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Endogenous Credibility and Wage-Price Spirals

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Abstract

Elevated inflation can threaten the credibility of central banks and increase the risk that inflation expectations do not remain anchored. Wage-price spirals might develop in such an environment, and high inflation could become entrenched. We quantitively assess the risks of a wage-price spiral occurring in Canada over history by using a medium-scale dynamic stochastic general equilibrium model enhanced with heterogenous expectation and learning. This mechanism generates time-varying propagation of inflationary shocks that improves forecasting performance of inflation and wage growth. Central bank credibility is endogenous in our model and depends on several notions of the learning mechanism. Weaker credibility and a higher risk of inflation expectations not remaining anchored increase the risk of a wage-price spiral.

Topics: Business fluctuations and cycles; Credibility; Inflation and prices; Monetary policy
JEL codes: E00, E7, E47, C22

Résumé

Une inflation forte menace la crédibilité des banques centrales et accentue le risque d'un désancrage des anticipations d'inflation. Un tel contexte peut mener à des spirales salaires-prix et à la normalisation d'une inflation élevée. Nous quantifions le risque de voir apparaître au Canada une spirale salaires-prix au fil du temps. Nous nous servons d'un modèle d'équilibre général dynamique et stochastique de taille moyenne qui intègre des anticipations hétérogènes et un processus d'apprentissage. Ce dispositif produit une diffusion des chocs inflationnistes au fil du temps qui permet d'améliorer la qualité des prévisions sur l'inflation et la croissance des salaires. Dans notre modèle, la crédibilité des banques centrales est endogène et dépend de la définition choisie pour décrire le processus d'apprentissage. La spirale salaires-prix devient plus probable lorsqu'il y a une érosion de la crédibilité des banques centrales et un risque accru de désancrage des anticipations d'inflation.

Sujets : Crédibilité; Cycles et fluctuations économiques; Inflation et prix; Politique monétaire
Codes JEL : E00, E7, E47, C22

1 Introduction

When inflation remains high for an extended period, households and firms may start expecting inflation to become persistent and remain high for a long time. This can put the credibility of monetary policy under threat and unanchor inflation expectations. As high inflation lowers the purchasing power of money, workers may start demanding higher wage increases to ensure their pay keeps up with the rising cost of living.[1] Firms with wage-setting based on annual or multi-year contracts can factor in the anticipated increase in wage costs and start charging higher prices for their products. Workers' higher expectations for inflation can raise their expectations for wages, which can further boost inflation. This process, often referred to as a wage-price spiral (henceforth WPS), could become a vicious circle that presents significant challenges for monetary policy.

We introduce a novel expectation formation mechanism with bounded rationality into a standard New Keynesian (NK) model (Smets and Wouters, 2007) to study the risk of unanchoring of inflation expectations and the role of central bank credibility in the development of WPS. We model inflation and nominal wage expectations using the heterogeneous agents model (HAM) approach, given its presented advantage for modelling expectations in environments with regime shifts by allowing agents to switch their forecasting models (Hommes and Lustenhouwer, 2019). Agents in our model can switch between forming expectations based on a fundamentalist forecasting rule and an adaptive learning rule, depending on the relative forecasting performance of these two alternative rules.

In our model, the evolving share of fundamentalist agents is closely related to the degree of central bank credibility. The fundamentalists understand how monetary policy works in a fully rational NK model. Their expectations internalize the stabilization properties of monetary policy as well as their belief in policy effectiveness in the future. In contrast, adaptive agents form their expectations based on historical data. Their beliefs are not based on understanding fundamentals of the economy or monetary policy. Instead,

[1] Recent examples from collective bargaining indicate workers' demands for higher pay increases, in some cases higher than recent inflation rates. For example, United Auto Workers had a strike in summer 2023 demanding wage and benefit improvements that resulted in 25% to 30% higher labour costs over the four years of the contract.

our adaptive agents are econometricians who update their beliefs about inflation and nominal wage growth based on an AR(1) model as more data become available (Evans and Honkapohja, 2012). In addition, our adaptive agents can respond more strongly to the data during regime shifts. We model this using an endogenous gain mechanism based on Kostyshyna (2012) that generates a time-varying response to data, similar to Carvalho et al. and Gati (Carvalho et al., 2023; Gati, 2023). Using a time-varying response to data is motivated by the evidence that the formation of expectations can vary depending on the economic environment. For example, the attention to inflation tends to increase with the level of inflation (Weber et al., 2023).

Our first finding is that the model with heterogeneous expectations and learning about inflation and nominal wages provides a better fit for the historical Canadian data for inflation and wage growth and their respective expectations, compared with the standard benchmark with rational expectations. Second, our model generates stronger and more persistent shock amplification relative to that in the benchmark. Since the heterogeneous expectation-switching mechanism and time-varying response to data give rise to endogenous central bank credibility, shock propagation in our model is time-varying in contrast to constant shock propagation in the benchmark. For instance, during the recent post-pandemic inflation surge, our model generates stronger responses to inflation and nominal wage growth to price and wage mark-up shocks than in the RE benchmark.

Third, we find that the risks of unanchoring inflation expectations and compromised credibility increase with the proportion of adaptive learners, the value of the endogenous gain and the degree of extrapolation in their beliefs. If people believe that inflation and nominal wages will remain elevated, temporary shocks may have a stronger and more persistent effect on both inflation and wage growth. As a result, our model generates higher risks of developing wage price spirals based on several different notions of WPS in the literature (Lorenzoni and Werning, 2023; Alvarez et al., 2022; Blanchard, 1986). And finally, in our model with heterogeneous and non-rational expectations and learning, monetary policy must act much more aggressively to guide the expectations to effectively mitigate the risks of unanchoring inflation expectations and preventing a WPS from developing.

The mechanism of expectation formation in our model contributes to our findings in the following way. When agents observe a shift in inflationary dynamics to a high inflation regime, several elements of expectation formation come together to amplify the economy's response to the underlying economic shocks. The share of adaptive agents increases as their forecasting rule tracks inflation better than the fundamentalists' forecasting rule during such a regime shift. At the same time, the adaptive agents start to react more strongly to recent inflationary dynamics as they quickly learn about the shift in inflation. Lastly, as adaptive agents learn from strong inflation, they start to extrapolate that the high inflation trend will persist into the future. The combination of developments in these elements of expectations is indicative of unanchoring of inflation expectations and eroding of central bank credibility. The novel heterogeneous expectation featuring the endogenous-gain learning mechanism strengthens shock propagation in our model. This makes inflation and wage growth responses to shocks more persistent and time-varying. During persistent high-inflation episodes, an increase in the share of adaptive agents who accelerate learning from the data and extrapolate trends contributes to a higher risk of developing a WPS than in the RE benchmark.

We contribute to the literature in the following ways. First, we develop a model with heterogeneous expectations and learning and provide empirical validation of the model against both data and the expectations for inflation and wage growth. The model with heterogeneous expectation and endogenous gain learning outperforms the model with rational expectation in matching historical data, including the post-pandemic high-inflation episode.

Second, our model provides a new analytical framework to assess the risks of wage price spirals associated with unanchored inflation expectations under endogenous central bank credibility. To the best of our knowledge, this is the first paper that uses a model with heterogeneous agents combined with learning to analyze WPS dynamics. Our analysis provides a generalized framework to compare different notions of WPS (Blanchard, 1986; Alvarez et al., 2022; Lorenzoni and Werning, 2023), and our results are robust across different definitions—the likelihood of a WPS is higher in our model with bounded rationality than in the RE benchmark.

Lastly, our model also provides a new tool for monetary policymakers to use to understand inflation and manage inflation expectations. In contrast to the literature where monetary policy is assumed to be static and perfectly credible at all times, we argue that central bank credibility is an endogenous outcome of how well inflation expectations are anchored. When inflation surges, an increasing number of adaptive agents who form extrapolative beliefs could drive inflation higher and even into a wage price spiral. This could greatly threaten central bank credibility and impair monetary policy transmission. We draw policy implications by showing that in the presence of boundedly rational expectations, monetary policy must act more aggressively to guide expectations and prevent the potential development of a WPS.

Related literature

Our paper is related to the literature on using behavioural models to study endogenous credibility and the unanchoring of inflation expectations, as well as to the literature on the formation of WPS. Deviating from the RE benchmark in the form of heterogeneous expectations is consistent with the extensive empirical evidence about heterogeneity in inflation and wage expectations.[2]

Heterogeneous agent models (HAMs), introduced by Brock and Hommes (1997, 1998), have been applied in macroeconomic studies using NK models (Hommes and Lustenhouwer, 2019; Hommes et al., 2023; Branch and McGough, 2010). An earlier example is Cornea et al. (2019), where the authors estimate a behavioural model of inflation dynamics with heterogeneous firm expectations. More recently, HAMs have been used to study endogenous monetary policy credibility when the policy rate is constrained at the effective lower bound (ELB) in Hommes and Lustenhouwer (2019). In their study, the share of fundamentalist agents, whose inflation expectations always correspond to the inflation target, is used as a measure of central bank credibility. Experimental evidence in McMahon and Rholes (2023) also provides support to a model with endogenous credibility. We contribute to this literature by introducing a generalized concept of central bank credibility that relates to the evolving share of fundamentalists, as well as the beliefs of

[2] See, for e.g., Branch (2004); Carroll (2003); Pfajfar and Santoro (2010); Madeira and Zafar (2015); Mankiw et al. (2003) for evidence about heterogeneity in inflation expectations and Jain et al. (2024); Coibion et al. (2021); Hajdini et al. (2022) for evidence about wage expectations of firms and workers.

adaptive agents.

Further, our modelling of endogenous credibility and unanchoring of inflation expectations is closely related to Carvalho et al. (2023) and Gati (2023). Carvalho et al. (2023) are the first to propose capturing unanchoring of inflation expectations through the dynamics of a discrete function of endogenous gain parameter. When the gain value increases, long-run expectations become more sensitive to short-term inflation forecast errors, thus indicating a potential unanchoring risk. Building on Carvalho et al. (2023), Gati (2023) introduces a continuous function of the endogenous gain parameter to study unanchoring inflation and optimal monetary policy. We use an adaptive step-size algorithm (Kostyshyna, 2012) to model the continuous evolution of gain based on recursive updating in response to forecasting errors. Compared with Carvalho et al. (2023) and Gati (2023), our model, in addition to using an evolving endogenous gain, includes two other factors contributing to the unanchoring of expectations—the evolving share of adaptive learners and the degree of extrapolation in their adaptive expectations. Time-varying sensitivity of expectations has been used in several other papers (Marcet and Nicolini, 2003; Cho and Kasa, 2015; Kostyshyna, 2012; Milani, 2014; Carvalho et al., 2023; Gati, 2023). Our approach is also related to the regime-switching literature on monetary policy; for example, Davig and Leeper (2007) examine stability conditions of standard rational expectation models when monetary policy switches between active and passive regimes. Our model is similar to an endogenous regime-switching approach, where credibility continuously evolves based on economic conditions and expectations.

Our paper is related to other studies using non-rational expectations in a New Keynesian model to study anchoring of inflation expectations and monetary policy. Orphanides and Williams (2005) show that in the presence of perpetual (constant gain) learning, economic shocks can give rise to endogenous unanchoring of inflation expectations. In addition, monetary policy must respond strongly to inflationary threats and must also respond to inflation expectations, not only to actual inflation. Adam and Woodford (2012) show that monetary policy must have greater resistance to inflation surprises in a model with model-inconsistent expectations. Our results also indicate more restrictive monetary policy in the model with learning than in the model with RE. Bianchi et al. (2022) show that introducing diagnostic expectations in an NK model replicates a business cycle response to a monetary policy shock.

The rest of the paper is organized as follows: Section 2 provides a brief discussion of the behaviour of inflation and wage growth expectations in Canada in recent history, including the post-pandemic inflation surge. Section 3 discusses the analytical environment, the learning mechanism and the empirical methodology. Section 4 provides the estimation results and the model validation exercise with survey data. Section 5 shows the key model dynamics related to endogenous central bank credibility, and discusses the risk of unanchoring of expectations and WPS in the model from a historical context. Section 6 concludes.

2 Inflation and Wage Expectations in Canada

In this section, we discuss the behaviour of inflation and wage growth expectations over the period 2014Q4–2022Q4, which includes both a period of stable and low inflation and the post-pandemic inflation surge starting in 2021Q2. Figures 1 and 2 present the results from the Bank of Canada's Canadian Survey of Consumer Expectations (CSCE, 2023) and Business Outlook Survey (BOS, 2024). Our focus is on the dynamics of inflation and wage expectations in different economic environments, such as periods of low and stable inflation compared with the dynamics in periods of high inflation. In addition, we examine the link between inflation and wage expectations and their intrinsic persistence.

Dynamics: Consumers' and firms' expectations for inflation and wages are relatively stable during the pre-pandemic period, characterized by low and stable inflation. As the inflation rate in Canada has steadily increased since 2021Q1, consumers' and firms' inflation expectations have started to rapidly increase. Figures 1 and 2 show that consumers and firms have quickly adapted their short-term inflation expectations in response to the post-pandemic inflation surge. There is evidence that inflation expectations have become extrapolative during this period (Kostyshyna et al., 2024).

