of an illiquidity premium and IRP in the actual BEIR may mask a genuine reading of inflation expectations, especially in the current macroeconomic environment.

[8] The earliest date the 10-year BEIR can be calculated from real return bonds is May 2008. The Government of Canada introduced real return bonds in 1998 and issued them in the 30-year sector regularly. BEIRs at shorter horizons are computed using the residual maturity of outstanding real return bonds.

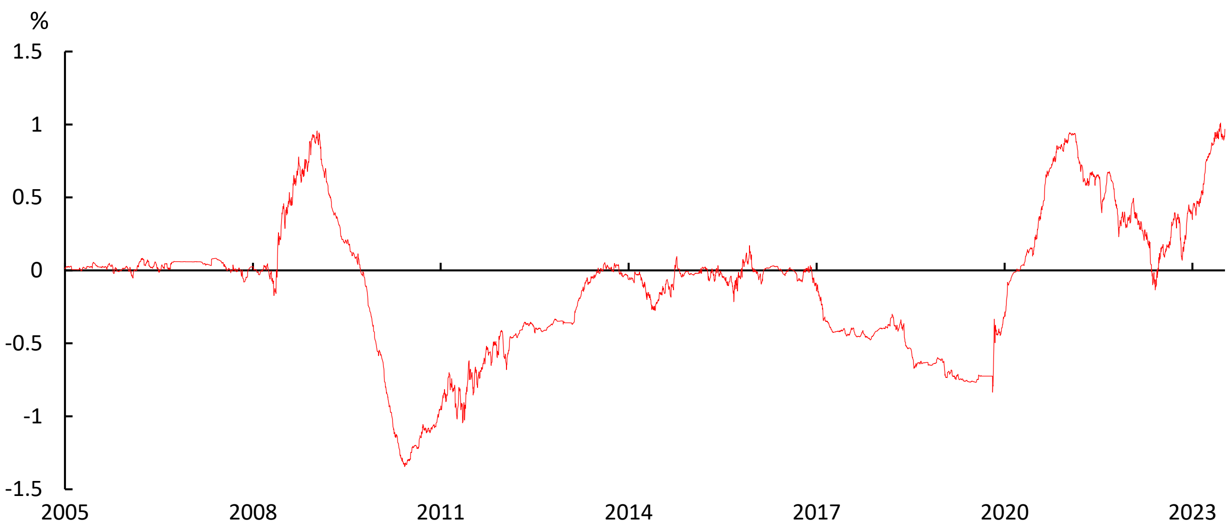
Chart 15: Variation in nominal term premium explained by inflation risk factors



Source: Bank of Canada calculations

Last observation: November 2023

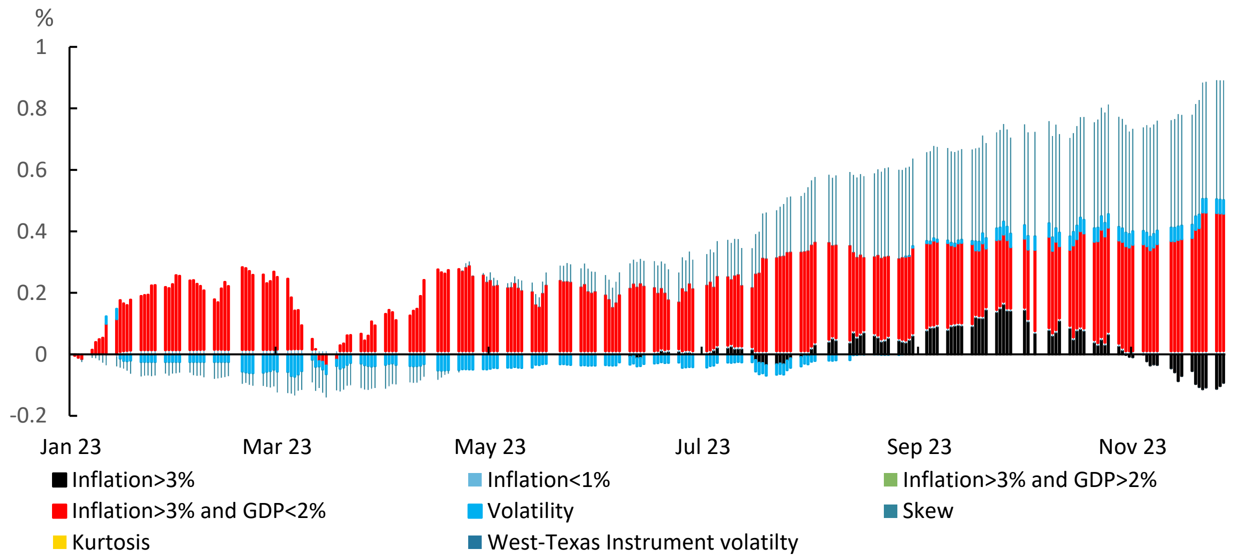
Chart 16: Estimated 10-year inflation risk premium



