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Risk Amplification Macro Model (RAMM)

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Abstract

The Risk Amplification Macro Model (RAMM) is a new nonlinear two-country dynamic model that captures rare but severe adverse shocks. Tail risk arises from heightened financial stress abroad or in Canada that triggers a regime change with a negative feedback loop to the real economy. We rely on a combination of sign, zero and elasticity restrictions to identify structural shocks. The foreign block (global and US variables) impacts the domestic block (a large number of Canadian macrofinancial variables), but not vice-versa. Simulations suggest that tighter financial conditions in the United States can spill over to Canada, and a regime change in macrofinancial elasticities provides a good replication of economic downturns. The RAMM can be used to assess the financial stability implications of both domestic and foreign-originated risk scenarios.

Topics: Financial stability, Business fluctuations and cycles, Econometric and statistical methods, Monetary policy transmission
JEL codes: C51, E37, E44, F44

Résumé

Le modèle macroéconomique d'amplification des risques (modèle RAMM) est un nouveau modèle dynamique non linéaire à deux pays qui rend compte de chocs négatifs rares, mais graves. Les risques extrêmes découlent de tensions financières accrues à l'étranger ou au Canada, lesquelles déclenchent un changement de régime ayant un effet de rétroaction négatif sur l'économie réelle. Nous comptons sur une combinaison de contraintes de signe, de nullité et d'élasticité pour cerner les chocs structurels. Le bloc de variables étrangères (mondiales et américaines) a une incidence sur le bloc de variables intérieures (un grand nombre de variables macrofinancières canadiennes), mais pas l'inverse. Des simulations donnent à penser qu'un resserrement des conditions financières aux États-Unis peut se répercuter sur le Canada, et qu'un changement de régime (et donc des « élasticités macrofinancières ») reproduit bien les ralentissements économiques. Le modèle RAMM peut être utilisé pour évaluer les implications de scénarios de risques d'origine intérieure ou étrangère pour la stabilité financière.

Sujets : Stabilité financière, Cycles et fluctuations économiques, Méthodes économétriques et statistiques, Transmission de la politique monétaire
Codes JEL : C51, E37, E44, F44

1 Introduction

Analyzing the impact of risks on the aggregate macroeconomic and financial variables is an integral part of assessing the resilience of the financial system. From a financial stability perspective, quantifying rare and severe but also plausible risks is of the utmost importance. The repercussions of these tail-risk events are transmitted and amplified through the financial system. One trigger for amplifications of the impacts of adverse shocks could be elevated financial stress in the economy. The literature supports this view by documenting empirical evidence that the economic downturns associated with high financial stress are more severe (Reinhart and Rogoff, 2009, 2014; Krishnamurthy and Muir, 2020; Adrian, Boyarchenko, and Giannone, 2019). Therefore, for risk scenario analyses and stress testing, it is important to have a model that takes risk amplification and financial stress into account.

In this paper, we present the Risk Amplification Macro Model (RAMM), a nonlinear two-country dynamic model that can generate macrofinancial risk scenarios. As a risk amplification mechanism, we use a threshold vector autoregressive (TVAR) model where high financial stress, i.e., systemic financial turmoil spanning several market segments, could trigger an endogenous regime change. This nonlinearity gives rise to different relationships between macrofinancial variables based on the financial stress level. As a result, the model allows for a higher reaction of macroeconomic aggregates to adverse shocks during high financial stress. To analyze international risk scenarios, the RAMM entails one TVAR model for foreign variables and another TVAR model for Canadian variables. Canada is modelled as a small open economy. Overall, the RAMM is a two-country and two-regime model, where elasticities and correlations between variables are allowed to differ in low and high financial stress regimes specific to each country.

Financial stress indices are an integral part of the RAMM's determination of the endogenous regime switching. We use the measures of Duprey, Klaus, and Peltonen (2017) and Duprey (2020) for the foreign and Canadian financial stress indices, respectively. These studies measure a systemic financial stress that causes financial turmoil spanning several market segments, asset classes and market prices. Time-varying correlations provide a system-wide financial stress index reflecting important simultaneous movements in the financial sector. Even though Canada's financial system is closely linked to that of the United States, the financial stress episodes in each country do not necessarily coincide. The correlation coefficient between the two indices is around 0.60. These facts constitute a rationale for having two separate TVAR models for each block where stress regimes are driven by the country-specific financial stress index.

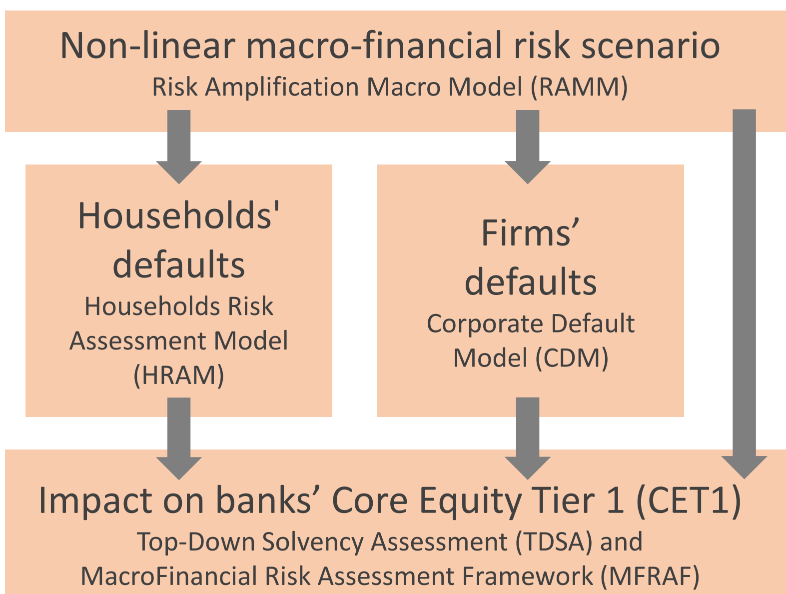
Containing 7 foreign and 24 Canadian variables together with regime-specific VAR coefficients, the RAMM can be considered a medium- to large-scale model. To estimate such a complex model, we use shrinkage techniques for large Bayesian TVAR models developed by Bruneau, Chapman, and Tuzcuoglu (forthcoming), who provide an extension of Bańbura, Giannone, and Reichlin (2010)'s efficient shrinkage estimation method for large Bayesian VAR models. The structural shock identification, both in the foreign and domestic blocks, relies generally on theory-driven sign and zero restrictions. We also discuss how extra restrictions, such as a Taylor rule, can be incorporated in the shock identification algorithm. Having identified the economically meaningful shocks, the TVAR models allow us to compute unconditional impulse responses as well as conditional forecasts of macro aggregates after imposing certain paths for policy variables. In all these exercises, the benefit of the RAMM over linear models is its ability to capture amplification effects that cannot be obtained from average historical relationships, should the financial stress level become high enough.

Finally, we conduct a simple scenario analysis exercise focusing on the 2008–09 global financial crisis episode. In contrast to a typical scenario analysis where one makes assumptions on the magnitude and duration of structural shocks, we simply take the observed foreign data as given. This approach corresponds to a scenario with perfect foresight of the foreign structural shocks. Then, we compare the RAMM's predictions for the Canadian variables when we allow regime switching versus when the model is restricted to the low-stress regime. Regime switching provides higher elasticities in high stress, with gross domestic product (GDP) reacting much more, creating a negative macro feedback loop with the financial stress. In contrast, not allowing regime switching, that is, mimicking the use of a single-regime VAR model, would lose the ability to explain a significant portion of the economic downturn despite having perfect foresight of the foreign shocks. For instance, at the trough, a single-regime model would miss an extra 5% drop in annualized real GDP growth, a nearly 1 percentage point additional increase in the unemployment rate, an extra $30\% - 40\%$ decrease in investment as well as sharp declines in other macrofinancial variables. In other words, a model without regime switching would have predicted a much milder recession than it actually was and also than a model with two-regimes could predict. This finding is also supported by Alessandri and Mumtaz (2017) who show that financial variables display significant predictive power over the global financial crisis period, particularly if used within a threshold model.

The RAMM is related to TVAR models in other studies analyzing the role of nonlinearities. For example, Balke (2000) examines the roles of credit conditions as a nonlinear propagator of shocks, Alessandri and Mumtaz (2019) find that the recessionary effects of uncertainty shocks are much larger when the economy is in high financial stress, and Chatterjee, Chiu, Duprey, and Hacıoğlu-Hoke (2022) use the UK financial stress index as a trigger to highlight the importance of shock amplification due to nonlinearities. The RAMM deviates from the models in this literature on various fronts: (i) it is a two-country model and thus includes four regimes, while other models focus on a single country with two regimes; (ii) it has a much larger scale—containing 31 variables that span a wider proportion of the economy—than other models; (iii) it has a richer set of identified shocks, which makes it more policy-relevant.

The RAMM can be utilized for several purposes: to assess the risks of international or domestic tail events, to conduct counterfactual analysis of different policies, and to generate macro risk scenarios for stress testing of banks. While the RAMM can operate as a stand-alone model, it can also be used in conjunction with other models. For instance, the RAMM is used as the first module in the Framework for Risk Identification and Assessment (FRIDA, MacDonald and Traclet (2018)), a suite of models developed at the Bank of Canada to quantify the impact of financial stability risks. The risk scenario generated by the RAMM is an input for other models in the FRIDA to conduct top-down stress testing of financial system participants, such as households, corporations and banks. In particular, the output of the RAMM—a severe tail-risk scenario—enters into the Corporate Default Model (CDM, Bruneau, Duprey, and Hipp (2022)) and the Household Risk Assessment Model (HRAM, Peterson and Roberts (2016)), which provide scenario-consistent default probabilities for corporate and household loans, respectively. Finally, the stress scenario and the default probabilities are all fed into banking sector models, such as the Top-Down Solvency Assessment (TDSA) and MacroFinancial Risk Assessment Framework (MFRAF, Fique (2017)) models, to assess the capital positions and solvency of the banks. The FRIDA can be illustrated as in Figure 1.

Figure 1: Framework for Risk Identification and Assessment:
An overview



The plan of the rest of the paper is as follows. The data are discussed in section 2. The econometric model is introduced in section 3 together with its estimation method.

The shock identification strategy is explained in section 4. The impulse responses are presented in section 5. A scenario analysis is illustrated in section 6. Finally, section 7 concludes. The appendix at the end of the paper contains further details on the data (section A) and some mathematical derivations (section B).

2 The data

The RAMM data set contains 7 foreign and 24 domestic variables at monthly frequency with a sample period of January 1983 to December 2019, i.e., 444 time-series observations for each variable. We end the dataset before the onset of the COVID-19 pandemic since the inclusion of the observations after 2020 could skew the results. Omitting the pandemic episode for estimation is an acceptable method because the primary aim of the RAMM is parameter estimation and structural analysis instead of forecasting (see Lenza and Primiceri (2020) and Ng (2021) for potential solutions). The growth rates are calculated as annualized quarter-over-quarter log differences. We opt not to use month-over-month growth rates since they are too noisy. As well, we prefer not to use quarterly frequency since we would miss intra-quarter variations and also end up with very few observations in high-stress regimes. We obtain real variables by dividing their nominal values by the US and Canadian consumer price index (CPI) for foreign and domestic variables, respectively. We seasonally adjust the series if necessary.

Foreign variables For oil prices and non-energy commodity prices, we use the refiner's acquisition cost of crude oil and the Fisher commodity price index (all sectors excluding energy), respectively. For the global term premium variable, we use the US five-year term premium.[1] For US macrofinancial variables, we use US real GDP, US CPI, the effective federal funds rate (FFR), and the US Financial Stress Index (USFSI) measured following the method of Duprey et al. (2017). Finally, we take the growth rates of real commodity prices, real GDP and CPI. We use the levels of the

[1] Note that we have also calculated a principal component from major economies' term premia. However, the resulting factor has a correlation coefficient of 0.97 with the US term premium. Hence, we treat the US term premium as the global term premium.

term premium, FFR and USFSI. The stress index is indexed to be between 0 and 100.

Canadian core variables We select the Canadian core variables in accordance with their US counterparts. Specifically, we use Canadian real GDP, CPI, the overnight rate and the Canadian Financial Stress Index (CFSI), which is provided by Duprey (2020) and is also indexed to be between 0 and 100. The housing activity variable is constructed as follows: we sum residential unit sales and housing starts in all of Canada, then we divide this sum by the population. The total of residential sales and new housing captures the total activity in the housing sector, while the division by the population makes this sum stationary. We also multiply it by 100 so that housing activity has a standard deviation of a similar magnitude as the other variables. As a result, the unit of this variable is 100 residential sales or new housing per capita within a month. Since its unit is complicated to interpret, we construct this variable as an index reflecting activity in the housing sector. Finally, we use the growth rate of real GDP and CPI, whereas levels are used for other variables.

Canadian satellite variables We use 19 macrofinancial variables: the unemployment rate, residential and non-residential investment, household disposable income, government spending, real imports from and exports to the United States, real house prices measured as the average residential sales price in Canada, the ratio of pre-tax net income to total assets of Canadian D-SIBs (domestic systemically important banks),[2] real residential mortgages from Canadian D-SIBs, real consumer loans from the Canadian D-SIBs, real business credit, consumer and business confidence indices, five-year Government of Canada bond yields, average five-year mortgage rates, corporate spreads, the nominal foreign exchange rate (CAD/USD) such that an increase indicates an appreciation in the Canadian dollar against the US dollar, and the real Toronto Stock Exchange composite index.

Further details on the data, such as their detailed definition, transformation and

[2]There are six D-SIBs in Canada: Bank of Montreal, Bank of Nova Scotia, Canadian Imperial Bank of Commerce, National Bank of Canada, Royal Bank of Canada and Toronto-Dominion Bank. The D-SIBs constitute a large majority of the banking system. They account for more than 90% of the total assets and total loans in the banking system.

sources, can be found in Table 4 in Section A of the appendix.

3 The model and estimation methodology

The RAMM is a two-country and two-regime Threshold Vector Autoregressive (TVAR) model. All of the VAR parameters, i.e., the autoregressive coefficients and the covariance matrices, are regime-dependent. The regimes are determined by the financial stress level in each country. Specifically, if the financial stress index for a country is above (below) a threshold level, which is a parameter to be estimated, then we say that this country is in a high-(low-)stress regime. In this sense, it is similar to a Markov-Switching VAR (MSVAR) model. However, one of the main differences arises from the fact that the regime switching in the TVAR model occurs because of an observed transition variable, namely the financial stress index, as opposed to a latent variable as in MSVAR models. Moreover, regime switching in the TVAR model is endogenous, whereas, in a typical MSVAR model, it occurs exogenously.