The response in workers' expectations for nominal wage growth have remained relatively moderate and have not kept up with actual inflation or inflation expectations.[3] But firms' wage expectations have reacted quickly to rising inflation by increasing. These quick upward dynamics in inflation expectations and wage expectations have given rise to concerns among policymakers and researchers about the development of a wage price spiral.

Link between wage and inflation expectations: Many firms cite the rising cost of living as an important driver of wage growth, suggesting a link between their inflation and wage growth expectations. Survey evidence indicates a backward-looking formation of firms' wage expectations: Most firms (about 75% of them) take into account past inflation when setting wages, whereas a small share take into account expected inflation (Amirault et al., 2013). The evidence from the micro-level data, however, suggests that workers' and firms' wage expectations are weakly linked to their inflation expectations across different countries—Canada (Jain et al., 2024), the United States (Hajdini et al., 2022) and France (Coibion et al., 2021). The historically weak link between expected wages and inflation may be interpreted as representing limited risks of developing a WPS. In Canada, this link has strengthened since 2021Q2 during the post-pandemic period, even though this correlation remains substantially below 1 (Jain et al., 2024).

Persistence: Lastly, empirical evidence suggests that inflation expectations exhibit substantial persistence with a strong link to consumers' own perceptions about past inflation (Jonung, 1981; D'Acunto et al., 2020). CSCE inflation expectations in Canada also have a strong backward-looking aspect, with the correlation between perceptions and expectations around 0.7–0.8 (Jain et al., 2024). In addition, workers' wage growth expectations in CSCE exhibit persistence, albeit somewhat lower than the persistence in inflation expectations, at about 0.5 (Jain et al., 2024).

Data from consumer and firm surveys indicate that inflation and wage expectations tend to remain stable during low and stable inflation, but they adjust upward quickly when a shift to a high inflation regime occurs. Empirical evidence from the literature suggests that the link between wage and

[3]Relatively muted dynamics in workers' wage expectations during the post-pandemic high-inflation period are most likely because CSCE elicits workers' expectations for their earnings *conditional* on workers staying in the same job and working the same hours. For more details, see Jain et al. (2024). Wage growth for new hires has been shown to be more cyclical than the wages of job stayers (Bils, 1985; Haefke et al., 2013; Gertler et al., 2020).

inflation expectations is relatively weak. And finally, both inflation and wage expectations exhibit a high degree of intrinsic persistence. Our modelling of inflation and wage expectations is based on this empirical evidence. Building on these findings, we introduce a behavioural New Keynesian model with boundedly rational agents that is designed to capture key features of expectations such as persistence and backward-lookingness.

3 Behavioural New Keynesian Model

In this section, we use the canonical Smets and Wouters (2007) NK model, henceforth SW07, to introduce heterogeneous expectations and adaptive learning in inflation and wage expectations. This is consistent with empirical evidence about expectations. We first describe the standard price and wage New Keynesian Phillips Curves (NKPCs) in the model. Further details of the model are omitted here and can be found in Appendix A.

3.1 Price and Wage Phillips Curves

The price NKPC is given by the following conventional form:

$$\Big\{\pi_t = \pi_1 \mathbb{E}_t \pi_{t+1} - \pi_2 \mu_t^p + \epsilon_t^p, \tag{1}$$

where π_t denotes inflation, $\mathbb{E}_t \pi_{t+1}$ denotes one-quarter ahead inflation expectations, μ_t^p is the real marginal cost and ϵ_t^p represents cost-push shocks. The wage NKPC has a similar structure and is given by:

$$\Big\{w_t = (\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) - w_2 \mu_t^w + \epsilon_t^w, \tag{2}$$

where w_t denotes real wages, $\mathbb{E}_t w_{t+1}$ are real wage expectations, μ_t^w refers to the wage mark-up and ϵ_t^w captures wage markup shocks.

In our model, we depart from the assumption of RE by introducing bounded rationality in price and wage expectations in the NKPCs. Our bounded rationality has three main components: 1) heterogeneous expectations with fundamentalist and adaptive agents, 2) AR(1) learning of adaptive agents and 3) an endogenous gain mechanism with time-varying attention of adaptive agents to the recent inflation and wage data. Each of these components can contribute to the unanchoring of expectations and evolving central bank credibility. We provide further details for each of these elements and how they can contribute to WPS risk in the following sections.

3.2 Heterogeneous Expectations

We first introduce a heterogeneous agents model (HAM) into the benchmark NK framework. HAM has the advantage of performing well in an environment characterized by regime shifts because it allows agents to switch between forecasting models depending on their forecasting performance. This mechanism is useful to explain inflation dynamics over a long period encompassing both high- and low-inflation episodes, periods of stable inflation and shifts in inflation dynamics.[4]

We specify two types of agents who have different expectation-formation rules for inflation and wages. The first type of agent is *fundamentalists*, who use the underlying RE solution of the NK model. Fundamentalists have full information about the model structure, including monetary policy, and form their expectations about inflation and wages consistent with rational expectations within the NK model. This type of agent uses the simple NK model to form their forecasts.

The second type is *adaptive* agents, who are not aware of the underlying model structure and instead learn about the structure through a simple AR(1) model as more data become available. They form their expectations based on the estimated AR(1) process for inflation and nominal wage growth.[5]

Note that fundamentalists in the model do not take into account the presence of adaptive agents when forming their expectations. We use this

[4] Our sample encompasses the high inflation of 1980s, the introduction of an inflation targeting monetary policy regime in 1991 and subsequent decline in inflation, relatively stable inflation in 1990s up to 2020 and the recent post-pandemic surge in inflation.

[5] Note that our goal is to analyze the marginal impact of bounded rationality in inflation and wage expectations. Therefore, the mechanism is only introduced for these two variables. For the rest of the variables, all agents form expectations based on the RE solution of the NK model.

assumption to ensure that these agents' expectations are characterized as a distribution centred around the average inflation target. These agents perceive any deviations from the inflation target as transitory. Thus, expectations of fundamentalists are anchored at the target regardless of the economic conditions, and these agents act as a stabilizing force in the economy. The decline in the share of fundamentalists indicates unanchoring of expectations and can present an upside risk of inflation. This is tied to a decline in central bank credibility, which is discussed in further detail in Section 5.3.

Formally, the model can be written in the standard matrix form as follows:

$$\begin{cases}\mathbf{A}X_t = \mathbf{B}X_{t-1} + \mathbf{C}\mathbb{E}_t X_{t+1} + \mathbf{D}\varepsilon_t,\end{cases} \tag{3}$$

with conformable structural matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. X_t denotes the vector of state variables and ε_t denotes the vector of iid shocks. Further, $X_t = [\tilde{X}_t', \pi_t, \tilde{w}_t]'$, where $\tilde{w}_t$ denotes nominal wage growth, defined as $\tilde{w}_t = w_t - w_{t-1} + \pi_t$ and $\tilde{X}_t'$ correspond to all state variables other than inflation and nominal wage growth in the model. The perceived law of motion of fundamentalist agents, together with their one-step-ahead expectations, are given by:

$$\begin{cases}X_t = \mathbf{b}^{RE} X_{t-1} + \mathbf{d}^{RE}\epsilon_t, \\ \mathbb{E}_t[X_{t+1}^{RE}] = \mathbf{b}^{RE} X_t,\end{cases} \tag{4}$$

with $\mathbb{E}_t[X_{t+1}^{RE}] = \left[\mathbb{E}_t[\tilde{X}_{t+1}^{RE}], \mathbb{E}_t[\pi_{t+1}^{RE}], \mathbb{E}_t[\tilde{w}_{t+1}^{RE}]\right]'$, and $\mathbf{b}^{RE}$ and $\mathbf{d}^{RE}$ corresponding to conformable matrices associated with the minimum state variable (MSV) RE solution of the model.

The adaptive agents do not know the rational expectations solution of the model $\mathbf{b}^{RE}$ and $\mathbf{d}^{RE}$. Instead, they use a simple AR(1) heuristic to learn from recent data when forming their expectations about inflation and nominal wage growth:

$$\begin{cases}x_t^L = \alpha_{t-1}^x + \beta_{t-1}^x x_{t-1}, \\ \mathbb{E}_t x_{t+1}^L = \alpha_{t-1}^x + \beta_{t-1}^x x_t,\end{cases} \tag{5}$$

with $x_t \in \{\pi_t, \tilde{w}_t\}$.[6] α_t^x and β_t^x denote agents' time-varying perceptions about

[6]Given the expectations about inflation $\mathbb{E}_t^L \pi_{t+1}$ and nominal wage growth $\mathbb{E}_t^L \Delta\tilde{w}_{t+1}$, real wage expectations are implicitly defined as $\mathbb{E}_t^L w_{t+1} = \Delta\tilde{w}_{t+1} - \mathbb{E}_t^L \pi_{t+1} + w_t$.

the mean and persistence of inflation and real wages.[7] Expectations about all other state variables follow the rational expectation rule, which we denote by $\mathbb{E}_t[X^L_{t+1}] = \Big[\mathbb{E}_t[\tilde{X}^{RE}_{t+1}], \mathbb{E}_t[\pi^L_{t+1}], \mathbb{E}_t[\tilde{w}^L_{t+1}]\Big]'$.[8]

The population shares of fundamentalist and adaptive agents evolve endogenously in response to forecasting errors of the respective type of agents, following the approach in Brock and Hommes (1997). Specifically, the shares of fundamentalist (n^{RE}_t) and adaptive agents (n^L_t) are given by:

$$n^{RE}_t = \frac{exp(-\chi\zeta^{RE}_t)}{exp(-\chi\zeta^{RE}_t) + exp(-\chi\zeta^L_t)}, \quad n^L_t = \frac{exp(-\chi\zeta^L_t)}{exp(-\chi\zeta^{RE}_t) + exp(-\chi\zeta^L_t)}, \tag{6}$$

where ζ^{RE}_t and ζ^L_t correspond to a fitness measure associated with the fundamentalist and adaptive forecasting rules respectively. χ is the intensity of choice indicating how strongly agents respond to the forecasting performance of two models, with a higher value indicating stronger reaction to fitness. The fitness measures are given by:

$$\begin{cases} \zeta^{RE}_t = (1-\omega)FE^{RE}_t + \omega\zeta^{RE}_{t-1}, \\ \zeta^L_t = (1-\omega)FE^L_t + \omega\zeta^L_{t-1}, \end{cases} \tag{7}$$

with FE^{RE}_t and FE^L_t denoting the inflation forecast errors for the fundamentalist and adaptive rules.[9] Given the population shares of the fundamentalist and adaptive agents, the aggregate expectations are determined as a weighted average every period:

$$\mathbb{E}_t[X_{t+1}] = n^L_{t-1}\mathbb{E}_t[X^L_{t+1}] + n^{RE}_{t-1}\mathbb{E}_t[X^{RE}_{t+1}].$$

A declining share of fundamentalist agents increases the likelihood that expectations may shift away from the central bank's target. Therefore the share of fundamentalist agents can be used as a proxy for central bank credibility and unanchoring of expectations.

[7]Note that agents have two separate AR(1) models for inflation and real wages, where perceived mean and persistence are updated according to (8), respectively. Following the recent literature on peoples' cognitive limitation in processing complexity, we assume that agents use the same endogenous gain process for both variables.

[8]Note that for both RE in equation (4) and learning in equation (5), we keep the standard timing assumption that expectations and state variables are jointly realized; i.e., expectations for period $t+1$ are a function of period t state variables. Beliefs for the learning model are updated sequentially, *after* the state variables $\boldsymbol{X}_t$ are realized.

[9]ω is a hyper-parameter that introduces persistence into the fitness measures. We refer to it as "memory" henceforth.

3.3 Adaptive Learning

The adaptive agents in our model are not aware of the underlying economic structure and monetary policy. Instead, they act as econometricians and estimate an AR(1) process for inflation and real wages. They use the estimated model to form their expectations about inflation and nominal wages (5). They update the coefficients in their AR(1) model—perceived mean and persistence—every period as new inflation and nominal wage data become available. Learning is modelled based on a recursive least squares algorithm as in Evans and Honkapohja (2012):

$$\begin{cases} R_t^x = R_{t-1}^x + \gamma_{t-1}(\tilde{\boldsymbol{x}}_{t-1}\tilde{\boldsymbol{x}}_{t-1}' - R_{t-1}^x), \\ \Phi_t^x = \Phi_{t-1}^x + \gamma_{t-1}(R_t^x)^{-1}\tilde{\boldsymbol{x}}_{t-1}(\tilde{\boldsymbol{x}}_t - \Phi_{t-1}\tilde{\boldsymbol{x}}_{t-1})', \end{cases} \tag{8}$$

with $\tilde{\boldsymbol{x}}_t' = [1, x_t]'$, $x_t \in \{\pi_t, \tilde{w}_t\}$. R_t^x denotes agents' perceptions about the volatility of inflation and nominal wage growth, γ_{t-1} denotes the endogenous gain and $\Phi_t^x = [\alpha_t^x, \beta_t^x]$, i.e., agents' perceived mean and persistence for each variable.

Our choice of PLM of the learning agents in the model is motivated by historical data and recent empirical evidence during the post-pandemic high inflation period discussed in Section 2. Historically, the correlation between inflation expectations and wage expectations is very weak, whereas a high degree of persistence characterizes both inflation expectations and wage expectations. Therefore, it is desirable to have a model that can encompass these features in these two variables. Learning literature has documented that introducing AR(1) learning rules in a DSGE model typically leads to considerable improvements over the RE benchmark.[10] We thus follow this modelling approach to assume that adaptive agents follow AR(1) learning rules.