Source: Bank of Canada calculations

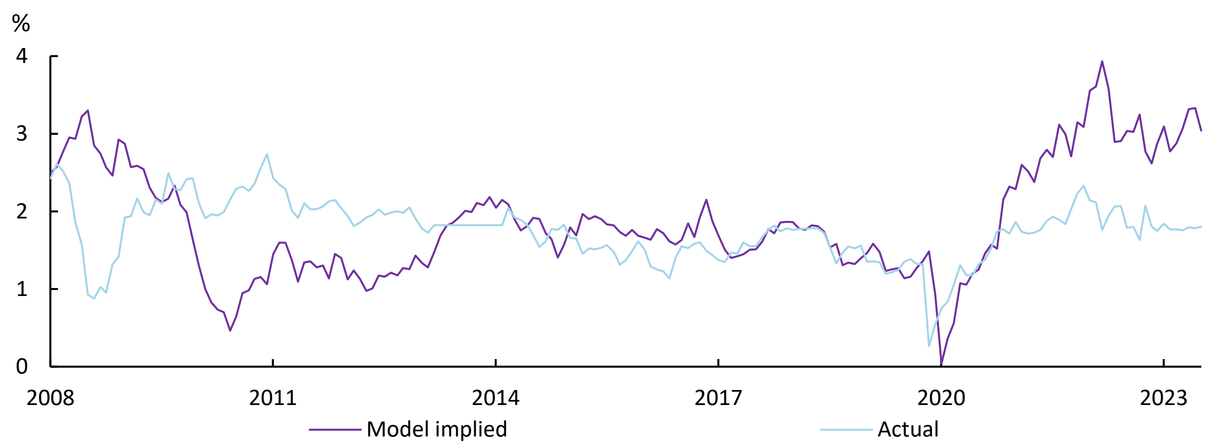
Last observation: November 2023

Chart 17: Drivers of estimated inflation risk premium



Source: Bank of Canada calculations
Last observation: November 2023

Chart 18: Comparison of 10-year actual and model-implied breakeven inflation rate



Sources: Bloomberg and Bank of Canada calculations
Last observation: November 2023

4. Conclusion

We present two simple models to capture inflation expectations and the IRP embedded in the Government of Canada bond yields and some macroeconomic variables. The model outputs are of interest to policy-makers and market participants. Used together, our tools can capture the four key drivers of nominal yields:

- real short-rate expectations
- inflation expectations
- IRP
- real term premium

Moreover, this allows us to construct a model-implied BEIR that does not rely on real return bonds, which will be particularly useful in a future without real return bonds in Canada.

With regards to the IRP, we highlight two potential avenues for future work. First, the unexplained variation in the term premium can be attributed to other factors that are not considered in this framework, such as changes in the supply and demand for bonds through quantitative easing, quantitative tightening, safe-haven flows and other technical factors affecting the term premium. This can be explored using different approach than the one presented here. Second, we explain drivers of the 10-year nominal term premium, but our driving variables reflect 1-year-ahead probabilities. Future improvement could incorporate measures proxying long-term risk factors.

References

Adrian, T., Crump, R., and E. Moench. 2013. "Pricing the term structure with linear regressions," Journal of Financial Economics, Elsevier, vol. 110(1), pp. 110-138

Ang, A., G. Bekaert and M. Wei. 2008. "The Term Structure of Real Rates and Expected Inflation." *Journal of Finance* 63 (2): 797–849.

Bekaert, G. and X. S. Wang. 2010. "Inflation Risk and the Inflation Risk Premium." SSRN: https://ssrn.com/abstract=1600312.

Bauer, M. D. and J. D. Hamilton. 2018. "Robust Bond Risk Premia." *Review of Financial Studies* 31 (2): 399–448.

Buraschi, A. and A. Jiltsov. 2005. "Inflation Risk Premia and the Expectations Hypothesis." *Journal of Financial Economics* 75 (2): 429–490.