3.1 The model

There are two blocks in the RAMM: foreign and Canadian. We model Canada as a small open economy. Foreign variables can affect the Canadian economy, but not vice versa. In other words, foreign variables enter the Canadian TVAR model as exogenous variables, but Canadian variables do not enter the foreign one. For the foreign block, we use only US variables and commodity prices (oil and non-energy). We do not include any other country in the foreign block because the United States practically serves the role of the "rest of the world" for Canada by constituting around 75% of total Canadian trade. The effects of the European Union and China—Canada's other two big trade partners—are mostly captured by commodity prices and the exchange rate. The exchange rate will be included in the Canadian block since it is not exogenous with respect to domestic economic policies.

The vector of foreign endogenous variables is denoted by $\mathbf{Y}_t^F$ and is of dimension $(N^F \times 1)$, where $N^F = 7$. It contains the growth rate of the real oil price, the growth rate of the real non-energy commodity price index, the US 5-year term premium, the growth rate of US real GDP, the growth rate of US CPI, the FFR and the USFSI. The piecewise linear TVAR(p^F) equation for the foreign block has p^F lags and is given as follows:

$$\mathbf{Y}_t^F = \begin{cases} \sum_{i=1}^{p^F} \mathbf{A}_{L,i}^F \mathbf{Y}_{t-i}^F + \mathbf{e}_{L,t}^F & \text{if} \quad USFSI_{t-1} \leq \gamma^F \\ \sum_{i=1}^{p^F} \mathbf{A}_{H,i}^F \mathbf{Y}_{t-i}^F + \mathbf{e}_{H,t}^F & \text{if} \quad USFSI_{t-1} > \gamma^F, \end{cases}$$

where $\mathbf{e}_{L,t}^F \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_L^F)$ and $\mathbf{e}_{H,t}^F \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_H^F)$. The subscripts L and H denote the low- and high-stress regimes, respectively, which are determined by the relative levels of the transition variable at a lag of one month, $USFSI_{t-1}$, and the threshold parameter, γ^F.[3] Note that the $(N^F \times N^F)$ autoregressive coefficient matrices $\{\mathbf{A}_{L,i}^F, \mathbf{A}_{L,i}^H\}$, for $i = 1, \ldots, p^F$, and the $(N^F \times N^F)$ covariance matrices $\{\mathbf{\Sigma}_L^F, \mathbf{\Sigma}_H^F\}$ are all regime-dependent. A nice feature of the RAMM is that it does not force the coefficients to differ but lets the data speak in finding the optimal threshold value and differences in the regime-specific parameters. Another benefit of regime-dependent parameters is that they partially capture time-varying heteroskedasticity, which is one of the stylized facts of many macroeconomic and financial variables.

For the Canadian block, the vector of endogenous variables is denoted by $\mathbf{Y}_t^C$ and is of dimension $(N^C \times 1)$, where $N^C = 24$. Hence, this constitutes a medium- to large-scale TVAR model.[4] The endogenous domestic variables include the unemployment rate; the growth rates of the residential investment, non-residential investment, household disposable income, government spending, imports, exports and house price index; the level of pre-tax net income as a percentage of total assets of the D-SIBs; the growth

[3] In principle, we can generalize the lag of the transition variable from $USFSI_{t-1}$ to $USFSI_{t-\delta}$ and estimate the time delay parameter δ in our MCMC sampling. However, it is economically more reasonable to have $\delta = 1$ since the most recent observation of the USFSI has the most up-to-date information about the state of the economy. Moreover, estimating another parameter brings about extra computational and convergence-related costs. Finally, the results are fairly robust to allowing a free time delay parameter.

[4] The modelling and estimation approach of the RAMM is flexible enough to include many more variables in the Canadian satellite block if needed for certain policy analyses.

rates of residential mortgages, consumer loans and business loans; the levels of the consumer confidence index, business confidence index, 5-year Government of Canada bond yields, the 5-year mortgage rate and the corporate spread; the growth rates of the nominal exchange rate (USD/CAD), stock prices, real GDP, housing activity and CPI; and the levels of the overnight rate and the Canadian Financial Stress Index. In addition to the endogenous variables, the Canadian TVAR(p^C, q^C) equation includes the contemporaneous and lagged foreign variables as exogenous regressors. As a result, the model can be written as

$$\mathbf{Y}_t^C = \begin{cases} \sum_{i=1}^{p^C} \mathbf{A}_{L,i}^C \mathbf{Y}_{t-i}^C + \sum_{j=0}^{q^C} \mathbf{B}_{L,j}^F \mathbf{Y}_{t-j}^F + \mathbf{e}_{L,t}^C & \text{if} \quad CFSI_{t-1} \leq \gamma^C \\ \sum_{i=1}^{p^C} \mathbf{A}_{H,i}^C \mathbf{Y}_{t-i}^C + \sum_{j=0}^{q^C} \mathbf{B}_{H,j}^F \mathbf{Y}_{t-j}^F + \mathbf{e}_{H,t}^C & \text{if} \quad CFSI_{t-1} > \gamma^C, \end{cases}$$

where $\mathbf{e}_{L,t}^C \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_L^C)$ and $\mathbf{e}_{H,t}^C \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_H^C)$. As in the foreign block, the low- and high-stress regimes for Canada are determined by a comparison of the level of the $CFSI_{t-1}$ to the threshold parameter γ^C. Similarly, the $(N^C \times N^C)$ autoregressive coefficient matrices $\{\mathbf{A}_{L,i}^C, \mathbf{A}_{L,i}^H\}$, for $i = 1, \ldots, p^C$, the $(N^C \times N^F)$ exogenous variables' coefficient matrices $\{\mathbf{B}_{L,j}^C, \mathbf{B}_{L,j}^H\}$, for $j = 0, \ldots, q^C$ and the $(N^C \times N^C)$ covariance matrices $\{\mathbf{\Sigma}_L^C, \mathbf{\Sigma}_H^C\}$ are all allowed to be different across regimes.

So far, we have described the reduced-form TVAR models for each country. But, to be able to perform any structural analysis, e.g., a policy exercise, we need to identify underlying structural shocks. Our identification strategy for each block is explained in detail in section 4.

3.2 The estimation

The RAMM's modelling and estimation techniques are mainly based on Bruneau, Chapman, and Tuzcuoglu (forthcoming), which proposes a Bayesian inference method on a one-country and multiple-regime TVAR model. We estimate the TVAR model of the foreign and domestic blocks separately. For each block, we use a Gibbs sampler with an adaptive random walk Metropolis-Hastings step. Since this is a large model, we apply some shrinkage on the parameters to avoid the curse of dimensionality. In summary, we estimate the threshold parameter via a random walk Metropolis-Hastings step, and within each regime, we apply Bańbura et al. (2010)'s efficient Bayesian shrinkage estimation method for large VAR models to estimate the VAR coefficients.

Note that, once the regime is determined, we are in a linear VAR model environment. Hence, for the VAR coefficients, we can utilize commonly used shrinkage priors that provide a closed analytical form for their posteriors so that we can simply use a Gibbs approach for sampling. In particular, we utilize Bańbura et al. (2010)'s methodology that relies on Litterman's priors but increases the shrinkage as the model gets larger. Specifically, we shrink the diagonal of the first autoregressive coefficient matrices $\{\mathbf{A}_{L,1}, \mathbf{A}_{H,1}\}$ toward the first unconditional autocorrelation coefficient values of each variable. We shrink the other coefficients toward zero, where the shrinkage gets tighter for distant lags and for off-diagonals. We also ensure the stability of the VAR since all the variables are stationary. Finally, inverse Wishart priors are used for the covariance matrix $\mathbf{\Sigma}$. Consequently, the VAR parameters can be efficiently sampled from their posteriors.

For the threshold parameter, Bruneau, Chapman, and Tuzcuoglu (forthcoming) propose a Beta distribution. Being bounded between 0 and 1, a Beta distribution provides an advantage in terms of stability compared with commonly used (unbounded) normal priors. A bounded prior together with a stationary transition variable prevent undesired cases such as the process being stuck in a regime forever. Another advantage of a Beta prior is that its hyperparameters can be easily set to be in line with our apriori beliefs. For instance, based on the total duration of recessionary episodes and financial crises in US and Canadian economic history, it is safe to say that the proportion of a high-financial-stress regime in either country is likely somewhere around 10%–15%. Finally, to sample from the posterior of the threshold parameter, we use an adaptive Metropolis-Hastings step, where the variance of the proposal is adjusted such that the acceptance rate is around the asymptotically optimal rate of 23.4% according to Gelman, Gilks, and Roberts (1997).

Regarding the lag lengths, we try $p^F = \{3, 6, 12\}$ for the foreign model. In terms of the sampling and convergence characteristics, $p^F = 12$ yields the best results. For the Canadian model, we try $p^C = \{3, 6, 12\}$ and $q^C = \{0, 1, 2, 3, 4, 5\}$. The model with $(p^C, q^C) = (6, 5)$ not only provides the best sampling and convergence properties but also better captures transmission mechanism dynamics in the impulse-response functions (IRFs).[5] As a result, the foreign model has 1,248 parameters while the Canadian one has 7,096.

For each block, we run a total of 200,000 iterations where the first 50,000 draws are discarded as burn-in and a thinning by a factor of 15 is applied. Eventually, for each country, we obtain a chain that consists of 10,000 iterations that do not exhibit any significant autocorrelation. Consequently, the calculated inefficiency factors are extremely small. We also perform convergence checks on parameter chains according to the Geweke's convergence diagnostics (Geweke, 1992). The results indicate that less than 10% of the parameters do not have convergent chains. These numbers are acceptable in such a large model with thousands of parameters. In terms of the computational time, 200,000 MCMC iterations for the foreign and Canadian block take approximately 1.5 and 9 hours, respectively.

Finally, as an alternative approach to modelling and estimation, it is noteworthy to compare a sequential estimation of the foreign and Canadian blocks with their joint estimation with block exogeneity. In the sequential estimation, which is the RAMM's current approach, we first estimate the foreign block, then the Canadian block, and, if desired, we obtain the structural shocks together with their associated IRFs separately for each block. If we want to obtain the responses of the Canadian

[5] For instance, when we assign $p^C = 12$, the model becomes vast with around 20,000 parameters. Hence, we need to apply a high level of shrinkage. But tighter priors push the coefficients of very distant lags extremely close to 0, rendering them useless in capturing any extra dynamics. In contrast, if we loosen the priors, then the model becomes less stable and it gets much harder to find a stationary draw for the autoregressive coefficients. As a result, such practicalities also play a role in choosing lag lengths in large models.

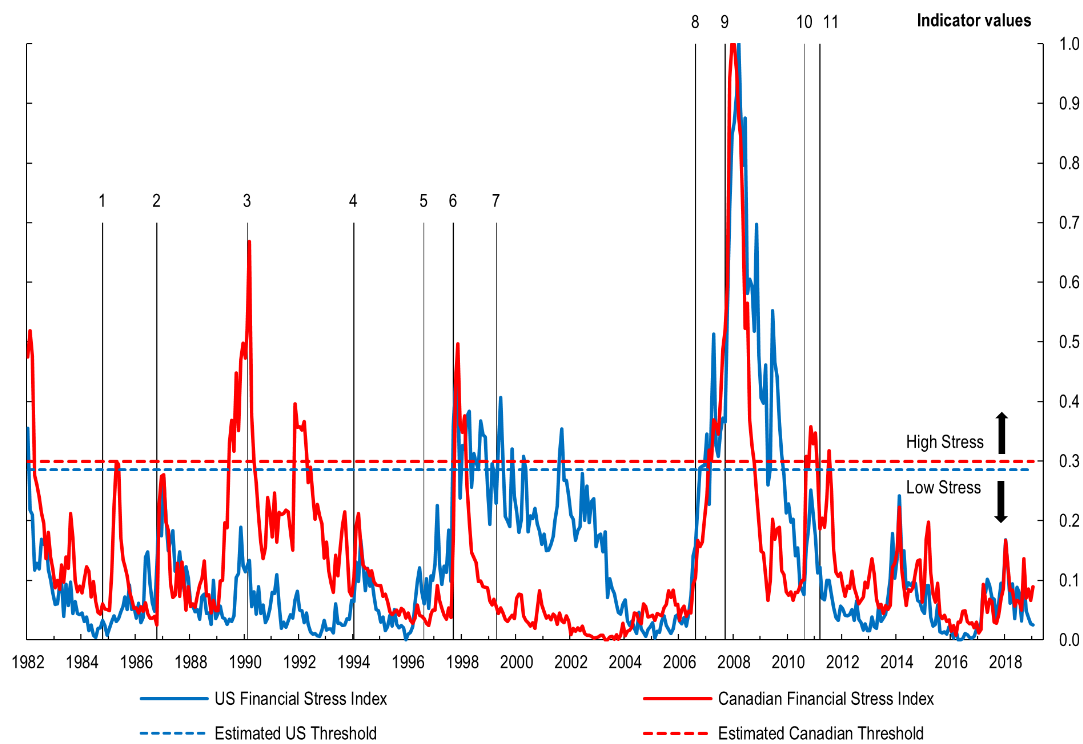
variables to foreign shocks, then we take the median IRFs of the foreign blocks and feed them into the exogenous variables in the Canadian TVAR equations. In contrast, the joint estimation provides parameter estimates and IRFs of both blocks simultaneously while imposing a block exogeneity on the autoregressive coefficient matrices as well as on the structural covariance matrices.

One of the advantages of the sequential estimation is the flexibility of choosing country-specific priors and hyperparameters—such as the intensity of the shrinkage or lag lengths. Hence, we have more control over each model relative to a joint estimation. There are also some computational advantages since estimation of individual models is faster than estimating a very large joint model. Additionally, it is easier to make changes to the set of domestic variables and re-estimating the Canadian block individually. In contrast, one of the advantages of the joint estimation is that it yields a more consistent estimation, in particular, for the structural analysis. Moreover, it provides a correct inference when calculating the transmission of the foreign shocks to the Canadian economy. However, one disadvantage of this approach could be the possibly small number of observations within each regime pair. This problem arises in the joint estimation—as opposed to the sequential one—since one can consider that there are indeed four regime pairs: low/low, low/high, high/low and high/high. For instance, when the USFSI is below its threshold and the CFSI is above its threshold, i.e., when the United States is in low stress and Canada is in high stress, estimation of the model would rely on only around 5% of the data, i.e., around 22 observations (discussed in Table 1 below). Hence, the gain in providing a theoretically correct inference might vanish due to the imprecise estimation of small samples within each regime pair. Finally, to the best of our knowledge, the literature has not provided a practical joint estimation methodology with block exogeneity for large TVAR models. For this, one could potentially merge the block exogeneity method of Zha (1999) with the efficient large VAR estimation method of Bańbura et al. (2010) and its extension to TVAR models by Bruneau, Chapman, and Tuzcuoglu (forthcoming); but this is still an open question in the literature.

3.3 The financial stress regimes

As a measure of the financial stress prevalent in an economy, we use financial stress indices based on the methodologies developed by Duprey et al. (2017) and Duprey (2020). Chart 1 shows the evolution of the US and Canadian financial stress indexes—scaled to $[0, 1]$—and highlights some significant events that affected financial stress in either economy throughout the last 40 years. We also show where the estimated thresholds stand for each country, indicating cutoff points for a regime switch as identified by the model.