The persistence coefficients in the AR(1) PLMs have important implications for the potential risk of unanchoring of expectations. When $\beta_t^x < 1$, the forecasting rule is mean-reverting, which means that the expectations of adaptive learners are anchored. When $\beta_t^x > 1$ instead, the forecasting rule becomes extrapolative and adaptive learners' expectations become unanchored.

[10] Some examples include Ormeño and Molnár (2015), Slobodyan and Wouters (2012) and Hommes et al. (2023), among many others.

The discussion in Section 3.2 has established the link between the share of adaptive agents and the potential risk of unanchoring of inflation expectations. The time-varying perceptions about persistence and mean in inflation and wage expectations constitute the second source of unanchoring inflation expectations and high inflation risk in our model.

3.4 Endogenous Gain

When learning agents update their model recursively, an important parameter is the gain, γ_t, that governs the strength of agents' reactions to the most recent data. In the literature, two approaches are generally used to model the gain process: a decreasing gain as in least-squares learning, and constant gain as in Evans and Honkapohja (2012). In a stationary environment, a decreasing gain approach is an appropriate choice as it gives equal weight to all past observations. However, in an environment with structural breaks, a constant gain assumption has some advantages as it allows agents to recognize regime changes and to respond more strongly to recent data.

We introduce an endogenous gain mechanism to allow for a time-varying response by adaptive agents to economic dynamics. The flexibility of the endogenous gain mechanism permits these agents to recognize regime switches and respond to them with quicker updating. The endogenous gain γ from (8) evolves based on an adaptive step-size algorithm given by Kostyshyna (2012):

$$\begin{cases} \gamma_t = \gamma_{t-1} + \mu R^{\gamma}_{t-1} FE^L_t, \\ R^{\gamma}_t = (1-\gamma_{t-1})R^{\gamma}_{t-1} + FE^L_t, \end{cases} \tag{9}$$

where μ is the step-size of the gain process, or "gain on the gain."[11]

The adaptive step-size algorithm has several important properties (Kostyshyna, 2012). The gain process γ_t changes in response to both past discounted errors $(R_t - 1)^{\gamma_t}$ and the most recent forecast error FE^L_t. Discounting past errors can represent agents' limited memory, cognitive ability or limited interest to keep remembering the distant past. When agents' most recent forecast error

[11] Kushner and Yin (2003) show that the performance of the step-size algorithm is much less sensitive to the choice of step-size μ, compared to the choice of a constant gain algorithm.

is in the same direction as past errors, agents realize that they are "repeating" the same mistakes and thus have an incentive to update their beliefs more quickly to correct these errors. To do so, they increase their gain γ_t and react more strongly to the most recent data. In contrast, if the most recent forecast error is in the opposite direction from past errors, agents attribute less urgency to correct their models. As a result, the gain decreases with weaker reaction to more recent data.

Endogenous gain learning presents a technical improvement over the constant gain approach and offers a behavioural intuition. In particular, Weber et al. (2023) show that attention to inflation depends on the level of inflation. In a high-inflation environment, people pay more attention to inflation. Our model of endogenous gain allows for such behaviour—when agents recognize a shift in the economic regime, they update their expectations more strongly in response to the recent data to catch up with the new regime. An increase in endogenous gain is another element in our expectations mechanism that can contribute to the higher risk of unanchoring of expectations.

Recent work by Carvalho et al. (2023) and Gati (2023) uses endogenous gain to model inflation expectations whose response to current inflation can vary. In these models, an increase in endogenous gain means a stronger response to recent inflation dynamics, which indicates unanchoring of inflation expectations. The evolution of endogenous gain in our model is similar to other endogenous gain mechanisms, in that it responds to past forecast errors. In Marcet and Nicolini (2003); Milani (2014); Carvalho et al. (2023), the endogenous process involves a switch between decreasing and constant gain, while Gati (2023) modifies Carvalho et al. (2023) by introducing a smooth gain function. The novel feature of our endogenous gain algorithm is that it is based on a continuous response of the gain to past and recent forecast errors, with recursive updating using its own gain.

The combination of heterogeneous expectations (6), adaptive learning (8) and the endogenous gain (9) can change model dynamics and its stability properties. Many papers in the literature have studied the theoretical properties of models with learning. For example, Evans and Honkapohja (2009) show that sufficiently high gain values can lead to a breakdown of the standard expectational stability (E-stability) condition in constant gain learning models. Hommes and Lustenhouwer (2019) find that a high share of boundedly rational agents is typically associated with unstable outcomes in a simple heterogeneous agent model.

While our full model is too large to derive analytically tractable stability results, in Appendix B we use a skeleton version of the model to sketch its E-stability properties. Stability of learning dynamics is closely linked with the notion of determinacy and multiplicity of equilibria. Many papers in the literature examine the interaction between determinacy and E-stability (McCallum, 2007; Bullard and Mitra, 2002; Eo and McClung, 2021). Other papers study how multiple E-stable equilibria can co-exist when expectations are not rational, and how this can result in a persistence amplification of inflation to shocks (Hommes and Zhu, 2014; Hommes et al., 2023). We instead focus on the interaction between the endogenous gain mechanism and the E-stability properties of learning dynamics. We find that the combination of a high gain, high share of adaptive learners and high inflation environment can lead to expectationally unstable (E-unstable) outcomes. This is consistent with previous studies in the literature that focus on stability properties under switching (Hommes and Lustenhouwer, 2019; Goy et al., 2022) or learning (Bullard and Mitra, 2002; Evans et al., 2022). When agents increasingly focus on recent inflation outcomes, learning dynamics can amplify the potential upside risk to inflation and nominal wage growth. The resulting risk of unanchoring of expectations and the formation of WPS poses a significant challenge to an inflation-targeting central bank in achieving its price stability objective. Building on this intuition, in the next section we construct a central bank credibility index that consolidates all elements of non-rational expectations in our model.

3.5 Central Bank Credibility Index

Several studies in the literature (Hommes and Lustenhouwer, 2019; Goy et al., 2022) have used the endogenous share of fundamentalists who follow rational expectations as a proxy for central bank credibility. In our model, central bank credibility and the risk of unanchoring of expectations depend not only on the share of fundamentalists, but also on the parameters that govern the adaptive learners' PLM. The combined effects of a lower share of fundamentalists, a higher perceived inflation persistence in AR(1) models and a higher gain all contribute to deteriorating central bank credibility and an increasing risk of WPS as inflation expectations become unanchored.

To capture the combined role of all components associated with updating expectations, we construct the following composite index C_t as our proxy for central bank credibility in the model:

$$C_t = \frac{exp(n_t^{RE})}{exp\Big(\frac{1}{3}(\hat{\mathbb{E}_t^L[\pi_{t+20}]} + \hat{\beta}_t^{\pi} + \hat{\gamma}_t)n_t^L\Big) + exp(n_t^{RE})}, \tag{10}$$

where the hatted variables correspond to their re-scaled counterparts fluctuating between 0 and 1.[12] The index C_t increases when there are more fundamentalists in the economy. Our setup nests within the RE benchmark case. In the limit case where everyone forms expectations based on the fundamentalist rule, the model simplifies to the standard RE equilibrium and central bank credibility reaches its highest level. As the share of fundamentalists decreases, the five-year inflation expectations of adaptive agents ($\hat{\mathbb{E}_t^L[\pi_{t+20}]}$), perceived inflation persistence ($\hat{\beta}_t^{\pi}$), and the endogenous gain ($\hat{\gamma}_t$) start to play an increasingly important role in the dynamics of the credibility index.[13] The variables based on AR(1) PLM and endogenous gain are all scaled to ensure they receive an equal weight in the composite index.

In the next section, we estimate the model with Canadian data and discuss our model-implied credibility index over the estimation sample period. We then conduct counterfactual experiments to examine the business cycle implications of central bank credibility (and its three components) within the context of inflation expectations and WPS risk.

4 Empirical Results

In this section, we apply the model to understand the dynamics of inflation and wage growth from 1973Q1 to 2023Q1. We focus on this sample since it includes several high inflation episodes before the introduction of the inflation targeting framework in Canada, along with the post-pandemic era experience. Ultimately, we want to provide relevant implications for con-

[12] Each variable x_t is normalized using $x_t = \frac{x_t - {x_t}^{max}}{{x_t}^{max} - {x_t}^{min}}$.

[13] Five-year inflation expectations of adaptive agents are computed iteratively based on the AR(1) learning rule (5). This is given by $\mathbb{E}_t\pi_{t+j}^L = \alpha_{t+j-1}^{\pi} + \beta_{t+j-1}^{\pi}\mathbb{E}_{t-1}\pi_{t+j-1}^L$.

ducting monetary policy to mitigate the risk of WPS.

We first estimate the baseline version of the model with RE using standard Bayesian likelihood methods over the sample period. We use Canadian data including growth rates of real GDP, real consumption, real business investment, core inflation, nominal interest rates, nominal wage growth and hours worked as observable variables.[14]

The parameters governing the adaptive learning processes in (6)–(9) are calibrated based on previous estimates in the relevant literature. Specifically, we use the estimates in Ozden (2024) for specifying the intensity of choice χ, the memory in switching ω and the gain value γ_0 to initialize the endogenous gain process. The step-size of the gain process is taken from Kostyshyna (2012), and the initial covariance matrix of the learning parameters R_0 in the endogenous gain process is an identity matrix scaled with a small coefficient.[15] All calibrated parameters, along with the priors and estimated posterior mode of the structural parameters, are reported in Table 1.

To empirically validate the importance of heterogeneous expectations and learning dynamics, we conduct a pseudo in-sample forecasting exercise using both the baseline RE and the learning models. Starting from 1976Q2, we generate forecasts up to eight quarters ahead using both models for every quarter.[16] We then compare the performance of the learning model relative to the RE benchmark. Table 2 shows the aggregated percentage RMSE improvements in the learning model relative to the RE benchmark for inflation, nominal wage growth, inflation expectations and wage expectations. For inflation and nominal wage growth expectations data, we use two-year-ahead inflation expectations and one-year-ahead wage growth expectations from the Business Outlook Survey (BOS). Since data availability differs for each variable under consideration, the RMSE's are based on a relevant sample period for the corresponding variable, as denoted in the "Evaluation Period"

[14]We use the same Bayesian priors for the estimated parameters as in Smets and Wouters (2007).

[15]A small R_0 corresponds to a diffuse prior about coefficients. It allows the agents to be conservative about updating their beliefs at the beginning of the sample, as they have a small number of observations and update their coefficients slowly.

[16]We use the first three years of the sample as a burn-in period, thus they are excluded from the evaluation exercise.

column of Table 2.

The learning model yields better or comparable forecasts relative to the RE benchmark across the board. In particular, for short-term inflation and inflation expectations, the results are better by up to 41% and 23% respectively, and the results are statistically significant at the 5% level. The improvements in wage and wage expectation forecasts are modest and insignificant at all forecast horizons.[17]

Figure 3a shows the one-step-ahead inflation forecasts in the learning model and RE baseline over the sample period. Our model better tracks inflation during the high inflation period in the 1970s and 1980s; following the introduction of inflation targeting in the 1990s; and during the recent run-up of inflation in the post-pandemic era. Figure 3b zooms in on the model-implied expectations of our model. During the recent inflation surge episode in 2022, the expectations of fundamentalists remain well anchored around the 2% inflation target, despite inflation rising persistently above target over this period. In contrast, expectations of adaptive learners increase much more compared with those of fundamentalists. The aggregated expectations, given as a weighted average of these two based on the population shares of adaptive learners and fundamentalists, track the rising expectations in survey data very well over this period, better than RE benchmark model.

5 Key Model Dynamics

As suggested by our empirical validation, the introduction of heterogeneous expectations and learning dynamics is crucial for improving model fitness. In this section, we examine the key dynamics and properties of our model and then explain the role of these properties in generating potential risks of a WPS.

[17] We abstract away from the forecasting performance of GDP and components, since our main focus is on price and wage dynamics. Other papers in the literature show that introducing learning dynamics on real-side variables and Euler equations can also lead to improvements in forecasting performance in terms of GDP and components; see, for e.g., Slobodyan and Wouters (2012) and Hommes et al. (2023).

5.1 The Role of Heterogeneity in Expectations in Shock Propagation

We first illustrate the role of heterogeneity in expectations for the propagation of shocks. To do so, we compute impulse responses of inflation and nominal wage growth to two inflationary supply shocks—price and wage mark-up shocks—as well as to a contractionary monetary policy shock. Figure 4 shows the impulse response functions (IRFs) in our model with endogenous shares (blue dotted line). To understand the role of heterogeneous expectations, we consider two extreme cases: 1) all agents are fundamentalist (yellow line), and 2) all agents are adaptive learners (red line).

Two key insights emerge from Figure 4. First, the impact of shocks on wage growth and inflation depends crucially on the proportion of fundamentalist and adaptive agents. When all agents are fundamentalist, temporary price and wage mark-up shocks lead to transitory increases in wages and inflation. Moving to the dynamics in our model where the share of these two types of agents are endogenous, the responses of inflation and nominal wage growth become more persistent. In the other limit, when all agents are adaptive, even temporary supply shocks can lead to near-permanent increases in both wage growth and inflation. Clearly, introducing heterogeneous expectations makes shocks more persistent, which is consistent with findings in other models with learning (Slobodyan and Wouters, 2012; Milani, 2014; Hommes et al., 2023).