Burkhardt, D. and H. Hasseltoft. 2012. "Understanding Asset Correlations." University of Zurich and Swiss Finance Institute manuscript.

Campbell, J. Y., R. J. Shiller and L. Viceira. 2009. "Understanding Inflation-Indexed Bond Markets." *Brookings Papers on Economic Activity* Spring: 79–120.

Campbell, J. Y., A. Sunderam and L. M. Viceira. 2017. "Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds." *Critical Finance Review* 6 (2): 263–301.

Cao, J., V. Dinger, T. Gómez, Z. Gric, M. Hodula, A. Jara, R. Juelsrud, K. Liaudinskas, S. Malovaná, and Y. Terajima. 2023. "Monetary Policy Spillover to Small Open Economies: Is the Transmission Different Under Low Interest Rates?" *Journal of Financial Stability* 65. 101116.

Cieslak, A. and P. Povala. ,2015. "Expected Returns in Treasury Bonds." *Review of Financial Studies* 28 (10): 2859–2901.

Cohen, B. H., P. Hördahl and D. Xia,2018. "Term Premia: Models and Some Stylised Facts." *BIS Quarterly Review* September: 79–91.

De Pooter, M., F. Ravazzolo and D. van Dijk,2010. "Term Structure Forecasting Using Macro Factors and Forecast Combination." Board of Governors of the Federal Reserve System International Finance Discussion Paper No. 993.

Dewachter, H. and M. Lyrio, 2006. "Macro Factors and the Term Structure of Interest Rates." *Journal of Money, Credit and Banking* 38 (1): 119–140.

Duffee, G. 2013. "Forecasting Interest Rates." In *Handbook of Economic Forecasting.* Edited by G. Elliott, C. Granger and A. Timmermann. Vol. 2 pp. 385-426.

Faust J., and J. Wright, 2013. "Inflation Forecasting", in Handbook of Economic Forecasting, Vol. 2, North-Holland, Amsterdam, pp. 2-56

Feunou, B., J.-S. Fontaine, A. Le and C. Lundblad,2015. "Tractable Term Structure Models". Bank of Canada Staff Working Paper 2015-46

Feunou, B. and J. Fontaine, 2012. “Forecasting Inflation and the Inflation Risk Premiums Using Nominal Yields”, Bank of Canada Working Paper 2012-37

Feunou, B., Azizova, C., and J. Kyeong, 2023. “Forecasting Risks to the Canadian Economic Outlook at a Daily Frequency”, Bank of Canada Staff Discussion Paper, 2023-19

García, A. J., and T. Werner,2010. “Inflation Risks and Inflation Risk Premia”. European Central Bank Working Paper No. 1162, 2010-03

Greenwood, R. and D. Vayanos ,2014. “Bond Supply and Excess Bond Returns,” Review of Financial Studies, ol. 27, pp. 663–713.

Grishchenko, O., and J. Huang ,2013. "The Inflation Risk Premium: Evidence from the TIPS market", Journal of Fixed Income, 22(4), pp.5-30

Joslin, S., Priebsch, M., and K. J. Singleton ,2014. “Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks,” Journal of Finance, vol. 69, pp. 1197–1233

Kim, D. H. and A. Orphanides, 2012.. “Term structure estimation with survey data on interest rate forecasts”. 47 (01), 241–272.

Kozicki S. and P.A. Tinsley,1998. “Moving Endpoints and the Internal Consistency of Agents’ Ex ante Forecasts”, Computational Economics, 11 , pp. 21-40

Kryzvtsov O., MacGee J., and L. Uzeda,2023. “The 2021-22 Surge in Inflation”, Bank of Canada Staff Discussion Ppaper No. 2023.

Ludvigson, S. C. and Serena Ng. 2009. “Macro Factors in Bond Risk Premia,” Review of Financial Studies, Vol. 22, pp. 5027–5067.

Ludvigson, S. C .and Serena Ng ,2010. “A Factor Analysis of Bond Risk Premia,” Handbook of Empirical Economics and Finance, p. 313

Lutz Kilian and D.P. Murphy .2012. "Why Agnostic Sign Restrictions Are Not Enough: Understanding The Dynamics Of Oil Market Var Models," Journal of the European Economic Association, European Economic Association, vol. 10(5), pp. 1166-1188.