Chart 1: Financial stress indexes for the United States and Canada



Events as numbered above are: 1. Canadian small banks failure 2. Stock Market Crash 3. Canadian trusts failure, ERM Crisis 4. Mexican Peso Crisis 5. Asian Financial Crisis 6. Russian Debt Default; Long-Term Capital Management Crisis 7. Dotcom Bubble Crash 8. Beginning of subprime crisis 9. Collapse of Lehman Brothers 10. European Sovereign Crisis (1st Greece Bailout) 11. European Sovereign Crisis (2nd Greece Bailout)

Source: Bank of Canada calculations | Last observation: December 31, 2019

In the RAMM, a two-country and two-regime model, there are four possibilities for the financial stress regimes, which are presented in Table 1 together with their estimated proportions in the data.

Table 1: Estimated stress regime configurations in the RAMM

		Canadian stress	
		Low	High
US stress	Low	80.1%	7.1%
	High	7.3%	5.5%

Note: The table presents the estimated percentage of the time periods when the US and Canadian economies are in a given regime. The time series span 444 observations.

4 Shock identification strategy

In this section, we discuss our strategy to identify country- and regime-specific structural shocks. We use combinations of recursive, sign, zero and elasticity restrictions that vary based on the country and regime. To explain the identification strategy more clearly, let us abstract from the dependence on the countries or regimes and consider a generic structural VAR(p) model, which can be written as

$$\mathbf{B_o Y_t} = \mathbf{B_1 Y}_{t-1} + \cdots + \mathbf{B_p Y}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{Y}_t$ is the vector of N endogenous variables, $\boldsymbol{\varepsilon}_t$ is the vector of structural shocks with an identity covariance matrix, and $\{\mathbf{B}_o, \ldots, \mathbf{B}_p\}$ are the structural coefficient matrices. We cannot estimate the structural shocks and parameters directly. However, multiplying both sides of the above equation by $\mathbf{B}_o^{-1}$ gives us the following reduced-form VAR model:

$$\mathbf{Y_t} = \mathbf{A_1 Y}_{t-1} + \cdots + \mathbf{A_p Y}_{t-p} + \mathbf{e}_t, \tag{2}$$

where $\{\mathbf{A}_1, \ldots, \mathbf{A}_p\}$ are the reduced-form coefficient matrices and $\mathbf{e}_t \sim (\mathbf{0}, \boldsymbol{\Sigma})$ are the reduced-form errors such that $\mathbf{e}_t = \mathbf{B}_o^{-1}\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\Sigma} = \mathbf{B}_o^{-1}\mathbf{B}_o^{-1\prime}$. The Bayesian estimation described in section 3.2 provides estimates for $\boldsymbol{\Sigma}$ and $\{\mathbf{A}_1, \ldots, \mathbf{A}_p\}$ (for each country and regime). However, without further restrictions on the model, these reduced-form estimates are not sufficient to uncover their structural counterparts, particularly the structural coefficient matrix $\mathbf{B}_o^{-1}$, which is the matrix of interest for identification.

The Cholesky decomposition, i.e., recursive identification, is one of the most common ways to identify $\mathbf{B}_o^{-1}$, where it is assumed to be lower triangular. In this identification scheme, the ordering of endogenous variables is decisive because it implies that a variable is not contemporaneously affected by variables that come after.

Another commonly used method is employing sign restrictions, which are theory-induced and economically meaningful conditions imposed on the relationships between variables and shocks (Canova and Nicolo, 2002; Uhlig, 2005). Given that there are infinite ways to factorize $\mathbf{\Sigma} = \mathbf{B}_o^{-1}\mathbf{B}_o^{-1\prime}$, how do we guarantee that $\mathbf{B}_o^{-1}$ would comply with the sign restrictions? To achieve a desired factorization, let us take an orthonormal matrix $\mathbf{\Xi}$, i.e., $\mathbf{\Xi}\mathbf{\Xi}' = \mathbf{I}_N$ is the $(N \times N)$ identity matrix, and write

$$\mathbf{\Sigma} = \mathbf{B}_o^{-1}\mathbf{B}_o^{-1\prime} = \mathbf{B}_o^{-1}\mathbf{\Xi}\mathbf{\Xi}'\mathbf{B}_o^{-1\prime} = (\mathbf{B}_o^{-1}\mathbf{\Xi})(\mathbf{B}_o^{-1}\mathbf{\Xi})' = \widetilde{\mathbf{B}}_o^{-1}\widetilde{\mathbf{B}}_o^{-1\prime}.$$

Hence, the identification lies in choosing $\mathbf{\Xi}$ such that $\widetilde{\mathbf{B}}_o^{-1}$ satisfies the desired sign restrictions. The way $\mathbf{\Xi}$ is generated relies on a QR decomposition (Rubio-Ramirez, Waggoner, and Zha, 2010), which is explained below.

Sign restrictions can also be combined with zero, elasticity and magnitude restrictions on certain elements of $\widetilde{\mathbf{B}}_o^{-1}$. While incorporating elasticity and magnitude restrictions is trivial—by checking whether $\widetilde{\mathbf{B}}_o^{-1}$ satisfies them—imposing additional zero restrictions is rather tricky. For the latter, following Baumeister and Benati (2013), one can multiply $\widetilde{\mathbf{B}}_o^{-1}$ by appropriate Givens matrices, which are simple rotational matrices using sine and cosine functions to get the desired elements in $\widetilde{\mathbf{B}}_o^{-1}$ to become zero.[6]

Overall, a sign restriction algorithm accompanied by zero, elasticity and magnitude restrictions can be implemented in the following steps. First, if needed, generate the necessary Givens matrices to rotate any $N \times N$ matrix to have zeros in the desired elements. The procedure to generate $\mathbf{G}$ matrices is explained in detail in section B of the Appendix. Then, for a given draw of $\mathbf{\Sigma}$ obtained from the Bayesian estimation, apply the Cholesky decomposition to get $\mathbf{\Sigma} = \mathbf{B}_o^{-1}\mathbf{B}_o^{-1\prime}$. Next,

[6] Alternatively, one can follow Arias, Rubio-Ramírez, and Waggoner (2018). However, in a large model like RAMM with several sign and zero restrictions, their algorithm appears to be computationally difficult due to the involvement of numerical derivatives.

1. Make a random draw from an $N-$dimensional standard normal distribution and take its QR decomposition. In other words, draw $\mathbf{\Psi} \sim \mathcal{N}(0, \mathbf{I}_N)$ and decompose it such that $\mathbf{\Psi} = \mathbf{\Xi}\mathbf{R}$ with $\mathbf{\Xi}\mathbf{\Xi}' = \mathbf{I}_N$. Define the candidate structural covariance matrix as $\widetilde{\mathbf{B}}_o^{-1} = \mathbf{B}_o^{-1}\mathbf{\Xi}$.

2. If there are any zero restrictions, apply the Givens matrices. That is, multiply $\widetilde{\mathbf{B}}_o^{-1}$ by the generated rotation matrices $\mathbf{G}$ and update it by $\widetilde{\mathbf{B}}_o^{-1} \leftarrow \widetilde{\mathbf{B}}_o^{-1}\mathbf{G}$. After the rotations, $\widetilde{\mathbf{B}}_o^{-1}$ automatically has zeros in its desired elements.

3. Finally, if the candidate matrix $\widetilde{\mathbf{B}}_o^{-1}$ satisfies the sign and elasticity restrictions and its elements are within the desired magnitude bounds, then keep this $\widetilde{\mathbf{B}}_o^{-1}$; otherwise, discard it.

4. Repeat the above steps $\mathcal{M}$ times, where $\mathcal{M}$ is a very large number, save each $\widetilde{\mathbf{B}}_o^{-1}$ matrix that satisfies the restrictions, and compute the associated IRFs.

As a result of this procedure, at each MCMC iteration, we obtain a collection of $\{\widetilde{\mathbf{B}}_o^{-1}(m)\}_{m=1}^{\mathcal{M}}$ matrices and $\mathcal{M}$ IRFs associated with each $\widetilde{\mathbf{B}}_o^{-1}(m)$ matrix. Having estimated the reduced-form residuals $\mathbf{e}_t$ and $\mathcal{M}$-many draws for $\mathbf{B}_o^{-1}$ matrices, one can easily compute the structural shocks by $\boldsymbol{\varepsilon}_t(m) = \widetilde{\mathbf{B}}_o(m)\mathbf{e}_t$ for every m.

There are two things that we adjust in this procedure specific to the RAMM. First, even though $\mathcal{M}$ is chosen to be large, we cease the steps as soon as we obtain one accepted $\widetilde{\mathbf{B}}_o^{-1}$ for an MCMC iteration. The aim is to have one $\widetilde{\mathbf{B}}_o^{-1}$ for each MCMC draw so that we do not over-represent certain MCMC draws in the structural analysis. Second, at each MCMC iteration, we have two covariance matrices corresponding to different regimes $\{\mathbf{\Sigma}_L, \mathbf{\Sigma}_H\}$. Since one of the purposes of the RAMM is to be able to conduct policy scenarios with potential regime switching, we want to make sure that we have an accepted $\widetilde{\mathbf{B}}_o^{-1}$ for each regime. Thus, for a given draw of $\{\mathbf{\Sigma}_L, \mathbf{\Sigma}_H\}$, if we can obtain an accepted $\widetilde{\mathbf{B}}_o^{-1}$ for the low-stress regime, let's denote it as $\widetilde{\mathbf{B}}_{o,L}^{-1}$, then we increase $\mathcal{M}$ for the high-stress regime to ensure that we can find an accepted $\widetilde{\mathbf{B}}_{o,H}^{-1}$. Eventually, we attain an accepted pair of $\{\widetilde{\mathbf{B}}_{o,L}^{-1}, \widetilde{\mathbf{B}}_{o,H}^{-1}\}$ for a given MCMC draw $\{\mathbf{\Sigma}_L, \mathbf{\Sigma}_H\}$. As mentioned above, the RAMM contains 10,000 MCMC draws for each country block, and for 8,726 of them, we are able to secure an accepted pair $\{\widetilde{\mathbf{B}}_{o,L}^{-1}, \widetilde{\mathbf{B}}_{o,H}^{-1}\}$. Ultimately, the shock identification strategy delivers us four sets of different structural shocks corresponding to two blocks and two regimes.

4.1 Shock identification in the foreign block

In this subsection, we present the restrictions imposed on the structural impact matrix of the foreign block, namely $(\mathbf{B}_o^F)^{-1}$. We identify global shocks by the recursive identification while US-specific shocks are identified by sign, zero and elasticity restrictions. The sign and zero restrictions are given in Table 2.

Global shocks While we assume that global shocks can affect US variables contemporaneously, US shocks can affect global variables only with a lag. The literature provides some evidence on commodity prices being predetermined with respect to the US economy (Kilian, 2009; Elder and Serletis, 2010; Kilian and Vega, 2011). We treat the US term premium as a global variable since it has a very high correlation (around 97%) with the principal component obtained from advanced economies' term premiums. This means that the US term premium can be viewed as the global term premium. Then, we impose a recursive identification to separate the shocks to global and US variables. Among the global variables, we assume that the term premium is the fastest moving, so we place it last. In addition, since the oil price is generally an input for the cost of production of non-energy commodities, we place the oil price first so that its shock can affect non-energy commodity prices contemporaneously. Alternatively, oil and non-energy commodity price shocks can be identified by sign restrictions as in Erten and Tuzcuoglu (2018), and US term premium shocks as in Zivanovic (2019). But this approach would complicate the sign identification algorithm in the foreign block. Thus, we keep the recursive identification approach for the global variables.

US shocks Identification of US shocks relies on a combination of sign, zero and elasticity restrictions. In particular, we assume that a US demand shock would increase economic activity while decreasing the stress level in the economy. Moreover, higher income pushes prices up, which, in turn, puts positive pressure on the policy rate. A negative (unfavourable) supply shock, such as supply chain issues or a reduction in productivity, has a negative impact on production while increasing financial stress. Additionally, such a supply shock increases prices, which the monetary authority is likely to respond to with an increase in the policy rate. A contractionary monetary policy has a negative effect on the economy by diminishing economic activity and increasing financial stress, while decreasing inflation. The effects of an unfavourable stress shock are assumed to be different in low- and high-stress regimes. While a stress shock is restricted to be contemporaneously muted in the low-stress regime—hence no effect on the economy on impact—it is assumed to have a negative effect on economic activity and inflation in the high-stress regime, which a central bank responds to with expansionary monetary policy. The underlying idea of this differential effect assumption is that we are interested in stress shocks that are more relevant to the economy during financial stress as opposed to tranquil times.

With these restrictions, we can identify all of the global shocks as well as the US supply and monetary policy shocks. Additionally, US demand and stress shocks can be identified during the low-stress regime thanks to the zero restrictions. However, it is not possible to separate stress shocks from negative demand shocks in the high-stress regime. Note that, in Table 2, the sign column of the stress shock is exactly opposite that of the demand shock in the high-stress regime. Hence, we need more assumptions to distinguish between an unfavourable demand shock and an unfavourable stress shock. In general, this is an intricate task since, by definition, stress shocks are types of negative demand shocks. To achieve identification, we rely on elasticity restrictions, which are commonly used in the literature (e.g., Kilian and Murphy (2014)). Specifically, we assume that the US Financial Stress Index responds more to an unfavourable stress shock than to an unfavourable demand shock. Mathematically, we impose a restriction on $(\mathbf{B}_o^F)^{-1}$ such that its $(7, 4)$ element, i.e., the response of the USFSI to a US demand shock, is smaller than its $(7, 7)$ element in absolute value.