Second, the presence of adaptive agents affects the efficacy and speed of monetary policy transmission. A contractionary monetary policy shock becomes less effective in lowering inflation and nominal wage growth when the share of adaptive learners increases (right panel of Figure 4). As adaptive learning introduces a greater degree of backward-lookingness in expectations, persistence in inflation increases. Greater intrinsic persistence in inflation, in turn, makes the output-inflation trade-off worse for a given policy action. Therefore contractionary monetary policy shocks achieve a smaller disinflationary effect in the presence of heterogeneous expectations, compared to the RE benchmark. In other words, lower credibility results in weaker monetary policy effectiveness.

5.2 Evolution of Expectations and Credibility

To examine the evolution of expectations in our model, it is useful to look under the hood to see how different elements of expectations contribute to our model dynamics and more persistent propagation of shocks discussed in the previous section. Figure 5 shows the model-implied estimates of the share of fundamentalist agents (top left panel), endogenous gain (top right), perceived inflation persistence (bottom left) and perceived average inflation (bottom right). We can distinguish several episodes with distinct dynamics in the key elements of expectation formation.

High-inflation period in 1970–1980: During this period of high and persistent inflation, the share of fundamentalist expectations is below the sample average (top left panel), as the forecasting performance of fundamentalist model is relatively weaker than the AR(1) adaptive learning model. During this period, adaptive learners react with increasing strength to the incoming data about rising inflation to catch up with it (top right panel). The beliefs of adaptive agents are extrapolative with persistence coefficient frequently reaching values above 1 (bottom left panel), and perceptions about average inflation are the highest during the sample period (bottom right panel).

Since both HAM and the endogenous gain mechanism are designed to capture and respond to regime switches, our model shows its unique strength during the periods featuring large shifts in inflation dynamics. We focus on the two most significant episodes: first, during early 1990s following the introduction of inflation targeting (IT) monetary policy framework in 1991 by the Bank of Canada, and second, during the post-pandemic increase in Canadian inflation in 2021Q2–2023Q1. We discuss the dynamics for each period below.

Introduction of inflation targeting: Following the introduction of IT in 1991 and contractionary monetary policy, realized inflation starts to decline sharply. In our model, agents respond to this regime shift in inflation in the following ways. First, the share of fundamentalists drops (top left panel of Figure 5) as, at the time of quickly declining inflation, the forecasting performance of adaptive agents is better than the performance of fundamentalists. The decline of the share of fundamentalist agents signifies a deterioration in the credibility of monetary policy. At this point, the credibility of the inflation targeting regime has not yet been established. Once inflation reaches 2% and stabilizes, the share of fundamentalist agents increases as the credibility of low and stable inflation has been established: first gradually, and then more decisively.

Second, as inflation drops sharply, adaptive agents over-forecast inflation and realize that there may be a regime shift. As a result, these agents understand that they need to quickly learn from the most recent data. Thus, they increase their endogenous gain to react more strongly to recent data (top right panel of Figure 5). A stronger response to recent data results in quick changes in the coefficients of the AR(1) model. The persistence coefficient initially increases substantially above 1 (bottom left panel of Figure 5), indicating extrapolative expectations and weaker credibility. However, there is counteracting development consistent with signs of developing credibility. As the adaptive agents observe declining inflation, they revise down their estimate of constant in AR(1) and it becomes *negative*. The combination of negative constant and above-one persistence results in the decline of perceived long-term average inflation; that is, adaptive agents revise their views about long-term inflation in the direction consistent with the introduced target. Interestingly, as adaptive agents over-react to the most recent data to follow the steeply declining inflation, the perceived average inflation temporarily falls below the inflation target (bottom right panel of Figure 5) before eventually settling around the target of 2%.

Post-pandemic inflation surge: During the recent post-pandemic inflation surge, we observe the same elements of expectations at play. The share of fundamentalist agents drops as inflation climbs up sharply starting in 2021Q2, signifying a decline in credibility. Agents realize that the fundamentalist rule does not describe the realized data well and switch to the learning rule. During the period of increasing inflation, adaptive agents can forecast more accurately using the AR(1) model than can fundamentalists. Adaptive agents quickly realize the shift in inflation dynamics: as inflation rises sharply, adaptive agents start to repeatedly under-forecast inflation and accumulate forecasting errors. This triggers their realization about the regime shift and a need to quickly adjust by learning more from the most recent data—endogenous gain increases quickly and substantially (top right panel). As adaptive agents learn from the increasing inflation and update their AR(1) coefficients, inflation persistence reaches its peak level, indicating that expectations have become extrapolative (bottom left panel). Extrapolative expectations feed further into increasing inflation. The perception about average inflation also increases during this time (bottom right panel).

During the period of increasing inflation, the decline in the share of fundamentalists, the increase in endogenous gain, the increase in the perceptions about persistence and the average inflation of adaptive learners act in concert to further fuel rising inflation. These dynamics of expectations are the primary reason behind the stronger propagation of shocks discussed above and illustrated in Figure 4. This is the main reason why our model is able to match inflation and inflation expectations data better than RE benchmark model.

When inflation starts to moderate in late 2022, credibility starts to improve. The share of fundamentalists is restored to around 50%, close to its historical average from the stable inflation period of 2000–2019. Endogenous gain declines as adaptive learners no longer need to respond more attentively to inflation in updating their expectations. During inflation run-up, adaptive agents make repeated errors under-forecasting. But with the sudden decline in inflation, their errors are in the opposite direction—over-forecasting. Adaptive agents slow down their response to recent data until the declining trend becomes more established (equation 9). However, other elements of expectations continue to show signs consistent with the unanchoring of inflation expectations. The expectations continue to be extrapolative as the persistence parameter in AR(1) remains above 1, despite some decline. And adaptive agents still perceive that average inflation is above 2%. Altogether, given that the beliefs of adaptive agents remain extrapolative, the danger of unanchoring of inflation expectations is not yet fully gone by the end of our sample in 2023Q1.

Credibility index: We illustrate the credibility index in Figure 6 by integrating the dynamics of different elements of expectation formation into a single indicator based on equation (10). The dynamics of the credibility index are broadly consistent with our discussion above. During the high-inflation period of 1970–1980, on average the credibility index is at its lowest relative to the rest of the sample. At the onset of the introduction of the inflation targeting framework, the credibility index plunges temporarily as agents need time to learn from the sharp downward shift in realized inflation. This is the period when the central bank gradually earns and establishes the credibility of the new monetary policy regime by demonstrating its ability to bring inflation down. As inflation stabilizes around the target of 2% by 1995, credibility is established and remains relatively stable with very small fluctuations from 1995 to 2021. Despite nearly three decades of successfully anchored inflation expectation in Canada, credibility should not be taken for granted. Evidently, as inflation surges starting in 2021Q2, the credibility index plunges quickly. However, when inflation moderates, credibility loss is reversed.

Different elements contribute to the dynamics of the credibility index. In Figure 7, we illustrate the marginal contribution of each modelling mechanism to the dynamics of central bank credibility over history.[18] The increase in the share of adaptive learners (yellow bars) contributes the most to the dynamics in the credibility index throughout our sample period, with the largest contributions to weak credibility occurring during the 1970s, 1980s and early 1990s. The increase of adaptive learners also helps explain the sudden drop of credibility during the recent high-inflation episode. Endogenous gain and AR(1) beliefs play prominent roles in the evolving credibility in the earlier part of the sample and during the recent run-up of inflation. But their contribution is small during the period of low and stable inflation in 1995–2020. The dynamics in the share of adaptive learners drive the evolution of credibility during the stable period.

5.3 Wage-Price Spirals

The presence of heterogeneous expectations and learning in our model results in time-varying credibility. What is the potential risk for a WPS to develop when heterogeneous expectations matter? In this section, we show that a WPS is more likely when central bank credibility is low and the risk of unanchoring of inflation expectations is high. This finding is robust to using different definitions of WPS.

[18]To construct the decomposition of different elements, we shut down each of the elements in the mechanism characterizing adaptive learning in the model (share of adaptive learners, endogenous gain, perceived mean and persistence) one at a time and compute the counterfactual. The contribution of each element to the loss of credibility is given by the difference between counterfactual and realized credibility indices.

There are several different notions of WPS used in the literature. The seminal paper of Blanchard (1986) defines WPS as a process where inflationary effects push workers and firms to temporarily increase their real wages and markups, which in turn fuels inflation further. This leads to a feedback loop where nominal wages and inflation reinforce each other, while real wages remain stable over time. Lorenzoni and Werning (2023) describe WPS as a propagation mechanism of the inflationary shock in a standard NK model. The common theme in both studies is that the standard rational expectations framework in a NK model with staggered prices and wages already embeds WPS. Alvarez et al. (2022) and Boissay et al. (2022) come up with a different notion of WPS, which is defined as an episode where consumer prices and nominal wages accelerate for at least three consecutive quarters in a year.

Using these alternative definitions of WPS, we assess the development of a WPS in our model using the following three approaches that encompass existing definitions of WPS in the literature: 1) a shock propagation mechanism based on impulse response functions, related to Lorenzoni and Werning (2023); 2) the prevalence of a WPS episode based on the definition from Alvarez et al. (2022) and Boissay et al. (2022); and 3) a correlation between inflation and wage growth, as discussed in Blanchard (1986). We discuss the results for each of these approaches in the next section.

5.3.1 Time-varying Shock Amplification

We assess the inflationary shocks propagation mechanism by comparing cumulative IRFs of inflation and wages in our model to that from the alternative model with rational expectations. We focus on price and mark-up shocks. This comparison allows us to isolate the contribution of heterogeneous expectations and evolving credibility to the internal propagation of shocks in these two models.

We examine the time variation in the impulse responses of inflation and wages to inflationary shocks over history. The summary measure of cumulative responses over two years offers two key advantages. First, this statistic measures the "risk build up" in the economy as it allows us to show how the economy responds to new inflationary shocks at each point in time. Second, and more importantly, we can quantify the relative amplification of price versus wage mark-up shocks, a natural step leading to the assessment of a wage price spiral. Lastly, this time-varying amplification of shocks also works as a proxy for the intrinsic persistence in inflation and wages (see, for example, Andrews and Chen, 1994; Kang et al., 2009), and the pass-through between prices and wages.

We present the responses of inflation and nominal wage growth to one standard deviation price mark-up shock in Figure 8 and wage mark-up shock in Figure 9. These figures illustrate three notable results. First, the introduction of heterogeneous expectations and learning yields the time-varying responsiveness of inflation and nominal wage growth (black lines) compared with the constant responsiveness in baseline NK model (red lines).

Second, these responses are strongest during the periods of large shifts in inflation—the decline in inflation in the mid-1980s and early 1990s and the post-pandemic surge in inflation. As discussed earlier in Section 5.3, these periods are characterized by large fluctuations in credibility that can be attributed to a decline in the share of fundamentalist expectations, stronger persistence of inflation perceptions and an increase in endogenous gain. These dynamics in expectations magnify the shock propagation mechanism of a NK model; that is, they strengthen the WPS mechanism built-in to standard NK models (Lorenzoni and Werning, 2023). During periods of low and stable inflation (1995–2020), impulse responses are close to or below those in the baseline rational expectation model, suggesting that the central bank has sustained stable credibility to prevent a WPS from developing.

Third, the response of nominal wage growth to wage mark-up shock is consistently stronger in our model (black line) than that in the benchmark model (red line). Nominal wage growth experiences sharp increases during the post-pandemic run-up of inflation. This result is consistent with Blanchard and Bernanke (2023), who suggest that labour market shocks have more persistent effects on inflation and wage growth, and with Aramonte (2022), who suggests that the perceived upside risk to inflation becomes more sensitive to labour market conditions when the labour market is tight.

5.3.2 Wage-Price Spiral Probability

In this section, we use the notion of WPS as an episode where both inflation and nominal wage growth are above 2% for at least three consecutive quarters in a year, following Alvarez et al. (2022) and Boissay et al. (2022). We use a density forecasting approach to compute a time series of historical WPS risk in our HAM model and its counterpart in the baseline RE model. We use the following approach to quantify the WPS risk: In both RE and HAM models, we generate density forecasts with stochastic price and wage mark-up shocks at each period.[19] Figure 10 shows the resulting WPS probability over history in our model (in blue) and in the benchmark RE model (in red).

In both models, the high inflation period of the 1970s, 1980s and early 1990s is characterized by a relatively high likelihood of WPS. Following the introduction of the inflation targeting mandate in 1991 and the subsequent stabilization of inflation, the risk of WPS declines and remains relatively low during the period of low and stable inflation. Accompanying the recent post-pandemic inflation surge, the risk of developing a WPS heightens again in both models.

Our HAM model suggests some asymmetry of WPS probability, conditional on the prevailing level of inflation. On one hand, the risks in RE and HAM are comparable during low-inflation periods. On the other hand, the risk of WPS is much higher in our model than the RE benchmark during high-inflation episodes, both in the early part of our sample in 1970–1980 and the post-pandemic inflation increase. Recently, our model indicates a very high likelihood, close to 80%, during the height of inflation in 2022Q3, compared with a significantly lower estimate (15%) in the benchmark model. In general, during high-inflation episodes, weakening credibility and unanchoring of inflation expectations along with a stronger shock amplification results in a higher likelihood that inflation and nominal wage growth remain elevated above the target (Figure 10). In other words, low and stable inflation, maintaining credibility and keeping expectations anchored are important in preventing a WPS from developing.