Ormeño A. and K. Molnár,2015. “Using Survey Data of Inflation Expectations in the Estimation of Learning and Rational Expectations Models”, Journal of Money, Credit and Banking, Vol. 47, No. 4 (June 2015), pp. 673-699.

Piazzesi M., and M.Schneider,2007. “Equilibrium Yield Curves,” NBER Chapters, in: NBER Macroeconomics Annual 2006, Volume 21, pp. 389-472.

Wright, J.H.,2013. “Evaluating Real-time VAR Forecasts with an Informative Democratic Prior”, Journal of Applied Econometrics, 28, pp. 762-776

Zhao, G,2020. “Ambiguity, Nominal Bond Yields, and Real Bond Yields”. American Economic Review: Insights 2, 177-92.

Appendix: Seasonal adjustment to the consumer price index

We derive month-over-month changes in the seasonally adjusted CPI from 1965–92. The year-over-year CPI between seasonally adjusted and non-adjusted follow the same trend (**Chart 1**).

We start with year-over-year inflation rate π_t^{yoy}can be expressed in month-over-month terms such that:

$$\pi_{t+\tau}^{yoy} = \sum_{j=1}^{\tau} \pi_{t+j}$$

$$\tau = 12$$

$$\pi_t^{yoy} = \sum_{j=1}^{\tau} \pi_{t-\tau+j} = \sum_{k=0}^{\tau-1} \pi_{t-k}$$

$$\pi_{t-1}^{yoy} = \sum_{k=0}^{\tau-1} \pi_{t-1-k} = \sum_{k=0}^{\tau-1} \pi_{t-(1+k)} = \sum_{j=1}^{\tau} \pi_{t-j} = \sum_{j=0}^{\tau-1} \pi_{t-j} + \pi_{t-\tau} - \pi_t == \pi_t^{yoy} - \pi_{t-\tau} - \pi_t$$

$$\pi_t^{yoy} - \pi_{t-1}^{yoy} = \sum_{k=0}^{\tau-1} \pi_{t-k} - \sum_{k=0}^{\tau-1} \pi_{t-1-k} = \sum_{k=0}^{\tau-1} \pi_{t-k} - \sum_{j=0}^{\tau-1} \pi_{t-(1+j)} = \sum_{k=0}^{\tau-1} \pi_{t-k} - \sum_{k=1}^{\tau} \pi_{t-k}$$

$$= \pi_t + \sum_{k=1}^{\tau-1} \pi_{t-k} - \sum_{k=1}^{\tau-1} \pi_{t-k} - \pi_{t-\tau}$$

$$\pi_t = \pi_t^{yoy} - \pi_{t-1}^{yoy} + \pi_{t-\tau} = \Delta\pi_t^{yoy} + \pi_{t-\tau}$$

$$\pi_t = \Delta\pi_t^{yoy} + \pi_{t-\tau}$$

Assuming we know,

$$\pi_{t0};\ \pi_{t+1};\ \ldots;\ \pi_{t0+\tau-1}$$

$$\pi_{t0+\tau} = \Delta\pi_{t0+\tau}^{yoy} + \pi_{t0}$$

$$\pi_{t0+\tau+1} = \Delta\pi_{t0+\tau+1}^{yoy} + \pi_{t0+1}$$

$$\vdots$$

$$\pi_{t0+2\tau-1} = \Delta\pi_{t0+2\tau-1}^{yoy} + \pi_{t0+\tau-1}$$

We can compute the following:

$$\pi_t = \Delta\pi_t^{yoy} + \pi_{t-\tau}$$

For $t \geq t_0 + \tau$

In our case, we move back in time:

$$t < t_0$$

$$\pi_t = \pi_{t+\tau} - \Delta\pi_t^{yoy}$$

⇨

- $\pi_{t0-1} = \pi_{t+\tau-1} - \Delta\pi_{t+\tau-1}^{yoy}$
- $\pi_{t0-2} = \pi_{t+\tau-2} - \Delta\pi_{t+\tau-2}^{yoy}$

$$\vdots$$

- $\pi_{t0-\tau} = \pi_{t0} - \Delta\pi_{t0}^{yoy}$

For

$$k > \tau$$

- $\pi_{t0-k} = \pi_{t+\tau-k} - \Delta\pi_{t+\tau-k}^{yoy}$

If

- $k = \tau + 1$ then $\pi_{t0-\tau\text{-}1} = \pi_{t-1} - \Delta\pi_{t0-1}^{yoy}$