This concludes the discussion of all identifying restrictions on the structural impact

Table 2: Recursive, sign, zero and elasticity restrictions in the foreign block

	Oil shock	Non-energy shock	Term premium shock	Demand shock	Supply shock	Monetary policy shock	Stress shock
Low-stress regime							
Oil price	x	0	0	0	0	0	0
Non-energy commodity price	x	x	0	0	0	0	0
Term premium	x	x	x	0	0	0	0
GDP	x	x	x	+	−	−	0
CPI	x	x	x	+	+	−	0
FFR	x	x	x	+	+	+	0
USFSI	x	x	x	−	+	+	+
High-stress regime							
Oil price	x	0	0	0	0	0	0
Non-energy commodity price	x	x	0	0	0	0	0
Term premium	x	x	x	0	0	0	0
GDP	x	x	x	+	−	−	−
CPI	x	x	x	+	+	−	−
FFR	x	x	x	+	+	+	−
USFSI	x	x	x	$-^*$	+	+	$+^*$

Note: The table presents the identification restrictions imposed on the contemporaneous responses of the variables (rows) to the structural shocks (columns) in each regime. Moreover, signs with asterisks ($*$) indicate that some elasticity restrictions are imposed on these responses. Finally, 'x' means that there is no restriction. GDP is gross domestic product; CPI is consumer price index; FFR is Federal Funds Rate; USFSI is US Financial Stress Index.

matrix $(\mathbf{B}_o^F)^{-1}$. But it is worth analyzing how the recursive ordering (Cholesky decomposition) and sign restrictions are combined since the linear algebra involved is not immediately clear. For this, ignoring regime dependence, we can write the covariance matrix $\mathbf{\Sigma}^F$ in four blocks corresponding to the variance of the world block shocks ($\mathbf{\Sigma}_W$), variance of the US block shocks ($\mathbf{\Sigma}_U$) and their covariances ($\mathbf{\Sigma}_{W,US}$) such that

$$\mathbf{\Sigma}^F = \begin{bmatrix} \mathbf{\Sigma}_W & \mathbf{\Sigma}_{W,US} \\ \mathbf{\Sigma}'_{W,US} & \mathbf{\Sigma}_{US} \end{bmatrix}.$$

Next, we want to decompose the reduced-form variance and covariance matrix $\mathbf{\Sigma}^F$

into structural matrices consisting of $(\mathbf{B}_o^W)^{-1}$ and $(\mathbf{B}_o^{US})^{-1}$, namely the structural covariance matrices for the world and US blocks, respectively, that are in accord with our restrictions depicted in Table 2. Specifically, we aim to have a $(\mathbf{B}_o^F)^{-1}$ matrix such that

$$(\mathbf{B}_o^F)^{-1} = \begin{bmatrix} (\mathbf{B}_o^W)^{-1} & \mathbf{0} \\ \mathbf{C} & (\mathbf{B}_o^{US})^{-1} \end{bmatrix},$$

where $\mathbf{C}$ is an unrestricted matrix capturing the contemporaneous response of the US variables to global shocks. In this setting,

$$\mathbf{\Sigma}^F = \left[(\mathbf{B}_o^F)^{-1}\right]\left[(\mathbf{B}_o^F)^{-1}\right]' = \begin{bmatrix} \left[(\mathbf{B}_o^W)^{-1}\right]\left[(\mathbf{B}_o^W)^{-1}\right]' & (\mathbf{B}_o^W)^{-1}\mathbf{C}' \\ \mathbf{C}\left[(\mathbf{B}_o^W)^{-1}\right]' & \mathbf{C}\mathbf{C}' + \left[(\mathbf{B}_o^{US})^{-1}\right]\left[(\mathbf{B}_o^{US})^{-1}\right]' \end{bmatrix},$$

implying that

$$\begin{aligned} \mathbf{\Sigma}_W &= \left[(\mathbf{B}_o^W)^{-1}\right]\left[(\mathbf{B}_o^W)^{-1}\right]', \\ \mathbf{\Sigma}_{W,US} &= \mathbf{C}\left[(\mathbf{B}_o^W)^{-1}\right]', \\ \mathbf{\Sigma}_{US} &= \mathbf{C}\mathbf{C}' + \left[(\mathbf{B}_o^{US})^{-1}\right]\left[(\mathbf{B}_o^{US})^{-1}\right]'. \end{aligned}$$

To obtain $(\mathbf{B}_o^W)^{-1}$, we simply use a Cholesky decomposition on $\mathbf{\Sigma}_W$ resulting in a lower diagonal $(\mathbf{B}_o^W)^{-1}$. To find $\mathbf{C}$, we post-multiply $\mathbf{\Sigma}'_{W,US}$ by $(\mathbf{B}_o^W)'$ such that $\mathbf{\Sigma}'_{W,US}(\mathbf{B}_o^W)' = \mathbf{C}\left[(\mathbf{B}_o^W)^{-1}\right]'(\mathbf{B}_o^W)' = \mathbf{C}$. Having obtained $\mathbf{C}$, we calculate $\mathbf{\Sigma}_{US} - \mathbf{C}\mathbf{C}'$ and apply our sign, zero and elasticity restrictions algorithm on this matrix to obtain $(\mathbf{B}_o^{US})^{-1}$. Eventually, we acquire a structural covariance matrix, $(\mathbf{B}_o^F)^{-1}$, that satisfies all of our restrictions and yields $\mathbf{\Sigma}^F = \left[(\mathbf{B}_o^F)^{-1}\right]\left[(\mathbf{B}_o^F)^{-1}\right]'$.

4.2 Shock identification in the Canadian block

In this subsection, we present the restrictions imposed on the structural impact matrix of the Canadian block, namely $(\mathbf{B}_o^C)^{-1}$. The Canadian block consists of two sets of variables: the *core* variables that are assumed to drive the underlying fundamental shocks of the economy and the *satellite* variables that help uncover transmission channels and various aspects of the economy. We identify the core structural shocks by sign, zero and elasticity restrictions, whereas there is no structural identification for the shocks of the satellite variables. We also assume that the core variables are predetermined with respect to the satellite ones, which separates the core structural shocks from the satellite shocks.

Core variables The shock identification for the Canadian core variables is very similar to that in the US block. In particular, we use the same sign and elasticity restrictions to identify the demand, supply, monetary policy and stress shocks. The only difference is the inclusion of a housing demand shock to the system. Note that house prices contribute significantly to vulnerabilities in the Canadian economy and, additionally, a house price correction poses a threat to the financial stability (see, e.g., Duprey and Roberts (2017); Duprey, Liu, MacDonald, van Oordt, Priazhkina, Shen, and Slive (2018); and the Bank of Canada's *Financial System Review* in June 2018). Therefore, we separately analyze the effects of housing demand and aggregate demand shocks in the Canadian block. We distinguish between them by allowing the CFSI to respond differently. Specifically, we assume that a positive housing demand shock, as opposed to a *regular* demand shock, increases the financial stress level—even though housing demand has a positive impact on the economy. The rationale is that housing market dynamics have substantial weight in the construction of the CFSI (Duprey, 2020). The disagreeing signs on the responses of the CFSI would disentangle the housing demand and aggregate demand shocks.

The restrictions on the responses of the housing variable are kept at minimum: a positive housing demand shock increases housing activity (housing starts and residential sales per capita) while contractionary monetary policy has a negative effect on it. We do not restrict the housing variable's response to an aggregate demand shock since there are two forces that could drive its response in opposite directions. A demand shock akin to a discount factor shock would make households want to anticipate consumption of both goods and housing services. This would increase overall economic activity—including housing starts and residential sales. In contrast, a demand shock akin to a preference shock that increases the utility derived from goods without affecting utility from housing services would make households shift their consumption away from housing and toward goods. This would decrease residential activity and increase goods production. Similarly, we do not restrict the response of housing to a stress shock since there are two opposing forces: elevated financial stress could make economic agents delay their residential activities, but at the same time, households could see housing as a safe investment amid an economic downturn.

Even though the imposed sign restrictions would identify the housing demand and aggregate demand shocks, we introduce extra elasticity restrictions to make the distinction stronger. Specifically, we assume that an aggregate demand shock increases the overall economic activity more than a housing demand shock, and similarly, a housing demand shock increases the housing activity more than an aggregate demand shock. Mathematically, we impose a restriction on $(\mathbf{B}_o^{core})^{-1}$ such that its $(1,1)$ element is larger than $(1,2)$ and its $(2,2)$ element is larger than $(2,1)$ in both regimes. Note that if the $(2,1)$ element is negative, then the second restriction would never bind. All these identification assumptions are presented in Table 3.

Satellite variables Even though the satellite variables are policy-relevant, they generally are of secondary importance compared with the core variables. Moreover, since there is a large number of variables in the satellite block, we do not perform any structural analysis of their shocks. Nevertheless, we apply a simple recursive identification scheme on the satellite shocks in case we need to use some judgment based on a specific policy scenario. To be more in line with recursive identification, we place the slow-moving variables before the fast-moving ones. Specifically, we place variables from the real side of the economy first—such as unemployment, investment, household income, imports, exports and credit-related variables. Then we place confidence indices, interest rates and spreads, and finally the exchange rate and the stock market index. By no means is this an ideal ordering for the slow- versus fast-moving variables, and thus, we do not put a lot of emphasis on their IRFs.

Table 3: Sign, zero and elasticity restrictions in the Canadian core block

	Demand shock	Housing demand shock	Supply shock	Monetary policy shock	Stress shock
Low-stress regime					
GDP	$+^*$	$+^*$	$-$	$-$	0
Housing activity		$+^*$		$-$	0
CPI	$+$		$+$	$-$	0
Overnight rate	$+$		$+$	$+$	0
CFSI	$-$	$+$	$+$	$+$	$+$
High-stress regime					
GDP	$+^*$	$+^*$	$-$	$-$	$-$
Housing activity		$+^*$		$-$	
CPI	$+$		$+$	$-$	$-$
Overnight rate	$+$		$+$	$+$	$-$
CFSI	$-^*$	$+$	$+$	$+$	$+^*$

Note: The table presents the identification restrictions imposed on the contemporaneous responses of the variables (rows) to the structural shocks (columns) in each regime. Moreover, signs with asterisks ($*$) indicate that some elasticity restrictions are imposed on these responses. Finally, an empty cell means that there is no restriction. GDP is gross domestic product; CPI is consumer price index; CFSI is Canadian Financial Stress Index.

Regarding the implementation of a combination of a Cholesky decomposition and sign identification in the Canadian block, we follow similar steps as in the foreign block. Ignoring the regime dependence, we write the Canadian covariance matrix $\mathbf{\Sigma}^C$ in four blocks corresponding to the variance of the satellite block "shocks" ($\mathbf{\Sigma}_s$), variance of the core block shocks ($\mathbf{\Sigma}_c$) and their covariances ($\mathbf{\Sigma}_{s,c}$) such that

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{\Sigma}_{s,c} \\ \mathbf{\Sigma}'_{s,c} & \mathbf{\Sigma}_c \end{bmatrix}.$$

Next, we decompose $\mathbf{\Sigma}$ into structural covariance matrices for the satellite and core blocks, $(\mathbf{B}_o^s)^{-1}$ and $(\mathbf{B}_o^c)^{-1}$ respectively, that are in line with our identifying restrictions.

Hence, our goal is to have a $(\mathbf{B}_o^C)^{-1}$ matrix such that $\mathbf{\Sigma}^C = \left[(\mathbf{B}_o^C)^{-1}\right]\left[(\mathbf{B}_o^C)^{-1}\right]'$ and

$$(\mathbf{B}_o^C)^{-1} = \begin{bmatrix} (\mathbf{B}_o^s)^{-1} & \mathbf{D} \\ \mathbf{0} & (\mathbf{B}_o^c)^{-1} \end{bmatrix},$$

where $\mathbf{D}$ is an unrestricted matrix representing the contemporaneous impact of the core shocks on the satellite variables. In this setting,

$$\mathbf{\Sigma}^C = \left[(\mathbf{B}_o^C)^{-1}\right]\left[(\mathbf{B}_o^C)^{-1}\right]' = \begin{bmatrix} [(\mathbf{B}_o^s)^{-1}]\,[(\mathbf{B}_o^s)^{-1}]' + \mathbf{D}\mathbf{D}' & \mathbf{D}\,[(\mathbf{B}_o^c)^{-1}]' \\ (\mathbf{B}_o^c)^{-1}\mathbf{D}' & [(\mathbf{B}_o^c)^{-1}]\,[(\mathbf{B}_o^c)^{-1}]' \end{bmatrix},$$

implying that

$$\begin{aligned} \mathbf{\Sigma}_s &= \left[(\mathbf{B}_o^s)^{-1}\right]\left[(\mathbf{B}_o^s)^{-1}\right]' + \mathbf{D}\mathbf{D}', \\ \mathbf{\Sigma}_{s,c} &= \mathbf{D}\left[(\mathbf{B}_o^c)^{-1}\right]', \\ \mathbf{\Sigma}_c &= \left[(\mathbf{B}_o^c)^{-1}\right]\left[(\mathbf{B}_o^c)^{-1}\right]'. \end{aligned}$$

First, we obtain $(\mathbf{B}_o^c)^{-1}$ by applying our sign, zero and elasticity restrictions algorithm on $\mathbf{\Sigma}_c$. Then, to find $\mathbf{D}$, we post-multiply $\mathbf{\Sigma}_{s,c}$ by $(\mathbf{B}_o^c)'$ such that $\mathbf{\Sigma}_{s,c}(\mathbf{B}_o^c)' = \mathbf{D}\,[(\mathbf{B}_o^c)^{-1}]'\,(\mathbf{B}_o^c)' = \mathbf{D}$. Afterward, we calculate $\mathbf{\Sigma}_s - \mathbf{D}\mathbf{D}'$ and use a Cholesky decomposition on this matrix to obtain $(\mathbf{B}_o^s)^{-1}$. Eventually, we acquire a structural covariance matrix, $(\mathbf{B}_o^C)^{-1}$, that satisfies all of our restrictions and yields $\mathbf{\Sigma}^C = \left[(\mathbf{B}_o^C)^{-1}\right]\left[(\mathbf{B}_o^C)^{-1}\right]'$.

4.3 Potential further restrictions

Note that the estimation period of the RAMM contains the high inflation and high interest rate episodes of late 1980s and early 1990s. In contrast, the Terms-of-Trade Economic Model (ToTEM III), the Bank of Canada's main dynamic stochastic general equilibrium (DSGE) model for projection and policy analysis (Corrigan, Desgagnés, Dorich, Lepetyuk, Miyamoto, and Zhang, 2021), uses a sample period that starts only after 1995. This means that the reaction of output and inflation to monetary policy shocks might be larger in the RAMM than in ToTEM III. Thus, we can introduce further restrictions on certain responses to be more in line with the conduct of monetary policy or with other policy models used at the Bank of Canada. For example, restrictions on the contemporaneous coefficients of output and inflation can be imposed according to an implied Taylor Rule for the policy rate equation. This would make the monetary policy responses to output and inflation more consistent with the rules embedded in DSGE models. Similarly (or alternatively), we can implement magnitude bounds on the responses of the policy rate, output and inflation so that the RAMM's key results are aligned with those of ToTEM III (or any other desired DSGE model). But these magnitude bounds should not be very tight since the RAMM entails a high-stress regime where assumptions of typical economic theories might be less relevant. Therefore, these bounds should be used to eliminate extreme responses and make the results of the RAMM more comparable with those of ToTEM III.