[19] We use a Monte Carlo approach with 1,000 simulations for each model at each time period.

5.3.3 Wage-Inflation Feedback

Finally, we assess the wage-inflation feedback as discussed in Blanchard (1986). For this exercise, we first construct density forecasts for inflation and wage growth for recent periods characterized by low inflation (2020Q2) and high inflation (2022Q3). We then compare correlations between inflation and wage growth in our model and in the benchmark RE model. Figure 11 shows the distribution of nominal wage growth (y-axis) against inflation (x-axis) in low-inflation and high-inflation episodes.

In the low-inflation environment, the correlations between wage growth and inflation are comparable in our model (60%, black dots) and the benchmark model (61%, pink). In contrast, in the high-inflation environment in 2022Q3, the correlation in our model increases to 95%, while it remains at 61% in the benchmark model. As discussed in Section 5.3.1, the introduction of heterogeneity of expectations and learning in our model makes shock propagation for both inflation and wage growth stronger and more persistent, which in turn makes the feedback channel between prices and wages much stronger and WPS more likely.

Note that during both low- and high-inflation episodes, real wages remain fairly stable. Furthermore, there is no systematic relationship between real wages and credibility in either case. What happens to real wages ultimately depends on whether inflation is increasing at a faster pace than nominal wage growth or vice versa. Therefore, the relative direction of real wages is not indicative of the likelihood and strength of a WPS. This result is in line with the theoretical findings of Blanchard (1986) and Lorenzoni and Werning (2023).

5.4 Monetary Policy to Manage Wage-Price Spiral

Our model with heterogeneous expectations and learning offers a novel framework to examine the potential risks of WPS in the context of endogenous central bank credibility. By generating time-varying transmission of shocks and thus time-varying risks to inflation, our model helps policymakers better understand the policy trade-offs. In this environment, managing inflation expectations is a critical task for monetary policy to maintain its credibility and to mitigate the risks of developing WPS.

In this section, we draw implications for monetary policy in low- and high-inflation episodes. We continue using the model-implied density forecasts for key macroeconomic variables of our model—inflation, nominal wage growth, GDP growth and policy rate (Figure 12). The forecasts of our HAM model and the benchmark RE model are very similar during the low-inflation period (Panel (a)), with 90% HPD bands well within the range of each other.

However, when we consider the density forecasts in the high-inflation episode, the upside risk for inflation and nominal wage growth is substantially higher in our model than in the benchamark RE model (Panel (b)). Higher expectations for inflation and nominal wage growth during high-inflation episode are the result of a stronger shock propagation mechanism in the high-inflation environment, as illustrated in Figure 13. In the high-inflation period, the share of fundamentalist expectations declines. Endogenous gain increases as agents with adaptive expectations respond more strongly to increasing inflation. Finally, adaptive agents view both inflation and nominal wage growth as more persistent. All these elements contribute to higher expectations for inflation and nominal wage growth.

What can monetary policy do in such circumstances? Figure 12 shows that the policy rate path in a high-inflation environment has to be higher in the model with heterogeneous expectations and learning than that in the benchmark RE model. More aggressive policy is necessary to guide expectations and to alleviate the risks of developing WPS. On the other hand, learning also implies a worse trade-off for monetary policy. As monetary policy tightens aggressively to combat rising inflation, GDP growth is weaker over the forecast horizon.

The results in this section underscore the key mechanism of our model, and the main symptoms of an increased risk of unanchored inflation expectations. The endogenous credibility mechanism leads to a time-varying pattern of feedback between inflation and wage growth. Episodes with high inflation are characterized by lower credibility and higher likelihood of extrapolative expectations.

6 Conclusions

In this paper, we propose a learning model with heterogeneous expectations, adaptive learning and endogenous gain to study the risks associated with unanchoring of inflation expectations and developing wage-price spirals. We show that the learning model provides a novel framework to assess the implications of endogenous central bank credibility. While the heterogeneity of expectations is the main driver of endogenous credibility, adaptive learning and endogenous gain can amplify the loss of credibility up to 30–35%. Empirically, our model produces a better fitness than the benchmark RE model for inflation and wage data over the historical sample, including high-inflation episodes. We find that low credibility in a high-inflation environment can drastically increase the risk of unanchoring inflation expectations and the risk of developing wage-price spirals. The role of monetary policy is to guide expectations by responding more strongly to inflationary shocks to prevent wage-price spirals from developing. The realistic inflation dynamics in our model makes it a good laboratory for policymakers during high-inflation episodes to study the risks associated with expectations. Our model focuses on understanding the role of expectations about inflation and nominal wages in the evolution of central bank credibility, dynamics of inflation and risks of development of WPS. It would be useful to study the role of fiscal policy in inflationary dynamics in future research.

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Figures and Tables

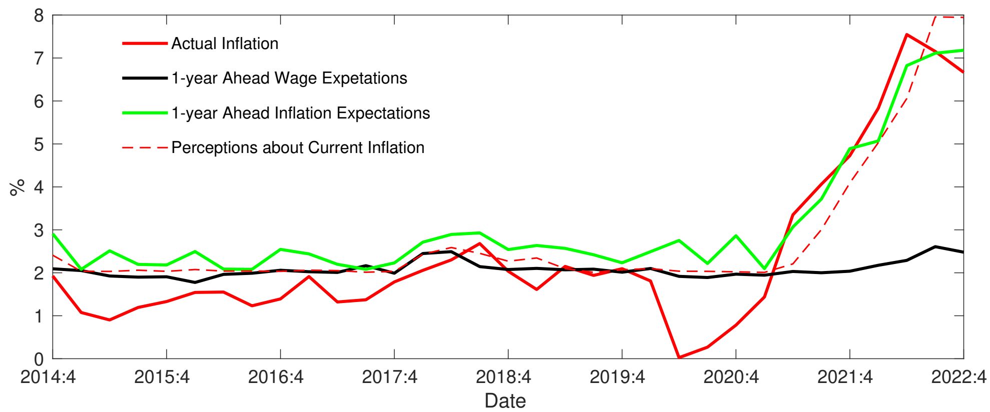


Figure 1: Inflation expectations and wage growth expectations of workers (CSCE).

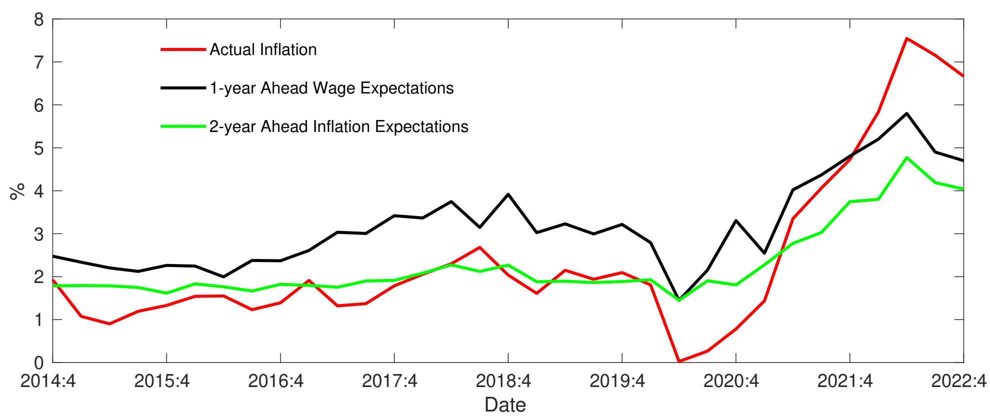
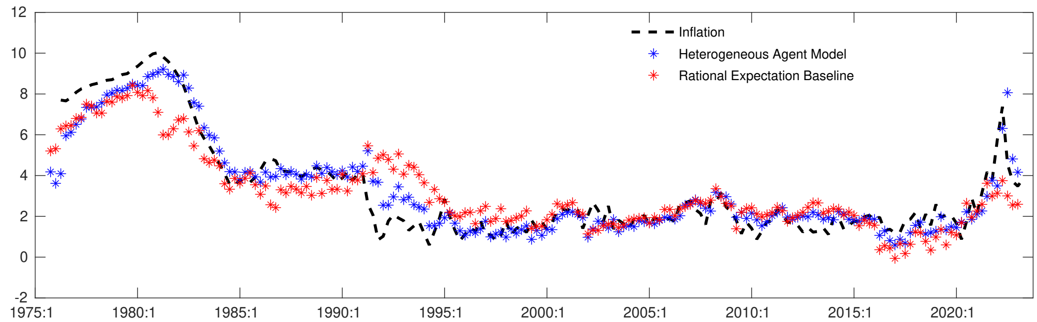
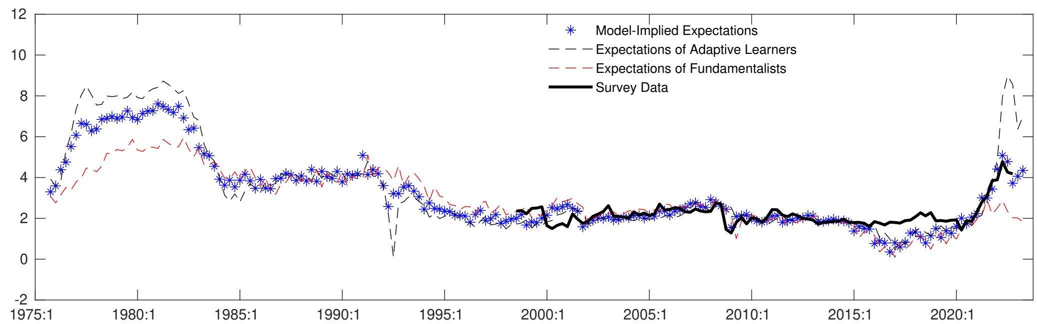


Figure 2: Inflation expectations and wage growth expectations of firms (BOS).

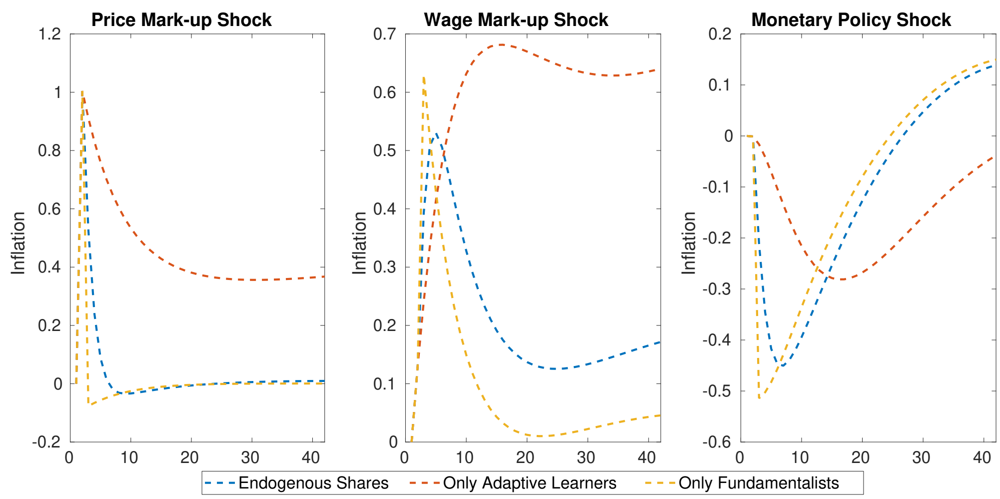


(a) Inflation

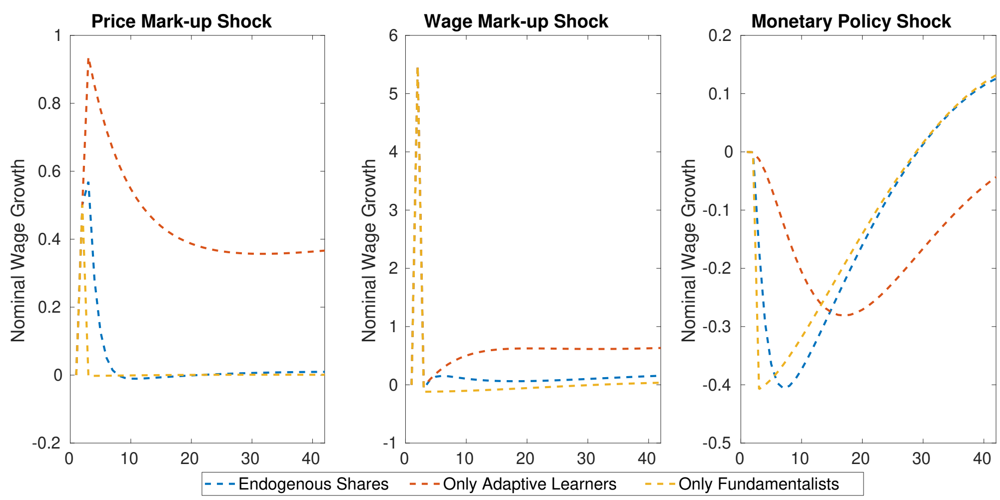


(b) Inflation Expectations

Figure 3: Top panel: In-sample one-step-ahead forecasts for inflation in the heterogeneous agent model and the RE benchmark. Bottom panel: Kalman filter estimates of adaptive learners' and fundamentalists' expectations, together with the model-implied aggregate expectations and the survey data.



(a) Impulse responses of inflation (y/y)



(b) Impulse responses of nominal wage growth (y/y)

Figure 4: Impulse response functions of inflation and nominal wage growth.

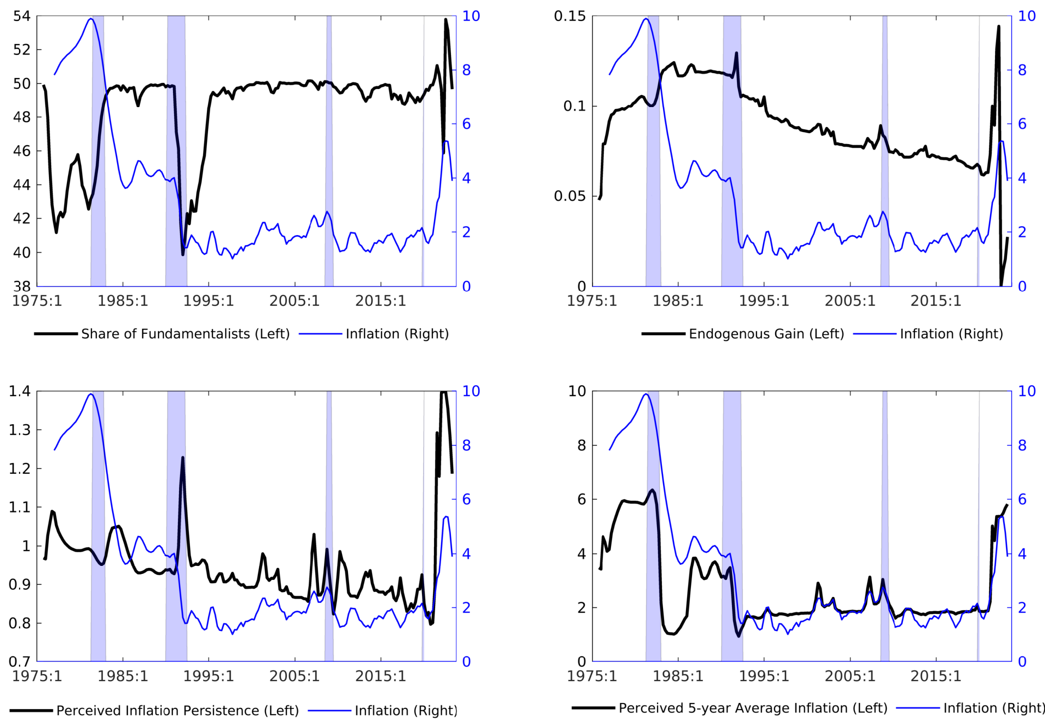


Figure 5: Estimated series of the share of fundamentalists (top left), endogenous gain (top right), perceived inflation persistence of adaptive learners (bottom left), perceived average inflation of adaptive learners (bottom right) and realized inflation (RHS) over the sample period 1975Q1–2023Q1.

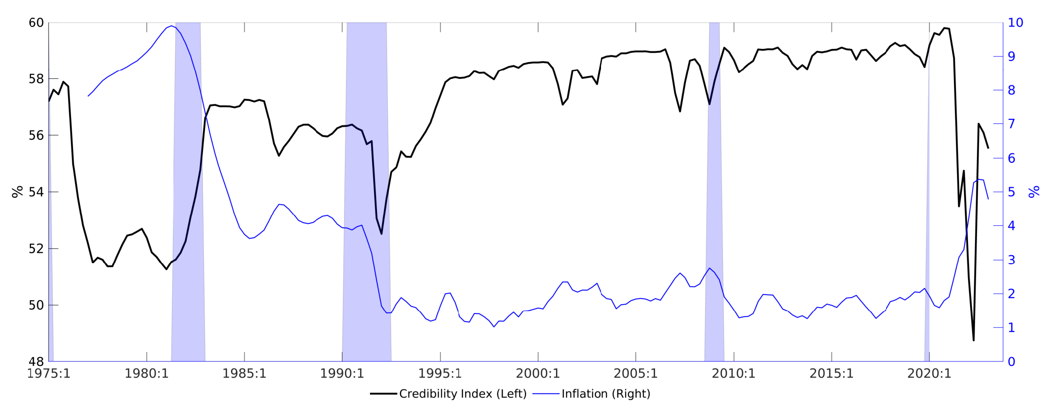


Figure 6: Estimated central bank credibility index and realized inflation (RHS) over the sample period 1975Q1–2023Q1.

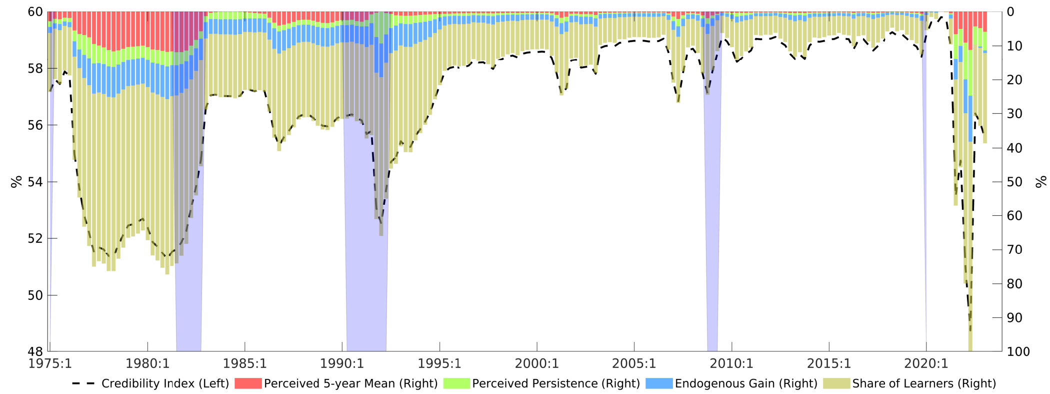


Figure 7: Estimated central bank credibility index (left scale), and the marginal contribution of each component of adaptive expectations to central bank credibility loss (right scale) over the sample period 1975Q1–2023Q1.

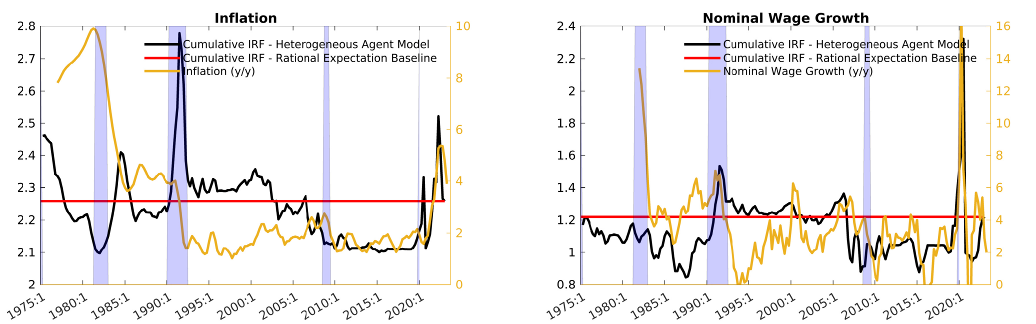


Figure 8: Cumulative responses of inflation and nominal wage growth to a one standard deviation price mark-up shock. The black line corresponds to the two-year cumulative responses in the heterogeneous agent model, while the dotted red line shows the response under the rational expectation baseline with full credibility. The shaded areas correspond to recessions that are characterized as episodes with two consecutive quarters of negative growth.

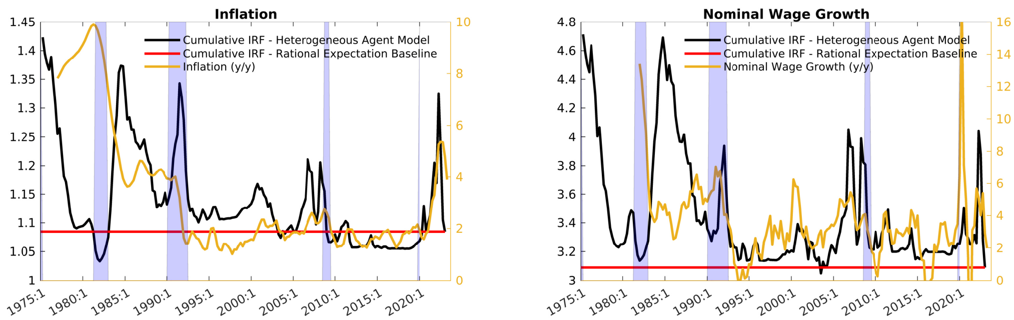


Figure 9: Cumulative responses of inflation and nominal wage growth to a one standard deviation wage mark-up shock. The black line corresponds to the two-year cumulative responses in the heterogeneous agent model, while the dotted red line shows the response under the rational expectation baseline with full credibility. The shaded areas correspond to recessions.

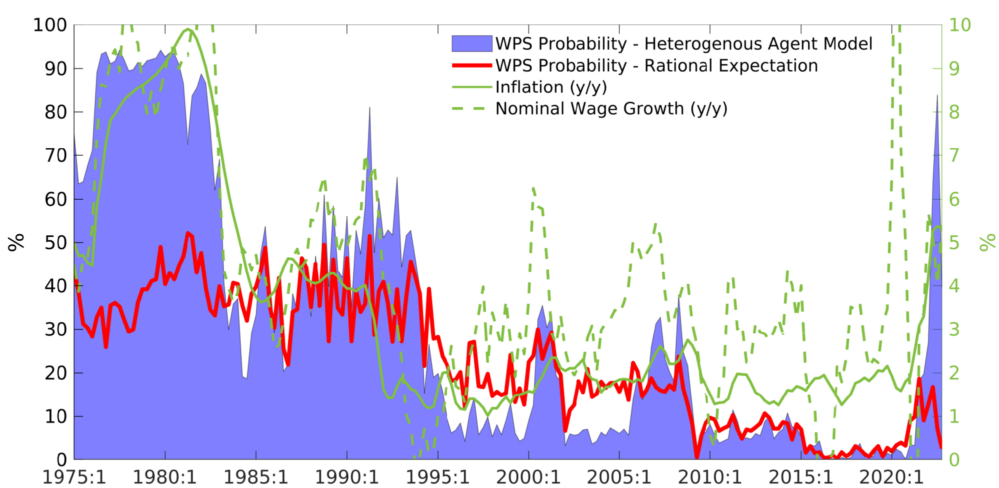
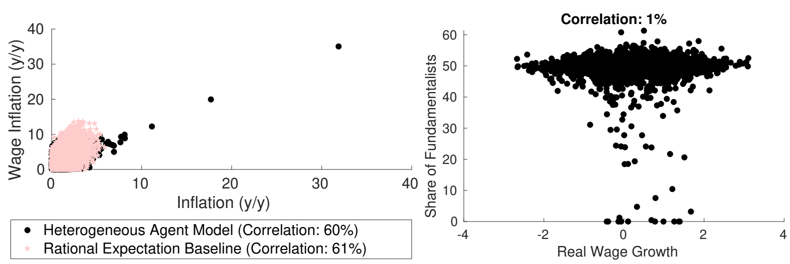
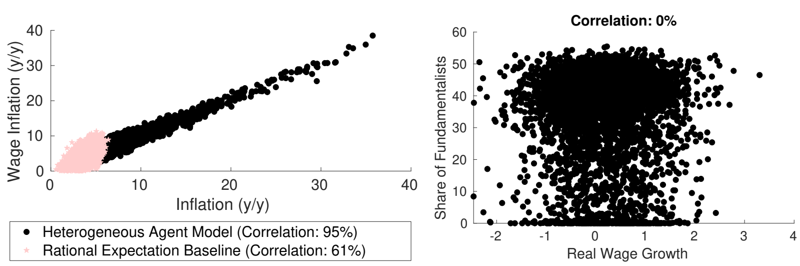


Figure 10: Probability of encountering a WPS episode over time.

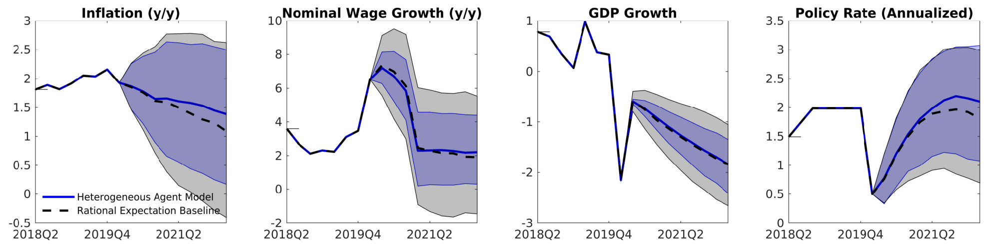


(a) Low inflation (2020Q2).

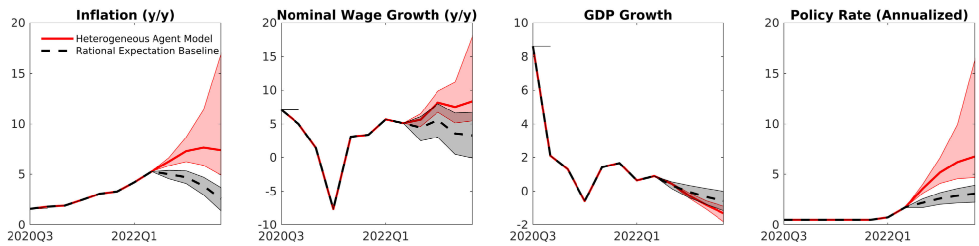


(b) High inflation (2022Q3).