4.3.1 Potential further restrictions on core responses

For the Taylor Rule–type restrictions, we can follow Arias, Caldara, and Rubio-Ramirez (2019) and Wolf (2020). Abstracting from countries and regimes, consider the structural VAR model given in equation (1) with the structural matrix $\mathbf{B}_o$. Note that the sign, zero and elasticity restrictions have been imposed on the structural covariance matrix $(\mathbf{B}_o)^{-1}$. However, the Taylor Rule restriction will be imposed on its inverse, namely the structural impact matrix $\mathbf{B}_o$. Let's denote the index of the policy rate, output growth and the inflation rate as i_r, i_y and i_π, respectively. Moreover, let $\mathbf{b}_{j,i_r}$ denote the transpose of the i_r^{th} row of $\mathbf{B}_j$ for $j = 0, 1, \dots, p$. Then, the i_r^{th} equation, i.e., the equation for the policy rate, in the structural VAR (SVAR) can be written as

$$\mathbf{b}'_{o,i_r}\mathbf{Y}_t = \sum_{j=1}^{p} \mathbf{b}'_{j,i_r}\mathbf{Y}_{t-j} + \varepsilon_{t,i_r}.$$

Next, ignoring the lagged and other contemporaneous terms, we can write

$$r_t = \frac{-b_{o,i_r i_y}}{b_{o,i_r i_r}} y_t + \frac{-b_{o,i_r i_\pi}}{b_{o,i_r i_r}} \pi_t + \cdots + \frac{1}{b_{o,i_r i_r}} \varepsilon_{t,i_r} = \phi_y y_t + \phi_\pi \pi_t + \cdots + \sigma \varepsilon_{t,i_r}, \quad (3)$$

where $b_{o,i_r i_r}$, $b_{o,i_r i_y}$ and $b_{o,i_r i_\pi}$ are the coefficients of r_t, y_t and π_t obtained from $\mathbf{b}'_{o,i_r}\mathbf{Y}_t$. Equation (3) is the implied Taylor Rule obtained from the SVAR model. While there are still debates about the optimal magnitudes of the sensitivity coefficients ϕ_y and ϕ_π, a typical Taylor Rule in DSGE models imposes at least positivity on them: $\phi_y > 0$ and $\phi_\pi > 0$. These restrictions imply that the short-term policy rate would contemporaneously react positively to changes in output and prices.[7]

Including a Taylor Rule–type restriction in the shock identification algorithms of the foreign and Canadian blocks eliminates implied monetary policy rules that are non-sensible. The resulting IRFs tend to have smaller confidence bands, indicating a possible elimination of implausible sign draws.

To implement magnitude bounds based on results of ToTEM III, we can impose another restriction in the shock identification algorithm accordingly. For instance, ToTEM III predicts that, after a 100 basis point monetary policy shock, output and year-over-year (core) inflation would decline by 0.35% and 0.10%, respectively, within a year (Corrigan et al., 2021, p. 31). This means that after identifying structural shocks and computing the IRFs, we can compute the total change in output and year-over-year inflation and discard a draw if it predicts extremely different magnitudes than those of ToTEM III. The lower and upper bounds for accepting a draw can be arranged such that the median responses are around the desired level.

4.3.2 Potential further restrictions on satellite responses

Given that there are two countries, two regimes, 12 structural shocks and 31 variables, the RAMM produces around 700 impulse responses. Hence, it would be unreasonable to expect all these IRFs to align with economic intuition. Most of the counterintuitive IRFs occur in the high-stress regime, suggesting that there could be some other

[7] Generally, Taylor Rules in DSGE models include the output gap instead of output growth.

forces in play—despite controlling for 24 domestic and 7 foreign variables—during turbulent times. These might include some stimulative government policies, increased unemployment insurance and eased regulations on financial firms during economic downturns. These variables are hard to control for. Therefore, we can further restrict the accepted sign draws to satisfy desired responses of satellite variables after core structural shocks. But if these restrictions become too restrictive and render shock identification infeasible, then, in scenario analysis, we can alternatively use either judgment or some satellite models to obtain some economically meaningful paths for these variables.

5 Impulse responses

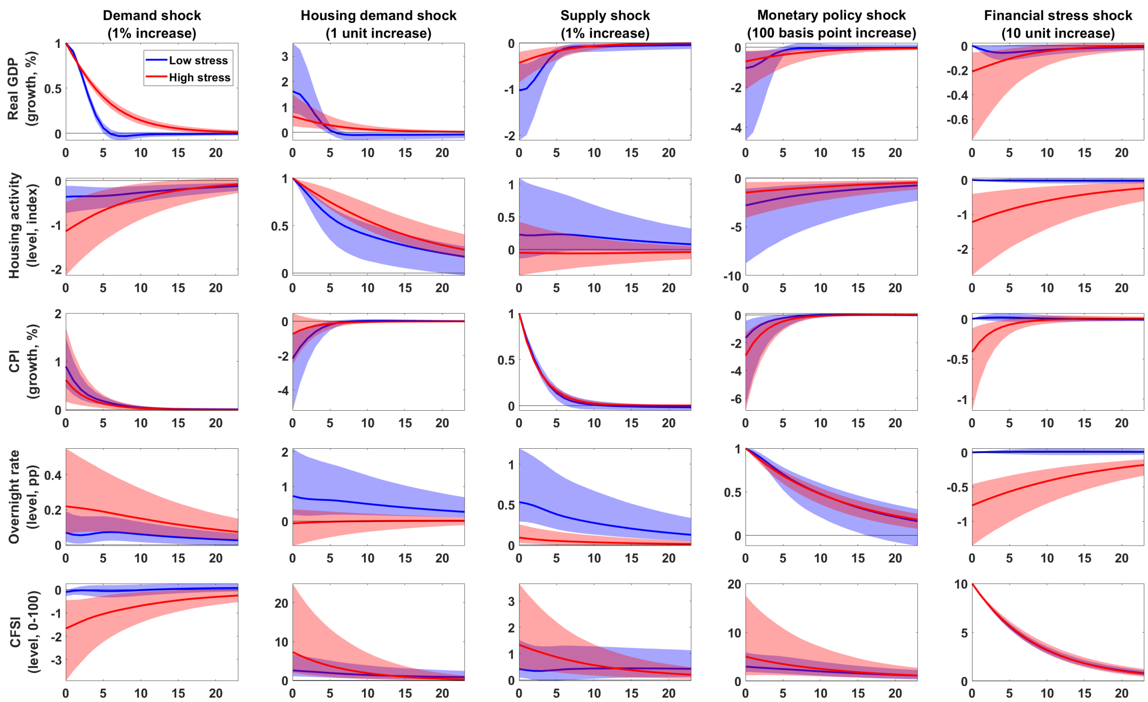
In this section, we present some selected impulse responses of key Canadian variables to both foreign and domestic shocks. The IRFs can be considered unconditional responses to unexpected structural shocks. They are derived assuming that the economy stays within the same regime as it encounters the initial shock. In other words, regardless of the level of financial stress, we trace the impact of a shock as if the economy stayed within the same regime. The horizon of the impulse responses is 24 months.

5.1 Impulse responses in the Canadian core block

Chart 2 presents the IRFs within the core block, where the imposed shock identification restrictions are given in Table 3. IRFs in the high-stress regime are red while those in the low-stress regime are blue. First, we feed an unexpected shock that contemporaneously increases its corresponding own variable by 1% for a demand shock, 1 unit for a housing demand shock, 1% for a supply shock, 100 basis points for a monetary policy shock and 10 units for a financial stress shock. Then we trace out the responses of the five core variables. Even though the IRFs are plotted with positive shocks, for consistency, we may switch to negative or unfavourable shocks in our discussions, particularly pertaining to the high-stress regime.

First, let's discuss the shock amplification mechanism that is present in the regime with high financial stress. The negative feedback loop between the financial stress and real GDP is depicted in the four IRFs at the four corners of Chart 2—responses of real GDP and CFSI to the demand and stress shocks. A negative demand shock elevates the CFSI, while an unfavourable financial stress shock decreases output. Hence, moving from the low- to high-stress regime triggers this negative feedback loop, where lower output and higher financial stress push the output further down and the stress higher. Unexpected accommodative monetary policy can be used to counteract this loop, decreasing financial stress and increasing economic activity.

Chart 2: Impulse responses in the Canadian core block



Note: The shocks are placed in the columns, while the responding variables are in the rows. Solid lines denote the median impulse responses, while the shaded regions denote the 68% equi-tailed confidence bands. The blue and red colours represent the low- and high-stress regimes, respectively. GDP is gross domestic product; CPI is consumer price index; CFSI is Canadian Financial Stress Index.

Regarding the effectiveness of monetary policy, the median IRFs provide a mixed result: during financial crises—in contrast to *tranquil* times—accommodative monetary policy is more effective at generating inflation but less effective at stimulating economic activity. However, given wide confidence bands, this difference might be insignificant.

The responses of housing activity are also noteworthy to discuss. Its negative response to aggregate demand shocks indicates that the types of demand shocks captured by the RAMM are more similar to preference shocks than to discount factor shocks. They decrease residential activity by shifting household consumption away from housing.[8] Moreover, this result is stronger in the high-stress regime. In contrast, the response of housing activity to stress shocks is just the opposite—an unfavourable stress shock lowers both output and housing activity. Meanwhile, a positive 1-unit housing demand shock increases the impact on real GDP by 0.8% (1.8%), real housing prices by 4% (1%) and residential investment by 4.5% (2%) in the low-(high-)stress regime (the last two IRFs are not reported here). Note that these are quarter-over-quarter annualized growth rates. For a quantitative comparison with other models, we may need to convert the responses to either year-over-year growth rates or cumulative effects.[9]

Overall, we can say that the RAMM captures three types of demand shocks: preference shocks that shift consumption toward goods and away from housing; housing-specific demand shocks that stimulate the overall economy; financial stress–related demand shocks that capture unfavourable aspects of aggregate demand, which contracts both housing and general economic activity.

5.2 Responses to Canadian financial stress shocks

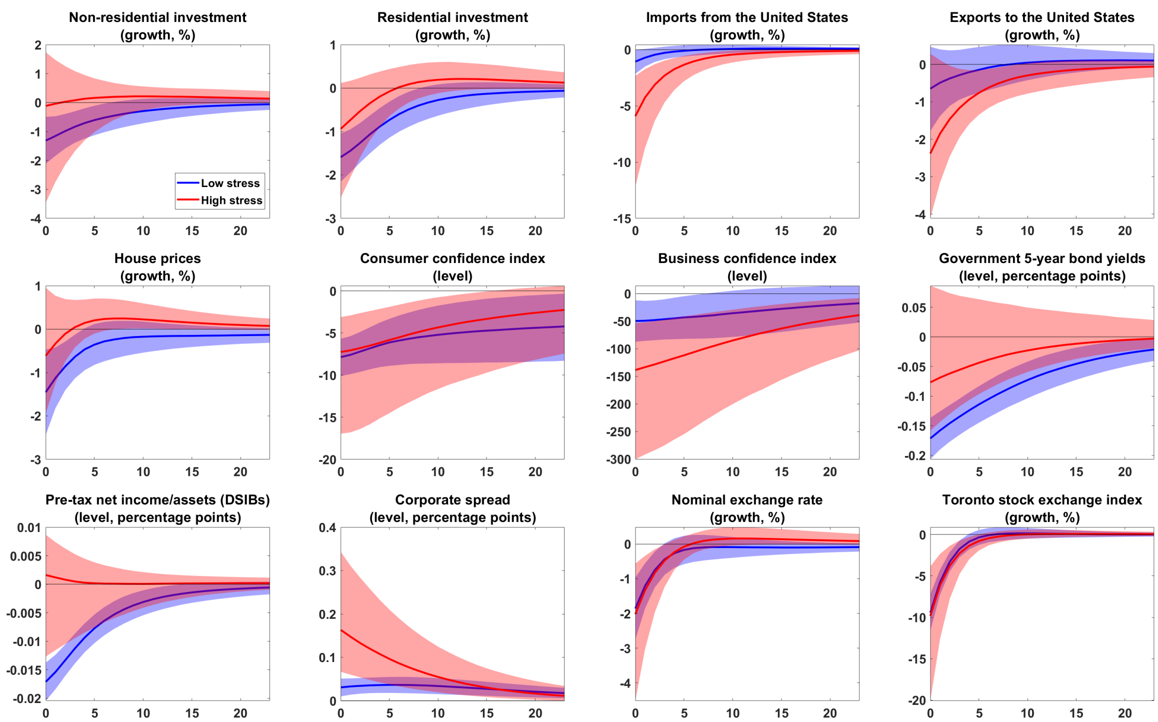
In this subsection, we focus primarily on financial stress shocks and their impact on the Canadian economy through various channels. Chart 3 shows the responses of some selected macrofinancial variables to an unexpected 10-unit increase in the Canadian Financial Stress Index, which simultaneously contracts real GDP by 0.2% (annualized) in the high-financial-stress regime. First, let's analyze responses of certain components of output. The top row of the figure shows the investment and trade channels through which a stress shock propagates across the real economy. Facing elevated financial stress, businesses and households cut back on their investments in both residential and

[8] This result agrees with the predictions of ToTEM III.

[9] The direction and magnitudes of these responses to a housing demand shock are generally in line with the results of Iacoviello and Neri (2010) and Abdallah and Lastrapes (2013).

non-residential structures. But we see that residential investment is a bigger source of this transmission than non-residential investment. In terms of the trade channel, we see that both imports and exports fall in the high-stress regime, while these drops are more muted in the low-stress regime. The net effect on the trade balance appears to be positive since imports fall more than exports—due to the depreciation in the Canadian dollar against the US dollar. The second row of the figure shows the decline in real house prices, deterioration in consumer and business expectations and a decline in long-term bond rates. Finally, the last row shows some financial channels where corporate spreads increase and real stock prices drop. Overall, we can say that an unfavourable financial stress shock deteriorates the economy in various ways through both financial and real channels despite an improvement in the trade balance.

Chart 3: Responses to a 10-unit Canadian financial stress shock



Note: This figure shows the responses of 12 macrofinancial variables to a 10-unit financial stress shock. Solid lines denote the median impulse responses, while the shaded regions denote the 68% equi-tailed confidence bands. The blue and red colours represent the low- and high-stress regimes, respectively. DSIBs are domestic systemically important banks.

5.3 Foreign shocks and their pass-through

In this subsection, we compute the response of the Canadian economy to foreign-originated shocks. First, we analyze the response of core Canadian variables to all foreign shocks. Then we focus particularly on the transmission channels of US financial stress shocks into the Canadian economy.