Figure 11: Left panel: Scattergram of inflation-nominal wage density forecasts starting from 2020Q2 and 2022Q3. Right panel: Scattergram of real wage growth and the share of fundamentalist agents.

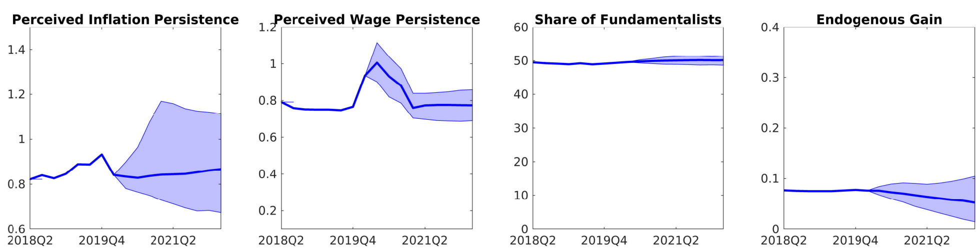


(a) Low inflation (2020Q2).

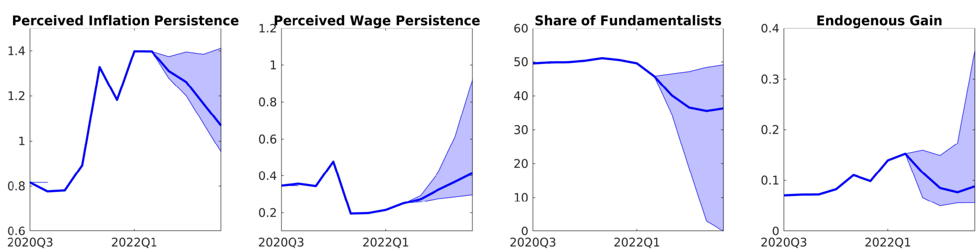


(b) High inflation (2022Q3).

Figure 12: Density forecasts of selected variables in the heterogeneous agent model (blue area) and the rational expectation baseline (grey area).



(a) Low inflation (2020Q2).



(b) High inflation (2022Q3).

Figure 13: Density forecasts of selected perception and learning variables in the heterogeneous agent model.

Table 1: Estimated and calibrated parameters for SW07 model using Canadian data over 1973Q1–2023Q1.

Parameters	Description	Prior Mean	Prior Std.	Post. Mode	Std. at Mode
ϕ	Capital adj. cost	4	1.5	7.08	0.89
σ_c	(Inv.) Elastiticty of subst.	1.5	0.38	1.41	0.07
λ	Cons. Habit	0.7	0.1	0.74	0.03
ξ_w	Wage Calvo	0.5	0.2	0.97	0.004
σ_l	Elasticity of labor	2	0.25	1.99	0.25
ξ_p	Price Calvo	0.5	0.2	0.88	0.03
ψ	Elasticity of cap. Util.	0.5	0.15	0.57	0.14
ϕ_p	Production fixed cost	1.25	0.13	0.97	0.04
r_π	MP inflation reaction	1.25	0.25	1.34	0.08
ρ	MP smoothing	0.75	0.05	0.79	0.02
$r_{\Delta y}$	MP output gap reaction	0.125	0.05	0.03	0.01
$\bar{\beta}$	discount factor	0.25	0.1	0.17	0.06
$\bar{l}$	Avg. growth of hours worked	0	2	0	0.02
$\bar{\gamma}$	Balanced growth rate	0.4	0.1	0.24	0.01
α	Capital share of output	0.3	0.05	0.12	0.01
ρ_a	Shock pers. - TFP	0.5	0.2	0.89	0.02
ρ_b	Shock pers. - Risk Premium	0.5	0.2	0.04	0.03
ρ_g	Shock pers. - Gov. Spending	0.5	0.2	0.99	0.004
ρ_i	Shock pers. - Investment	0.5	0.2	0.97	0.02
ρ_r	Shock pers. - Mon. Policy	0.5	0.2	0.26	0.08
ρ_{ga}	TFP/Gov. feedback	0.5	0.25	0.27	0.05
σ_a	Shock std. - TFP	0.1	4	0.87	0.05
σ_b	Shock std. - Risk Premium	0.1	4	6.16	0.69
σ_g	Shock std. - Gov. Spending	0.1	4	0.63	0.03
σ_i	Shock std. - Investment	0.1	4	0.28	0.04
σ_r	Shock std. - Mon. Policy	0.1	4	0.24	0.01
σ_p	Shock std. - Price mark-up	0.1	4	0.32	0.02
σ_w	Shock std. - Wage mark-up	0.1	4	0.74	0.04
Fixed structural parameters		Value			
ι_p	Price indexation	0			
ι_w	Wage indexation	0			
ρ_p	Price mark-up persistence	0			
ρ_w	Wage mark-up persistence	0			
$\bar{\pi}$	Inflation target	2% (y/y)			
Calibrated Learning/Switching parameters					
ω	Switching Memory	0.62	(Ozden, 2023)		
χ	Intensity of Choice	0.51	(Ozden, 2023)		
γ_0	Initial gain	0.059	(Ozden, 2023)		
μ	Gain step-size	0.001	(Kostyshyna, 2012)		
R_0	Initial cov. matrix	0.001	(Hommes et al., 2023)		

Table 2: RMSE for the learning model relative to the RE benchmark.

Variable	Evaluation Period	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
Inflation (y/y)	1976Q2-2022Q4	41.2*	29.3*	21.3*	14.9*	7.5*	4.4*	3.9*	3.9*
Inflation Expectations	1998Q3-2022Q4	23.0*	14.4*	13.1	7.1	2.9	-0.8	-1.9	-3
Nominal Wage Growth (y/y)	1981Q2- 2022Q4	-0.6	1.6	3.8	7.0	3.8	1.8	0.8	-0.1
Wage Expectations	2006Q4-2022Q4	-15.6	4.5	6.3	10.1	13.4	14.5	15.8	15.8

Notes: This table presents percentage improvements or deteriorations (indicated by -) in RMSE for the learning model relative to the RE benchmark. The period of evaluation for each variable is indicated in the second column. Starred values indicate statistical significance at 5% level according to the Diebold-Mariano test (2015).

Table 3: Marginal contribution of different behavioral features to inflation forecasts.

	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
HAM	41.2	29.3	21.3	14.9	7.5	4.4	3.9	3.9
Constant gain	40.5	28.8	20.5	13.8	5.4	2.9	2.9	3.3
No switching	38.5	27.1	19.9	14.2	6.7	4.5	4.5	4.9
No learning	30.5	17.3	8.7	2.3	-4.9	-6.1	-5.4	-4.5

Notes: The numbers correspond to percentage improvements or deteriorations (indicated by -) in inflation RMSE for each model, relative to the RE benchmark. **HAM** refers to the baseline heterogeneous agent model with all heterogeneous expectations and learning features. **Constant gain** model turns off the endogenous gain feature in (9) and keeps the gain value fixed at its initial value. **No switching** turns off the endogenous switching rule in (6) and (7), and fixes the shares of fundamentalist and adaptive agents at 50%. **No learning** turns off the adaptive learning dynamics in 5) and fixes beliefs of adaptive learners with a static AR(1) rule.

Appendix

A Full Model Equations

This appendix provides the full model equations. The model consists of 13 linearized equations and 7 exogenous AR(1) shocks. Micro-foundations of the model can be found in Smets and Wouters (2007). The aggregate resource constraint is given by:

$$\begin{cases} \tilde{y}_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g, \\ \epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g, \end{cases} \tag{11}$$

where $\tilde{y}_t, c_t, i_t$ and z_t are the output, consumption, investment and capital utilization rate, respectively. c_y, i_y and z_y are the steady-state shares in output of the respective variables. The second equation in (11) defines the AR(1) government spending shock. The consumption Euler equation is given by:

$$\begin{cases} c_t = c_1 c_{t-1} + (1-c_1)\mathbb{E}_t c_{t+1} + c_2(l_t - \mathbb{E}_t l_{t+1}) - c_3(r_t - \mathbb{E}_t \pi_{t+1}) + \epsilon_t^b, \\ \epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b, \end{cases} \tag{12}$$

with $c_1 = \frac{\lambda}{\gamma}/(1+\frac{\lambda}{\gamma})$,$c_2 = (\sigma_c - 1)(w_{ss} l_{ss}/c_{ss})/(\sigma_c(1+\frac{\lambda}{\gamma}))$,$c_3 = (1-\frac{\lambda}{\gamma})/((1+\frac{\lambda}{\gamma})\sigma_c)$, where λ, γ and σ_c denote the habit formation in consumption, steady-state growth rate and the elasticity of intertemporal substitution. x_{ss} corresponds to the steady-state level of a given variable x. ϵ_t^b is the AR(1) risk premium shock. The investment Euler equation is defined as:

$$\begin{cases} i_t = i_1 i_{t-1} + (1-i_1)\mathbb{E}_t i_{t+1} + i_2 q_t + \epsilon_t^i, \\ \epsilon_t^i = \rho_i \epsilon_{t-1}^i + \eta_t^i, \end{cases} \tag{13}$$

with $i_1 = \frac{1}{1+\bar{\beta}\gamma}, i_2 = \frac{1}{(1+\bar{\beta}\gamma)(\gamma^2\phi)}$, where $\bar{\beta} = \beta\gamma^{-\sigma_c}$, ϕ is the steady-state elasticity of capital adjustment cost and β is the household discount factor. q_t denotes the real value of existing capital stock. ϵ_t^i represents the AR(1) investment shock. The capital-arbitrage equation is given by:

$$q_t = q_1 \mathbb{E}_t q_{t+1} + (1-q_1)\mathbb{E}_t r_{t+1}^k - (r_t - \mathbb{E}_t \pi_{t+1}) + \tfrac{1}{c_3}\epsilon_t^b, \tag{14}$$

with $q_1 = \bar{\beta}(1-\delta)$. The production function is characterized as:

$$\begin{cases} \tilde{y}_t = \phi_p(\alpha k_t^s + (1-\alpha)l_t + \epsilon_t^a), \\ \epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a, \end{cases} \tag{15}$$

where k_t^s denotes the capital services used in production, α is the share of capital in production and ϕ_p is the share of fixed costs in production. ϵ_t^a is the AR(1) total factor productivity (TFP) shock. Capital is given by:

$$k_t^s = k_{t-1} + z_t. \tag{16}$$

The degree of capital utilization is a function of the degree of rental rate, $z_t = z_1 r_t^k$, with $z_1 = \frac{1-\psi}{\psi}$, and ψ the elasticity of capital utilization adjustment cost. Installed capital is given by:

$$k_t = k_1 k_{t-1} + (1-k_1)i_t + k_2 \epsilon_t^i, \tag{17}$$

with $k_1 = \frac{1-\delta}{\gamma}, k_2 = (1 - \frac{1-\delta}{\gamma})(1+\bar{\beta}\gamma)\gamma^2\phi$. The price mark-up equation is given by:

$$\mu_t^p = \alpha(k_t^s - l_t) + \epsilon_t^a - w_t. \tag{18}$$

The NKPC is given by:

$$\begin{cases} \pi_t = \pi_1 \mathbb{E}_t \pi_{t+1} - \pi_2 \mu_t^p + \epsilon_t^p, \\ \epsilon_t^p = \rho_p \epsilon_{t-1}^p + \eta_t^p, \end{cases} \tag{19}$$

with $\pi_1 = \bar{\beta}\gamma$,$\pi_2 = (1-\beta\gamma\xi_p)(1-\xi_p)/[\xi_p((\phi_p-1)\epsilon_p+1)]$, where ξ_p corresponds to the degree of price stickiness, while ϵ_p denotes the Kimball goods market aggregator. ϵ_t^p is the AR(1) price mark-up shock. The rental rate of capital is given by:

$$r_t^k = -(k_t - l_t) + w_t. \tag{20}$$

The wage mark-up is given by:

$$\mu_t^w = w_t - (\sigma_l l_t + \frac{1}{1-\lambda/\gamma}(c_t - \frac{\lambda}{\gamma}c_{t-1}), \tag{21}$$

where σ_l denotes the elasticity of labour supply. The real wage equation is given by:

$$\begin{cases} w_t = w_1 w_{t-1} + (1-w_1)(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) - w_2 \mu_t^w + \epsilon_t^w, \\ \epsilon_t^w = \rho_w \epsilon_{t-1}^w + \eta_t^w, \end{cases} \tag{22}$$

with $w_1 = 1/(1+\bar{\beta}\gamma)$ and $w_2 = ((1-\bar{\beta}\gamma\xi_w)(1-\xi_w)/(\xi_w(\phi_w - 1)\epsilon_w + 1))$. ϵ_t^w denotes the wage mark-up shock. Monetary policy follows a standard Taylor rule with:

$$\begin{cases} r_t = \rho r_{t-1} + (1-\rho)(\phi_\pi \pi_t + \phi_y y_t) + \phi_{\Delta y}(\Delta y_t) + \epsilon_t^r, \\ \epsilon_t^r = \rho_r \epsilon_{t-1}^r + \eta_t^r, \end{cases} \tag{23}$$

where y_t denotes the output gap and ϵ_t^r is the AR(1) monetary policy shock. We do not consider the flexible-price economy of the model to define the output gap. Instead, we assume that output gap is given by the deviation of output from the underlying TFP process, $y_t = \tilde{y}_t - \Phi_p \epsilon_t^a$.