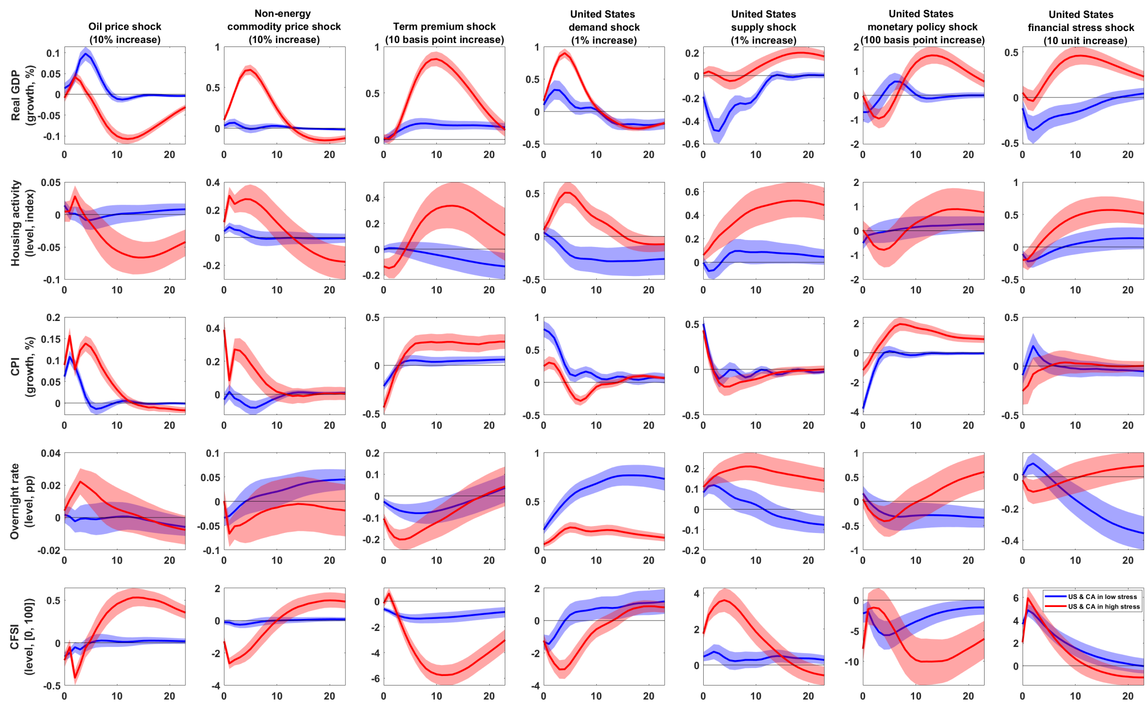
Chart 4 presents the responses of the five core variables (rows) to seven foreign structural shocks (columns). As shown in Table 1, there are four possible stress regime combinations in a two-country and two-regime setting. However, here we plot only when both countries are in the same regime, that is, low/low and high/high stress regimes for both the United States and Canada. The structural shocks increase their own corresponding foreign variables by 10% for the commodity price shocks, 10 basis points for the US term premium, 1% for the US demand and supply shocks, 100 basis points for the US monetary policy shock, and 10 units for the US financial stress shock.

The IRFs show that both of the commodity price shocks have generally expansionary effects on the Canadian economy. They boost real GDP and generate inflation that is particularly persistent in the high-stress regime. Regarding the global term premium shock, its overall impact on the Canadian economy appears to be positive, but the results are likely driven by the accommodative response of domestic monetary policy.

The responses to the US demand shock are expansionary despite a contractionary monetary policy response. Their peak pass-through to Canadian real GDP appears to be 40% and 100% in the low- and high-stress regime, respectively. This means that a 1% decline in US output creates a much larger deterioration in Canadian output when both countries are in a financial crisis. In contrast, the pass-trough of the US supply shocks are at 50%, regardless of the financial regime, but they are much shorter-lived, possibly thanks to the contractionary monetary policy response to control inflation.

There are two standard international transmission channels of US monetary policy to the Canadian economy: the US demand channel and exchange rate channel. After

Chart 4: The impact of foreign shocks on the Canadian core block



Note: The shocks are placed in the columns, while the responding variables are in the rows. Solid lines denote the median impulse responses, while the shaded regions denote the 68% equi-tailed confidence bands. The blue and red colours represent the low- and high-stress regimes, respectively, in both the United States and Canada. GDP is gross domestic product; CPI is consumer price index; CFSI is Canadian Financial Stress Index.

a contractionary US monetary policy shock, the US demand channel is transmitted through exports and commodity prices and has contractionary effects, while the exchange rate channel works through the trade balance and import prices and has expansionary effects. The response of Canadian monetary policy depends on which channel dominates. The RAMM IRFs indicate that the demand channel is the dominating one such that Canadian real GDP and inflation initially decline with some rebound effects later on. As an inflation-targeting central bank, the Bank of Canada cuts back the overnight rate to stimulate the economy but increases the rate later to control overshooting in inflation and output. Hence, as a result, there is no significant pass-through from US monetary policy to Canadian—if anything, the pass-through is in fact negative. These results are in line with the literature and two important policy models of the Bank of Canada, namely ToTEM III and LENS (Large Empirical and

Semi-structural model).[10] For a better comparison with single-regime models, we can take a weighted average of the low- and high-stress regime IRFs of the RAMM based on their observed frequencies (around 86% and 14%). The resulting average IRFs would be closer to those of the LENS, which is also an empirically driven model, in contrast to ToTEM III (see Gervais and Gosselin (2014, p. 33–38) for a comparison of IRFs between ToTEM III and LENS).

Finally, the pass-through of US financial stress shocks to the CFSI is around 50%−60%. Even though foreign stress shocks increase the stress in Canada, the responses of the core variables are driven more by the monetary policy reaction. In the low-stress regime, US financial stress increases inflation but decreases real GDP. Monetary policy responds with an increase followed by a large decrease in the overnight rate, which controls inflation while improving economic activity. In the high-stress regime, the results are opposite: US financial stress decreases inflation but does not affect real GDP. Monetary policy responds with a drop followed by a rise in the overnight rate, which generates some inflation and boosts economic activity. In either case, the impact on housing activity appears to be positive after the initial small negative effect. As a result, even though one might expect ex ante contractionary responses of the Canadian economy to unfavourable US financial stress shocks, the ultimate results are driven by the endogenous response of domestic monetary policy.

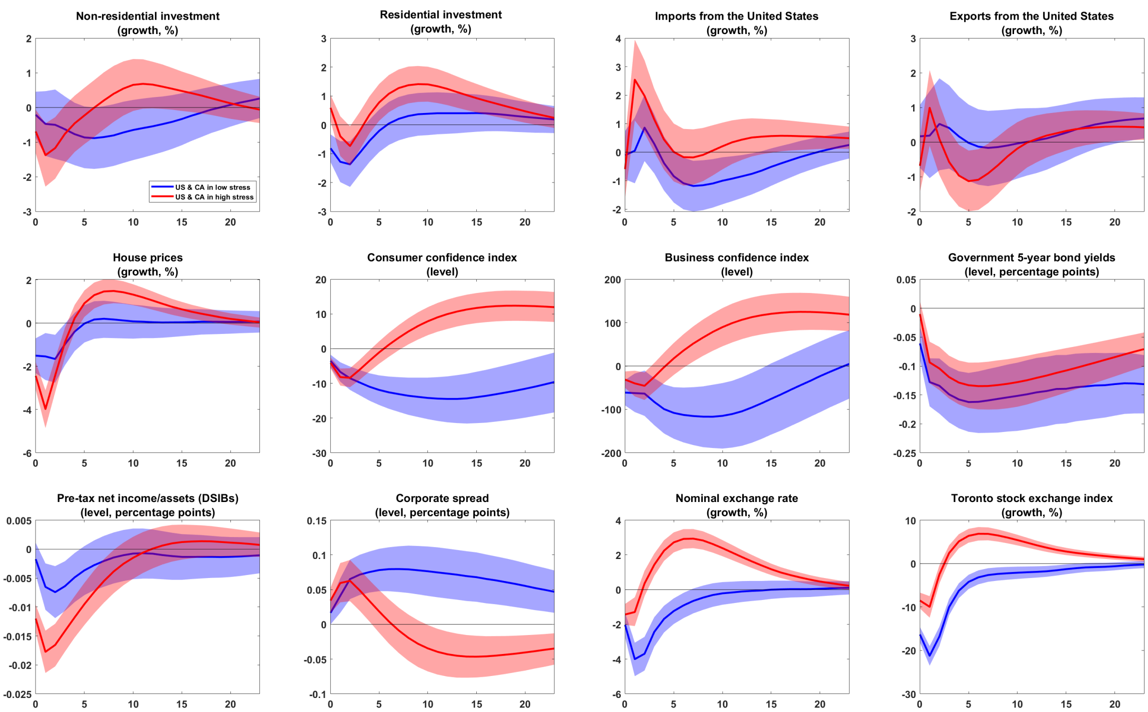
Next, let us focus on the responses of some Canadian satellite variables to the US stress shocks, which are given in Chart 5. Elevated foreign and domestic stress immediately deteriorate financial conditions and expectations. The effect on financial conditions can be inferred from an initial drop in asset prices, such as the TSX and house prices, a decline in the net income of D-SIBs and an increase in corporate spreads. The effect on expectations can be inferred from a fall in consumer and business confidence indexes. The worsening of these two aspects yield a contraction in investment.

However, the late responses of these fast-moving variables are driven mostly by the

[10] See Gervais and Gosselin (2014) for details on the LENS model.

reaction of domestic monetary policy since their elasticity is higher with respect to Canadian monetary policy than to foreign stress. We then see an improvement in all variables, where the rebound effect is much stronger in the high-stress regime thanks to the initial accommodative monetary policy that counteracts financial stress. Finally, we see an increase in imports in the high-stress regime due to the strong appreciation of the Canadian dollar. However, despite the appreciation, the response of exports is mixed since the US financial stress also contracts foreign demand.

Chart 5: Responses to a 10-unit US financial stress shock



Note: This figure shows the responses of 12 macrofinancial variables to a 10-unit financial stress shock in the United States. Solid lines denote the median impulse responses, while the shaded regions denote the 68% equi-tailed confidence bands. The blue and red colours represent the low- and high-stress regimes, respectively, in both the United States and Canada. DSIBs are domestic systemically important banks.

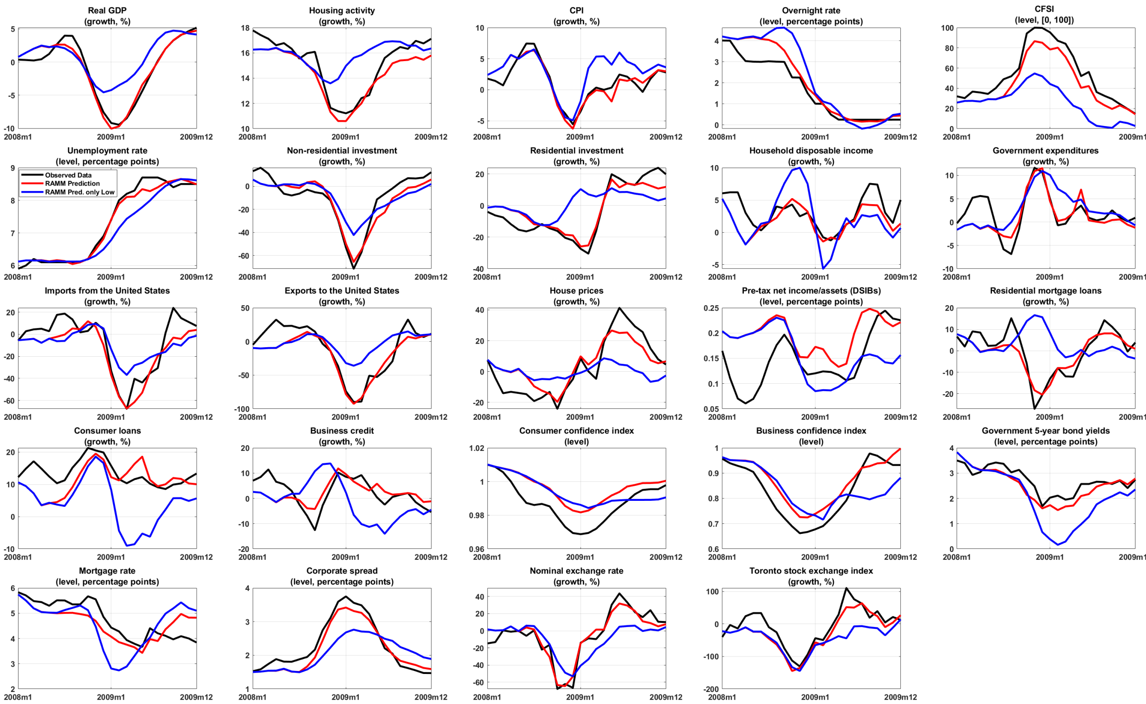
6 Scenario analysis

In this section, we illustrate the importance of shock amplification and the prediction accuracy of the RAMM through a scenario analysis exercise. We can also consider this a conditional forecasting exercise. For the scenario analysis, in contrast to the IRFs, we first need to specify the initial conditions and magnitudes of shocks. In nonlinear models, they both matter for shock propagation and regime switching. Specifically, we conduct a scenario analysis by using the conditions of the 2008–09 financial crisis. We will show that even if we had perfectly accurate knowledge about the shocks, it would not be enough to capture a significant portion of downside risks. We would also have to take into account shock amplifications due to regime changes.

This scenario analysis exercise focuses only on the responses of the Canadian block and covers the episode between January 2008 and December 2009. As the initial conditions, we use the observed data until December 2007. Then we make an extreme supposition and assume perfect foresight regarding the foreign shocks. Specifically, we feed the observed foreign data between January 2008 and December 2009 into the Canadian block as the exogenous variables. This is in contrast to a typical scenario analysis where one usually needs to make assumptions on the size and duration of the scenario-specific shocks. Finally, we trace the responses of the Canadian variables under two scenarios: (i) we simply obtain the predictions of the RAMM, and (ii) we do not allow regime switching to occur and use only the low-stress regime coefficients to obtain the predictions as though the RAMM was a single-regime model. A comparison between these two scenarios shows the significance of having a shock amplification mechanism.

Chart 6 shows the paths for all 24 Canadian variables. The first row plots the five core variables, and the other rows plot the satellite ones. The black lines represent the observed data between January 2008 and December 2009. The red line represents the median prediction of the RAMM. We choose the red colour for this because the Canadian economy moves into a high-stress regime, according to the RAMM's predictions, soon after the scenario starts. The blue line represents the median predictions of the RAMM under the second scenario, that is, when we prevent the regime switching. We choose the blue colour for this since the model is restricted to use the low-stress regime parameters regardless of the level of the CFSI.

Chart 6: Scenario analysis on the Canadian block



Note: This figure shows the responses of 24 Canadian macrofinancial variables under different scenarios between January 2008 and December 2009. The black line is the path of the observed data, the red line shows the median prediction of the RAMM, and the blue line is the median prediction of the RAMM when regime switching is not allowed. GDP is gross domestic product; CPI is consumer price index; CFSI is Canadian Financial Stress Index; DSIBs are domestic systemically important banks.

First, the observed data (the black lines) show that the 2008–09 financial crisis was detrimental for the Canadian economy. For instance, on the macroeconomy side, annualized real GDP growth declined to as low as -9%, inflation was around -5%, the unemployment rate increased by almost 3 percentage points, annualized non-residential and residential investment shrank by almost 70% and 30%, respectively, and total trade plummeted while the Canadian dollar lost significant value against the US dollar. Moreover, both short- and long-term interest rates declined between 2 to 4 percentage points. On the financial side, we see a big stock market crash, a 2 percentage point increase in corporate spreads, significant deterioration in confidence indices, a house price correction of around 20% and sharp decreases in mortgage loans and business credit. In contrast, the net income of D-SIBs exhibited a robust path, reflecting the resilience of the Canadian banking system.

Second, let's focus on the red lines, i.e., median predictions of the RAMM. The most striking result is the prediction accuracy of the model: the red and black lines are very close to each other, even overlapping for most of the time periods and variables. This could also reflect the fact that, throughout the financial crisis, most of the structural shocks came from abroad and the Canadian economy simply responded to them as opposed to generating its own domestic shocks. Remember that, even though this episode is in the estimation sample, we feed only foreign data into the scenario, not Canadian data.

There are a few variables and instances where the RAMM's predictions are not close to the observed data. One example is the prediction for the overnight rate in the early onset of the crisis. By mid-2008, there is a small rebound in important macroeconomic variables such as real GDP, inflation, net trade and even the stock market—possibly due in part to elevated commodity prices, which are not shown here. Due to all these temporary positive outlooks, the RAMM suggests holding the policy rate steady until September 2008—the bankruptcy of Lehman Brothers—followed by a steep decline in the predicted rates that catch up with the real data. The discrepancy might have also occurred since the RAMM is a backward-looking model while policy-makers set the policy rate using a more forward-looking approach. Other variables with poor predictions include the confidence indexes, which are also forward-looking; some credit variables, especially in the first nine months of the crisis; and the net income of DSIBs.

Third, let's focus on the blue lines, i.e., the median predictions of the RAMM when it is restricted to operate only with the low-stress regime parameters. One striking result is the poor predictive power of the model, which can be inferred by the difference between the blue and black lines. The other noticeable result is the difference between the red and blue lines, which shows us the importance of allowing regime switching, and thus the significance of taking the shock amplification into account. In terms of mean squared errors, the red line is around four times closer to the observed data than the blue line. Hence, if we had used a single-regime model, we may have missed up to a 5% further drop in annualized real GDP growth. Moreover, we would have wrongly predicted a recovery in inflation two quarters earlier than when the recovery actually happened. Similarly, we would have predicted a nearly 1 percentage point smaller unemployment rate, $30\% - 40\%$ more investment, 30% higher imports, 50% higher exports, and around 1 percentage point narrower corporate spread. Furthermore, we would have completely missed the house price correction as well as the sharp declines in mortgages and business credit. Hence, as a result, a model without regime switching would have provided a much milder recession than it actually was and also than a model with two regimes could predict. This comparison could be somewhat unfair to single-regime models since their predictions would likely be a weighted average of the blue and red lines. This would make their predictions better than the RAMM that is restricted to be in the low-stress regime. However, their predictions would be much closer to the blue ones since the economy is in low stress for around 87% of the entire time period.

Overall, this scenario analysis exercise emphasizes how crucial it is to allow for shock amplification and regime switching in financial stability models. Moreover, it shows that, with single-regime models, one could easily miss the identification of rare but severe downturns.

7 Conclusion

In this technical report, we present the Risk Amplification Macro Model (RAMM) that is used for financial stability analyses at the Bank of Canada. The RAMM is a semi-structural nonlinear model encompassing a large number of foreign and Canadian variables. It features regime switching between low and high financial stress, which could generate shock amplification that may not be captured in linear models. Thus, for financial stability considerations, the RAMM can be used as a stand-alone model as well as a tool to generate tail-risk scenarios in macro stress testing.

There are two blocks in the model: foreign and Canadian. Canada is modelled as a small open economy. Both blocks contain two financial stress regimes that are determined by the level of their country-specific financial stress index and thresholds. Within each block, the relationships between variables (e.g., coefficients and elasticities) are allowed to be different across stress regimes. For the structural shock identification, we employ theory-driven sign, zero and elasticity restrictions. We then present impulse-response results. A negative feedback loop between financial stress and real GDP is a particularly important feature of the model: an unfavourable stress shock reduces output, which in return increases financial stress further. We also analyze the pass-through of foreign shocks to the Canadian economy. The results indicate that there is a significant pass-through of US demand, supply and stress shocks.

Finally, a scenario analysis exercise for the 2008–09 financial crisis episode indicates that not allowing for regime switching would result in an overprediction in annualized real GDP growth by 5%, an underprediction of the unemployment rate by nearly 1 percentage point, an overprediction of inflation by 30%–40% at annualized rates and a false early recovery in inflation by 2 quarters. This exercise illustrates the importance of the risk amplification mechanism for financial stability purposes.

A Data

Table 4: Foreign and Canadian variables: Definitions and sources

Category/Variable	Definition	Source
Global and US variables		
Oil price	US refiner's acquisition cost of crude oil (including transportation and other fees, dollars per barrel).	Energy Information Administration
Non-energy commodity price	Fisher commodity price index for Canada, all sectors excluding energy (US dollar terms, index: 1972 = 100).	Bank of Canada
US term premium	US 5-year government bond yields - US 3-month T-Bill yields.	Bank of Canada
Real GDP	Real gross domestic product (chained (2021) United States dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a cubic method to match the average of the converted series to the original observations in each period.	Haver Analytics
CPI	Consumer price index for all urban consumers for all items (CPIAUCSL) (index: 1982-84 = 100, s.a.).	FRED
FFR	Effective federal funds rate.	FRED
US financial stress index	See Duprey et al. (2017).	Bank of Canada

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Table 4 – continued from the previous page

Category/Variable	Definition	Source
Canadian core variables		
Real GDP	Expenditure-based gross domestic product at market prices (chained 2012 dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a cubic method to match the average of the converted series to the original observations in each period.	Statistics Canada
Housing activity	The ratio of residential unit sales and Canada Mortgage and Housing Corporation housing starts series to population estimates in Canada as a percentage.	Statistics Canada
CPI	Consumer price index for all items (base year: 2002 = 100, s.a.).	Statistics Canada
Overnight rate	Overnight money market financing; seven-day call loan.	Bank of Canada
Canadian financial stress index	See Duprey (2020).	Bank of Canada
Canadian satellite variables		
Unemployment rate	Unemployment rate (both sexes, 15 years and over, s.a.)	Statistics Canada
Non-residential investment	Expenditure-based gross domestic product for non-residential structures, machinery and equipment in Canada (chained 2012 dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a linear method to match the average of the converted series to the original observations in each period.	Statistics Canada
Residential investment	Expenditure-based gross domestic product for residential structures in Canada (chained 2012 dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a linear method to match the average of the converted series to the original observations in each period.	Statistics Canada
Household disposable income	Household disposable income of Canadian households (chained 2012 dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a cubic method to match the average of the converted series to the original observations in each period.	Statistics Canada

Continued on next page

Table 4 – continued from the previous page

Category/Variable	Definition	Source
Canadian satellite variables		
Government spending	Expenditure-based gross domestic product for general governments final consumption expenditure in Canada (chained 2012 dollars). Seasonally adjusted at annual rates; converted from quarterly to monthly using a linear method to match the average of the converted series to the original observations in each period.	Statistics Canada
Imports	Three-month running average of merchandise imports from the United States to Canada.	Statistics Canada
Exports	Three-month running average of total exports and domestic exports from Canada to the United States.	Statistics Canada
House prices	Average residential sales price in Canada.	Haver Analytics
Pre-tax net income	The ratio of net income before provision for income taxes to total assets of Canadian domestic systemically important banks (DSIBs).	OSFI
Residential mortgages	Residential mortgages from Canadian DSIBs. It includes insured, uninsured and reverse mortgages, as well as mortgages less provision for losses and residential mortgages less allowance for impairment.	OSFI
Consumer loans	Consumer loans to individuals for non-business purposes from Canadian DSIBs. It includes all private loan balances to individuals in agriculture, fishing and trapping, logging and forestry, mining and quarrying, and manufacturing sectors.	OSFI
Business credit	Total credit liabilities and equity securities of private non-financial corporations. Data after 1990 are from Statistics Canada but those before January 1990 are extrapolated from values after 1990 by using growth rates of ceased early business credit data.	Statistics Canada
Consumer confidence index	Until April 2020, the series uses the Consumer Confidence Indicator for Canada (OECD). Data after April 2020 were filled using linear extrapolation with data from 2015 to 2021 by using the Conference Board of Canada's consumer confidence index.	CBOC and Haver Analytics

Continued on next page

Table 4 – continued from the previous page

Category/Variable	Definition	Source
Canadian satellite variables		
Business confidence index	Index of Business Confidence for Canada (base year: 2014 = 100).	CBOC
Five-year government bond yields	Selected five-year Government of Canada benchmark bond yields.	Bank of Canada
Five-year mortgage rates	Five-year mortgage rates in Canada. Before the end of August 1998, conventional five-year mortgage rates were used. Between September 1998 and December 2005, ING direct five-year fixed mortgage rates were used. From 2005 forward, average five-year fixed mortgage rates among national mortgage brokers were used.	Bank of Canada
Corporate spread	The corporate spread is created by the difference between two series: (1) yield-to-maturity (YTM) on corporate bonds and (2) yield on government bonds. The YTM on corporate bonds: before January 1995, average weighted yield on all mid-term Canadian corporates; after January 1995, average YTM on 3- to 5-year index and 5- to 7-year index of Canadian corporates. The yield on government bonds: before October 1980, yield on 3- to 5-year Government of Canada bonds; after October 1980, yield on 5-year Government of Canada bonds.	Bank of Canada, Intercontinental Exchange, and Bank of America Merrill Lynch
Nominal Exchange Rate	The foreign exchange rate between CAD and USD, where an increase means appreciation of CAD.	Bank of Canada
TSX	Toronto Stock Exchange composite index (base year: 2000 = 1,000).	Statistics Canada

B Imposing zero restrictions with Givens matrices

Givens matrices provide rotations on a matrix to achieve zero entries in the desired elements of that matrix. First, let us give a simple example of how Givens matrices are employed. Let's say that we want to impose a zero restriction on the $(1,4)$ element of a (4×4) matrix $\mathbf{B}$. Consider the following system of equations in terms of the unknown rotation angle ξ:

$$\mathbf{BG} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \cos(\xi) & 0 & 0 & -\sin(\xi) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\xi) & 0 & 0 & \cos(\xi) \end{bmatrix}$$

Equating the $(1,4)$ element of this multiplication to 0 and solving for the unknown ξ yields

$$-b_{11}\sin(\xi) + b_{14}\cos(\xi) = 0 \quad \Longrightarrow \quad \frac{\sin(\xi)}{\cos(\xi)} = \tan(\xi) = \frac{b_{14}}{b_{11}} \quad \Longrightarrow \quad \xi = \arctan\left(\frac{b_{14}}{b_{11}}\right).$$

Hence, setting $\xi = \arctan(b_{14}/b_{11})$, generating $\mathbf{G}$ as above, and multiplying $\mathbf{B}$ by $\mathbf{G}$ will turn the $(1,4)$ element of $\mathbf{B}$ into zero. However, this multiplication does not have any impact on $\mathbf{BB}'$ since $(\mathbf{BG})(\mathbf{BG})' = \mathbf{BGG}'\mathbf{B}' = \mathbf{BB}'$. In other words, $\mathbf{G}$ is an orthonormal matrix because

$$\mathbf{G}\mathbf{G}' = \begin{bmatrix} \cos(\xi) & 0 & 0 & -\sin(\xi) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\xi) & 0 & 0 & \cos(\xi) \end{bmatrix} \begin{bmatrix} \cos(\xi) & 0 & 0 & \sin(\xi) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\xi) & 0 & 0 & \cos(\xi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2(\xi) + \sin^2(\xi) & 0 & 0 & \cos(\xi)\sin(\xi) - \sin(\xi)\cos(\xi) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cos(\xi)\sin(\xi) - \sin(\xi)\cos(\xi) & 0 & 0 & \cos^2(\xi) + \sin^2(\xi) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

What if we want to impose zero restrictions on both b_{14} and b_{24}? In this case, we need to generate two Givens matrices, hence, solve two equations in two unknowns. The matrices $\mathbf{G}_{14}$ and $\mathbf{G}_{24}$, respectively, rotate the $\mathbf{B}$ matrix in a way to make the $(1, 4)$ and $(2, 4)$ elements equal to 0. Note that, the rotations are not independent since the zero restrictions are on the same column. Therefore, we need to solve for the rotation angles ξ_1 and ξ_2 jointly in the following system of equations:

$$\mathbf{B}\mathbf{G}_{14}\mathbf{G}_{24} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \cos(\xi_1) & 0 & 0 & -\sin(\xi_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\xi_1) & 0 & 0 & \cos(\xi_1) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\xi_2) & 0 & -\sin(\xi_2) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(\xi_2) & 0 & \cos(\xi_2) \end{bmatrix}$$

Equating the $(1, 4)$ and $(2, 4)$ elements of this multiplication to 0 and solving for the unknown ξ's yields

$$-b_{12}\sin(\xi_2) + (-b_{11}\sin(\xi_1) + b_{14}\cos(\xi_1))\cos(\xi_2) = 0$$
$$-b_{22}\sin(\xi_2) + (-b_{21}\sin(\xi_1) + b_{24}\cos(\xi_1))\cos(\xi_2) = 0.$$

Solving for $\sin(\xi_2)/\cos(\xi_2)$ yields

$$\frac{\sin(\xi_2)}{\cos(\xi_2)} = \frac{-b_{11}\sin(\xi_1) + b_{14}\cos(\xi_1)}{b_{12}} = \frac{-b_{21}\sin(\xi_1) + b_{24}\cos(\xi_1)}{b_{22}}.$$

Note that the first equality will provide a solution for ξ_2 in terms of ξ_1, while the last equality will give a solution for ξ_1 by rearranging as

$$\frac{\sin(\xi_1)}{\cos(\xi_1)} = \tan(\xi_1) = \frac{b_{22}b_{14} - b_{12}b_{24}}{b_{11}b_{22} - b_{12}b_{21}} \quad \Longrightarrow \quad \xi_1 = \arctan\left(\frac{b_{22}b_{14} - b_{12}b_{24}}{b_{11}b_{22} - b_{12}b_{21}}\right).$$

Hence, having solved for ξ_1, we can solve for ξ_2 as

$$\frac{\sin(\xi_2)}{\cos(\xi_2)} = \frac{-b_{11}\sin(\xi_1) + b_{14}\cos(\xi_1)}{b_{12}} \quad \Longrightarrow \quad \xi_2 = \arctan\left(\frac{-b_{11}\sin(\xi_1) + b_{14}\cos(\xi_1)}{b_{12}}\right).$$

Hence, setting ξ_1 and ξ_2 as above, generating $\mathbf{G_{14}}$ and $\mathbf{G_{24}}$ accordingly, and multiplying $\mathbf{B}$ by $\mathbf{G_{14}}$ and $\mathbf{G_{24}}$ will turn the $(1,4)$ and $(2,4)$ elements of $\mathbf{B}$ into zero.

Finally, we are ready to tackle the zero restrictions we impose on our structural $\mathbf{B}_o^{-1}$ matrices for each block given in Section 4. In particular, we want $\mathbf{B}_o^{-1}$ matrices of the foreign and Canadian blocks during low stress regimes to satisfy

$$\left(\mathbf{B}_o^{US}\right)^{-1} = \begin{bmatrix} x & x & x & 0 \\ x & x & x & 0 \\ x & x & x & 0 \\ x & x & x & x \end{bmatrix} \qquad \text{and} \qquad \left(\mathbf{B}_o^{CA}\right)^{-1} = \begin{bmatrix} x & x & x & x & 0 \\ x & x & x & x & 0 \\ x & x & x & x & 0 \\ x & x & x & x & 0 \\ x & x & x & x & x \end{bmatrix}.$$

Imposing these zero restrictions would require generating the associated Givens matrices and solving a system of equations of 3 and 4 unknown angles, respectively, for the foreign and Canadian blocks. Here, we provide only the solutions for the associated angles for each country.