B E-Stability Properties under Endogenous Gain

In this appendix, we examine the Expectational Stability (E-stability) properties of the endogenous gain learning mechanism presented in Section 3. Consider the NKPC shown in (1):

$$\begin{cases} \pi_t = \pi_1 \mathbb{E}_t \pi_{t+1} - \pi_2 \mu_t^p + \epsilon_t^p. \end{cases} \tag{24}$$

For tractability, we consider a skeleton version of the SW07 model and focus on the E-stability of inflation dynamics. Accordingly, assume that the law of motion for real marginal cost μ_t^p takes the form of an IS curve as in the standard three-equation New Keynesian model (Gali, 2008), and monetary policy reacts to inflation only:

$$\begin{cases} -\mu_t^p = \mathbb{E}_t \mu_{t+1}^p - \frac{1}{\tau}(r_t - \mathbb{E}_t \pi_{t+1}), \\ r_t = \phi_\pi \pi_t. \end{cases} \tag{25}$$

Real marginal cost expectations $\mathbb{E}_t \mu_{t+1}^p$ follow the fundamentalist rule, in line with the full quantitative version of the model in Section 3. Given the absence of lagged state variables in (24) and (25), expectations associated with the fundamentalist rule take the form of $\begin{bmatrix} \mathbb{E}_t \mu_{t+1}^p \\ \mathbb{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{\mu}^p \\ \bar{\pi} \end{bmatrix}$ for some constants $\bar{\mu}^p$ and $\bar{\pi}$. We use $\bar{\mu}^p = 0$ and $\bar{\pi} = 0$ without loss of generality, which reduces the three-equation system to:

$$\begin{cases} -\mu_t^p = -\frac{1}{\tau}(r_t - \mathbb{E}_t \pi_{t+1}), \\ r_t = \phi_\pi \pi_t, \\ \pi_t = \pi_1 \mathbb{E}_t \pi_{t+1} - \pi_2 \mu_t^p + \epsilon_t^p. \end{cases} \tag{26}$$

The system in (26) can be re-written in terms of inflation and inflation expectations:

$$\pi = \Gamma \mathbb{E}_t \pi_{t+1}, \tag{27}$$

where $\Gamma = \frac{\pi_1 \tau + \pi_2}{\tau + \pi_2 \phi_\pi}$. Now consider the AR(1) expectation formation rule of adaptive learners in (5). We abstract away from learning about the mean and focus on the perceived persistence β_t^π, so that $\alpha_t^\pi = 0$. It follows that the law of motion for one-step-ahead expectations of adaptive learners is given by:

$$\mathbb{E}_t^L \pi_{t+1} = \beta_{t-1}^\pi \pi_{t-1}. \tag{28}$$

Noting that the fundamentalist rule for inflation expectations is given by $\mathbb{E}_t^R \pi_{t+1} = \bar{\pi} = 0$, the aggregate expectations take the form of:

$$\mathbb{E}_t \pi_{t+1} = n_{t-1}^L \beta_{t-1}^\pi \pi_{t-1}. \tag{29}$$

Plugging this into (27), the implied actual law of motion for inflation becomes:

$$\pi_t = \Gamma n_{t-1}^L \beta_{t-1}^\pi \pi_{t-1}. \tag{30}$$

The corresponding mapping from agents' PLM to the implied ALM, that is, the T-map (Evans and Honkapohja, 2012), is given by:

$$\beta_{t-1}^\pi \Rightarrow T(\beta_{t-1}^\pi) = \Gamma n_{t-1}^L \beta_{t-1}^\pi \tag{31}$$

The associated E-stability condition for the T-map depends on the underlying gain value. For a sequence of decreasing gain values, the E-stability condition is satisfied if the following condition holds:

$$\frac{\partial T(\beta_{t-1}^\pi)}{\partial \beta_{t-1}^\pi} = \Gamma n_{t-1}^L < 1. \tag{32}$$

In the absence of a decreasing sequence of gain values, Evans and Honkapohja (2009) show that the E-stability condition becomes stricter. Our endogenous gain learning environment falls under this category, therefore we follow the same steps as in Evans and Honkapohja (2009) to outline the stability conditions. To derive an analytical expression, we make a few simplifying assumptions. Specifically, we assume that $R_t^\pi = 1$ in the learning recursions in (8), which is known as stochastic gradient learning (Evans et al., 2010). Using $\mathbb{E}_{t-1}^L \pi_t = \beta_{t-2}^\pi \pi_{t-1}$, we get the following law of motion for the perceived persistence:

$$\beta_t^\pi = \beta_{t-1}^\pi + \gamma_{t-1}\pi_{t-1}\Big(\pi_t - \beta_{t-2}^\pi \pi_{t-1}\Big). \tag{33}$$

Replacing π_t with the expression in (30) yields:

$$\beta_t^\pi = \beta_{t-1}^\pi + \gamma_{t-1}\pi_{t-1}\Big(\Gamma n_{t-1}^L \beta_{t-1}^\pi \pi_{t-1} - \beta_{t-2}^\pi \pi_{t-1}\Big). \tag{34}$$

Re-arranging, we obtain:

$$\beta_t^\pi = (1 + \Gamma\gamma_{t-1} n_{t-1}\pi_{t-1}^2)\beta_{t-1}^\pi - (\gamma_{t-1}\pi_{t-1}^2)\beta_{t-2}^\pi. \tag{35}$$

Re-write the expression in (35) in companion VAR(1) form:

$$\tilde{\beta}_t = \tilde{\Gamma}\tilde{\beta}_{t-1}, \tag{36}$$

where $\tilde{\beta}_t = \begin{bmatrix} \beta_t^\pi \\ \beta_{t-1}^\pi \end{bmatrix}$, $\tilde{\gamma} = \begin{bmatrix} a_t & -b_t \\ 1 & 0 \end{bmatrix}$, $a_t = (1 + \Gamma\gamma_{t-1} n_{t-1}\pi_{t-1}^2)$, and $b_t = (\gamma_{t-1}\pi_{t-1}^2)$. The eigenvalues associated with the companion form VAR(1) are given by:

$$\begin{cases} \lambda_{1,t} = \frac{1}{2}\Big(a_t - \sqrt{a_t^2 - 4b_t}\Big), \\ \lambda_{2,t} = \frac{1}{2}\Big(a_t + \sqrt{a_t^2 - 4b_t}\Big). \end{cases} \tag{37}$$

The E-stability condition for the perceived persistence of inflation is satisfied whenever $\rho(\beta_t^\pi) = max(|\lambda_{1,t}|, |\lambda_{2,t}|) < 1$. Thus, the E-stability condition for the inflation law of motion in (30) is satisfied whenever $\Gamma n_{t-1}^L \rho(\beta_t^\pi) < 1$. Based on the expressions for a_t and b_t, E-stability depends on:

- past values of inflation π_{t-1},
- the structural parameters in Γ,
- the share of adaptive learners n_{t-1}^L, and
- the endogenous gain value γ_{t-1}.

We focus on the relationship between E-stability and the endogenous gain for selected values of the share of learners and the level of inflation. The structural parameters are set to conventional values at $\tau = 2$, $\pi_1 = 0.99$, $\pi_2 = 0.05$ and $\phi_\pi = 1.5$. We fix the values for inflation and the share of adaptive learners at specific values of interest. In particular, we focus on the following four cases:

1. Low inflation ($\pi_{t-1} = 2.5\%$) and a low share of adaptive learners $n_t^L = 0.5$
2. Low inflation ($\pi_{t-1} = 2.5\%$) and a high share of adaptive learners $n_t^L = 0.95$
3. High inflation ($\pi_{t-1} = 5\%$) and a low share of adaptive learners $n_t^L = 0.5$
4. High inflation ($\pi_{t-1} = 5\%$) and a low share of adaptive learners $n_t^L = 0.95$

This offers four snippets that illustrate the interaction between endogenous gain and E-stability conditions within the space of inflation and the population shares of agents. Figure 14 shows the corresponding largest eigenvalues of the system in these four cases as a function of endogenous gain. The figure illustrates two important results:

1. Higher inflation pushes the perceived persistence and inflation law of motion toward the E-unstable region, which makes explosive outcomes more likely.
2. A higher share of adaptive learners make E-unstability more likely when the endogenous gain is sufficiently high.

Overall, both high inflation and a high share of adaptive learners make E-unstable outcomes more likely. Therefore, both layers of learning (endogenous gain and the time-varying shares of agents) can act as a persistence amplification mechanism during high inflation episodes to generate a larger upside risk on inflation.

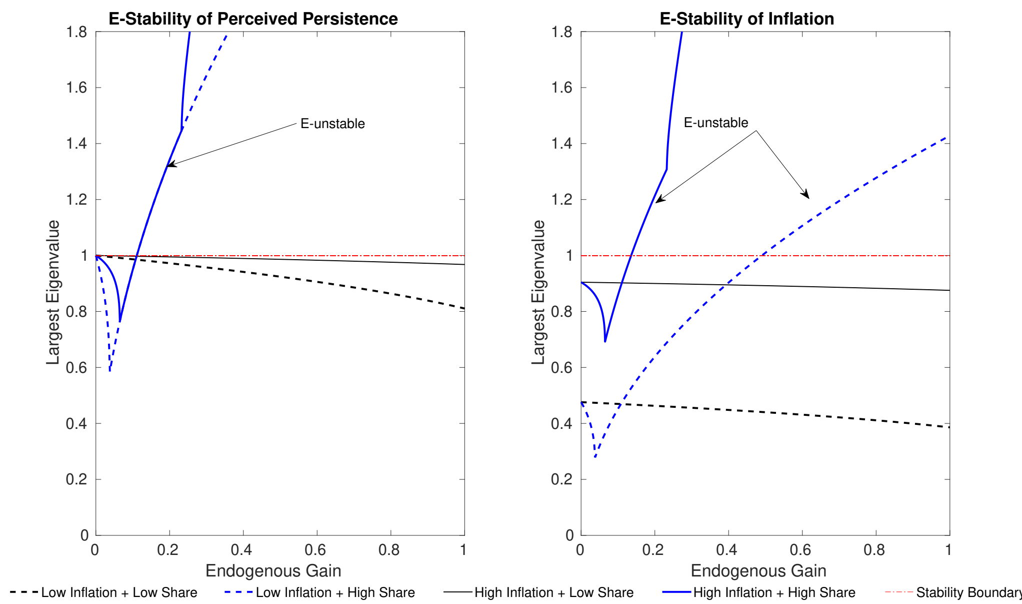


Figure 14: E-stability dynamics as a function of endogenous gain for selected values of inflation and the share of adaptive learners.

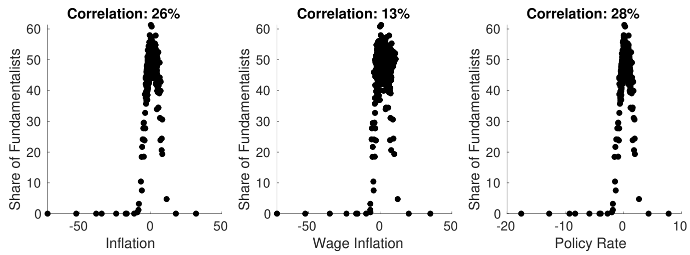
C Additional Distributions in Density Forecasts

To further examine the connection between shares of fundamentalist and adaptive agents and inflation and nominal wage growth, we present the distribution of simulated outcome in Figure 15. In a low-inflation period (2020Q2), the distributions of inflation, wage growth and policy rate are fairly symmetric, with a low frequency of extreme outcomes when the share of fundamentalists approaches zero levels.[20]

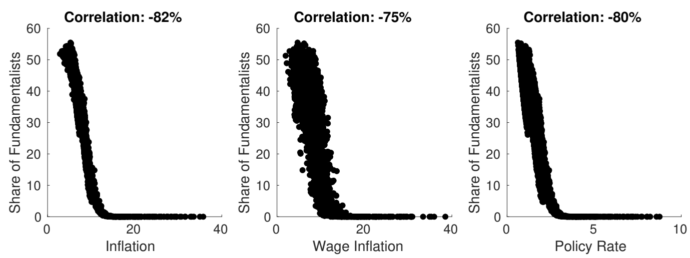
In contrast, in a high-inflation episode (2022Q3), the distributions become skewed and one-sided, where extreme outcomes with high inflation, nominal wage growth and policy rate are much more frequent. There is a very strong relationship between the proportion of fundamentalists and these extreme

[20]Note that we do not incorporate the ELB constraint on nominal interest rates in our model, hence monetary policy can become negative during deflationary traps. Ozden (2024) shows that the risk of deflationary spirals may become amplified in the presence of the ELB constraint. We abstract away from these outcomes in our paper since our focus is on the upside risk of inflation and inflation expectations, rather than deflationary spirals.

outcomes: The decline of the share of fundamentalists contributes to weaker central bank credibility and leads to a higher likelihood of high inflation and nominal wage growth.



(a) Low inflation (2020Q2).



(b) High inflation (2022Q3).

Figure 15: Scattergram of selected variables against central bank credibility using density forecasts starting from 2020Q2 (left panel) and 2022Q3 (right panel)

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