Foreign block Generate the three Givens matrices, namely $\mathbf{G_{14}}$, $\mathbf{G_{24}}$ and $\mathbf{G_{34}}$, as

$$\begin{bmatrix} \cos(\xi_{14}) & 0 & 0 & -\sin(\xi_{14}) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\xi_{14}) & 0 & 0 & \cos(\xi_{14}) \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\xi_{24}) & 0 & -\sin(\xi_{24}) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(\xi_{24}) & 0 & \cos(\xi_{24}) \end{bmatrix}, \text{ and } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\xi_{34}) & -\sin(\xi_{34}) \\ 0 & 0 & \sin(\xi_{34}) & \cos(\xi_{34}) \end{bmatrix},$$

respectively, where we solve for each angle recursively as

$$\xi_{14} = \arctan\left(\frac{\frac{b_{24}/b_{23}-b_{14}/b_{13}}{b_{12}/b_{13}-b_{22}/b_{23}} - \frac{b_{34}/b_{33}-b_{14}/b_{13}}{b_{12}/b_{13}-b_{32}/b_{33}}}{\frac{b_{21}/b_{23}-b_{11}/b_{13}}{b_{12}/b_{13}-b_{22}/b_{23}} - \frac{b_{31}/b_{33}-b_{11}/b_{13}}{b_{12}/b_{13}-b_{32}/b_{33}}} \right)$$

$$\xi_{24} = \arctan\left(\frac{(b_{31}/b_{33} - b_{11}/b_{13})\sin(\xi_{14}) - (b_{34}/b_{33} - b_{14}/b_{13})\cos(\xi_{14})}{b_{12}/b_{13} - b_{32}/b_{33}} \right)$$

$$\xi_{34} = \arctan\left(\frac{[b_{14}\cos(\xi_{14}) - b_{11}\sin(\xi_{14})]\cos(\xi_{24}) - b_{12}\sin(\xi_{24})}{b_{13}} \right).$$

Consequently, post-multiplying $\mathbf{B}_o^{-1}\boldsymbol{\Xi}$ by the Givens matrices would restrict the desired entries to zero. Specifically, the candidate matrix $\widetilde{\mathbf{B}}_o^{-1} = \mathbf{B}_o^{-1}\boldsymbol{\Xi}\mathbf{G_{14}}\mathbf{G_{24}}\mathbf{G_{34}}$ will have zeros at its $(1,4)$, $(2,4)$ and $(3,4)$ entries, where $\mathbf{B}_o^{-1}$ is the Cholesky decomposition of $\boldsymbol{\Sigma}$, the matrix $\boldsymbol{\Xi}$ comes from the QR decomposition of a random draw from a four-dimensional normal distribution. Having satisfied the zero restrictions, we are ready to check whether the candidate matrix $\widetilde{\mathbf{B}}_o^{-1}$ satisfies the desired sign, elasticity and magnitude restrictions.

Canadian block Generate the four Givens matrices $\{\mathbf{G_{15}}, \mathbf{G_{25}}, \mathbf{G_{35}}, \mathbf{G_{45}}\}$ with their associated angles $\{\xi_{15}, \xi_{25}, \xi_{35}, \xi_{45}\}$ in a similar way as in the foreign block. The solution for ξ_{15} is

$$\xi_{15} = \arctan\left(\frac{A-B}{C-D}\right),$$

where

$$A = \frac{\frac{b_{15}/b_{14}-b_{35}/b_{34}}{b_{33}/b_{34}-b_{13}/b_{14}} - \frac{b_{25}/b_{24}-b_{45}/b_{44}}{b_{43}/b_{44}-b_{23}/b_{24}}}{\frac{b_{12}/b_{14}-b_{32}/b_{34}}{b_{33}/b_{34}-b_{13}/b_{14}} - \frac{b_{22}/b_{24}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{23}/b_{24}}}, \qquad B = \frac{\frac{b_{15}/b_{14}-b_{25}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{35}/b_{34}-b_{45}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}{\frac{b_{12}/b_{14}-b_{22}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{32}/b_{34}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}},$$

$$C = \frac{\frac{b_{11}/b_{14}-b_{31}/b_{34}}{b_{33}/b_{34}-b_{13}/b_{14}} - \frac{b_{21}/b_{24}-b_{41}/b_{44}}{b_{43}/b_{44}-b_{23}/b_{24}}}{\frac{b_{12}/b_{14}-b_{32}/b_{34}}{b_{33}/b_{34}-b_{13}/b_{14}} - \frac{b_{22}/b_{24}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{23}/b_{24}}}, \qquad D = \frac{\frac{b_{11}/b_{14}-b_{21}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{31}/b_{34}-b_{41}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}{\frac{b_{12}/b_{14}-b_{22}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{32}/b_{34}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}.$$

Next, the recursive solutions for other angles are

$$\xi_{25} = \arctan\left(\cos(\xi_{15})\frac{\frac{b_{15}/b_{14}-b_{25}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{35}/b_{34}-b_{45}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}{\frac{b_{12}/b_{14}-b_{22}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{32}/b_{34}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}} - \sin(\xi_{15})\frac{\frac{b_{11}/b_{14}-b_{21}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{31}/b_{34}-b_{41}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}{\frac{b_{12}/b_{14}-b_{22}/b_{24}}{b_{23}/b_{24}-b_{13}/b_{14}} - \frac{b_{32}/b_{34}-b_{42}/b_{44}}{b_{43}/b_{44}-b_{33}/b_{34}}}\right)$$

$$\xi_{35} = \arctan\left(-\cos(\xi_{25})\frac{(b_{15}/b_{14} - b_{25}/b_{24})\cos(\xi_{15}) - (b_{11}/b_{14} - b_{21}/b_{24})\sin(\xi_{15}))}{b_{23}/b_{24} - b_{13}/b_{14}} + \sin(\xi_{25})\frac{[b_{12}/b_{14} - b_{22}/b_{24}]}{b_{23}/b_{24} - b_{13}/b_{14}}\right)$$

$$\xi_{45} = \arctan\left(-\cos(\xi_{35})\frac{[b_{12}\sin(\xi_{25}) + \cos(\xi_{25})\left(-b_{15}\cos(\xi_{15}) + b_{11}\sin(\xi_{15})\right)]}{b_{14}} - \sin(\xi_{35})\frac{b_{13}}{b_{14}}\right).$$

Consequently, post-multiplying $\mathbf{B}_o^{-1}\mathbf{\Xi}$ by the Givens matrices would restrict the desired entries to zero. Specifically, the candidate matrix $\widetilde{\mathbf{B}}_o^{-1} = \mathbf{B}_o^{-1}\mathbf{\Xi}\mathbf{G_{15}}\mathbf{G_{25}}\mathbf{G_{35}}\mathbf{G_{45}}$ will have zeros at its $(1,5)$, $(2,5)$, $(3,5)$ and $(4,5)$ entries, where $\mathbf{B}_o^{-1}$ is the Cholesky decomposition of $\mathbf{\Sigma}$, the matrix $\mathbf{\Xi}$ comes from the QR decomposition of a random draw from a five-dimensional normal distribution. Having satisfied the zero restrictions, we are ready to check whether the candidate matrix $\widetilde{\mathbf{B}}_o^{-1}$ satisfies the desired sign, elasticity and magnitude restrictions.

The algebra here might seem involved, but one needs to do these calculations only once. One can either use pen and paper or computer programs such as Mathematica to solve the linear system of five unknown angles. After obtaining their analytical solutions and coding them on MATLAB, the computation of Givens matrices and incorporating them into the sign restriction algorithm take a few milliseconds.

References

Abdallah, C. S. and W. D. Lastrapes. 2013. "Evidence on the Relationship Between Housing and Consumption in the United States: A State-Level Analysis." *Journal of Money, Credit and Banking* 45 (4): 559–590.

Adrian, T., N. Boyarchenko and D. Giannone. 2019. "Vulnerable Growth." *American Economic Review* 109 (4): 1263–1289.

Alessandri, P. and H. Mumtaz. 2017. "Financial Conditions and Density Forecasts for US Output and Inflation." *Review of Economic Dynamics* 24 (March): 66–78.

Alessandri, P. and H. Mumtaz. 2019. "Financial Regimes and Uncertainty Shocks." *Journal of Monetary Economics* 101 (January): 31–46.

Arias, J. E., D. Caldara and J. F. Rubio-Ramírez. 2019. "The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure." *Journal of Monetary Economics* 101 (January): 1–13.

Arias, J. E., J. F. Rubio-Ramírez and D. F. Waggoner. 2018. "Inference Based on Structural Vector Autoregressions Identified with Sign and Zero Restrictions: Theory and Applications." *Econometrica* 86 (2): 685–720.

Balke, N. S. 2000. "Credit and Economic Activity: Credit Regimes and Nonlinear Propagation of Shocks." *Review of Economics and Statistics* 82 (2): 344–349.

Bańbura, M., D. Giannone and L. Reichlin. 2010. "Large Bayesian Vector Auto Regressions." *Journal of Applied Econometrics* 25 (1): 71–92.

Baumeister, C. and L. Benati. 2013. "Unconventional Monetary Policy and the Great Recession: Estimating the Macroeconomic Effects of a Spread Compression at the Zero Lower Bound." *International Journal of Central Banking* 9 (2): 165–212.

Bruneau, G., J. Chapman and K. Tuzcuoglu. Forthcoming. "Bayesian Inference of Threshold Vector Autoregressive Models." Bank of Canada Staff Working Paper.

Bruneau, G., T. Duprey and R. Hipp. 2022. "Forecasting Bank's Corporate Loan Losses Under Stress: A New Corporate Default Model." Bank of Canada Technical Report No. 122.

Canova, F. and G. D. Nicolo. 2002. "Monetary Disturbances Matter for Business Fluctuations in the G-7." *Journal of Monetary Economics* 49 (6): 1131–1159.

Chatterjee, S., C.-W. (J.) Chiu, T. Duprey and S. Hacıoğlu-Hoke. 2022. "Systemic Financial Stress and Macroeconomic Amplifications in the United Kingdom." *Oxford Bulletin of Economics and Statistics* 84 (2): 380–400.

Corrigan, P., H. Desgagnés, J. Dorich, V. Lepetyuk, W. Miyamoto and Y. Zhang. 2021. "ToTEM III: The Bank of Canada's Main DSGE Model for Projection and Policy Analysis." Bank of Canada Technical Report No. 119.

Duprey, T. .2020. "Canadian Financial Stress and Macroeconomic Condition." *Canadian Public Policy* 46 (S3): S236–S260.

Duprey, T., B. Klaus and T. Peltonen. 2017. "Dating Systemic Financial Stress Episodes in the EU Countries." *Journal of Financial Stability* 32 (October): 30–56.

Duprey, T., X. Liu, C. MacDonald, M. van Oordt, S. Priazhkina, X. Shen and J. Slive. 2018. "Modelling the Macrofinancial Effects of a House Price Correction in Canada." Bank of Canada Staff Analytical Note No. 2018-36.

Duprey, T. and T. Roberts. 2017. "A Barometer of Canadian Financial System Vulnerabilities." Bank of Canada Staff Analytical Note No. 2017-24.

Elder, J. and A. Serletis. 2010. "Oil Price Uncertainty." *Journal of Money, Credit and Banking* 42 (6): 1137–1159.

Erten, B. and K. Tuzcuoglu. 2018. "Output Effects of Global Food Commodity Shocks." *Journal of Globalization and Development* 9 (1): 1–18.

Fique, J. 2017. "The Macrofinancial Risk Assessment Framework (MFRAF), Version 2.0." Bank of Canada Technical Report No. 111.

Gelman, A., W. R. Gilks and G. O. Roberts. 1997. "Weak Convergence and Optimal Scaling of Random Walk Metropolis Algorithms." *Annals of Applied Probability* 7 (1): 110–120.

Gervais, O. and M.-A. Gosselin (2014): "Analyzing and Forecasting the Canadian Economy Through the LENS Model." Bank of Canada Technical Report No. 102.

Geweke, J. 1992. "Evaluating the Accuracy of Sampling-Based Approaches to the Calculations of Posterior Moments." In *Bayesian Statistics*, Vol. 4., edited by J. M. Bernardo, J. O. Berger, P. Dawid and A. F. M. Smith. Oxford: Clarendon Press: 169–193.

Iacoviello, M. and S. Neri. 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics* 2 (2): 125–164.

Kilian, L. 2009. "Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market." *American Economic Review* 99 (3): 1053–1069.

Kilian, L. and D. P. Murphy. 2014. "The Role of Inventories and Speculative Trading in the Global Market for Crude Oil." *Journal of Applied Econometrics* 29 (3): 454–478.

Kilian, L. and C. Vega. 2011. "Do Energy Prices Respond to US Macroeconomic News? A Test of the Hypothesis of Predetermined Energy Prices." *Review of Economics and Statistics* 93 (2): 660–671.

Krishnamurthy, A. and T. Muir. 2020. "How Credit Cycles Across a Financial Crisis." National Bureau of Economic Research Working Paper No. 23850.

Lenza, M. and G. E. Primiceri. 2020. "How to Estimate a VAR After March 2020." European Central Bank Working Paper No. 2461.

MacDonald, C. and V. Traclet. 2018. "The Framework for Risk Identification and Assessment." Bank of Canada Technical Report No. 113.

Ng, S. 2021. "Modeling Macroeconomic Variations after COVID-19." National Bureau of Economic Research Working Paper No. 29060.

Peterson, B. and T. Roberts. 2016. "Household Risk Assessment Model." Bank of Canada Technical Report No. 106.

Reinhart, C. M. and K. S. Rogoff. 2009. "The Aftermath of Financial Crises." *American Economic Review* 99 (2): 466–472.

Reinhart, C. M. and K. S. Rogoff. 2014. "Recovery from Financial Crises: Evidence from 100 Episodes." *American Economic Review*: 104 (5): 50–55.

Rubio-Ramírez, J. F., D. F. Waggoner and T. Zha. 2010. "Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference." *Review of Economic Studies* 77 (2): 665–696.

Uhlig, H. 2005. "What Are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure." *Journal of Monetary Economics* 52 (2): 381–419.

Wolf, C. K. 2020. "SVAR (Mis)identification and the Real Effects of Monetary Policy Shocks." *American Economic Journal: Macroeconomics* 12 (4): 1–32.

Zha, T. 1999. "Block Recursion and Structural Vector Autoregressions." *Journal of Econometrics* 90 (2): 291–316.

Zivanovic, J. 2019. "What Does Structural Analysis of the External Finance Premium Say About Financial Frictions?" Bank of Canada Staff Working Paper No. 2019-38.