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Production Networks and the Propagation of Commodity Price Shocks

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Abstract

We examine the macro implications of commodity price shocks in a general equilibrium model with input-output linkages for a commodity-exporting small open economy. In the model, fluctuations in commodity price affect aggregate output not only through resource reallocation, currency value changes and monetary policy reaction, but also through upstream and downstream input-output linkages (both domestically and with the rest of the world). Calibrated to the Canadian economy, our model is able to explain a large part of the decline in real gross domestic product (GDP) in 2015 and 2016 following the sharp drop in commodity prices. We find that as the model economy adjusts to a commodity price shock, domestic downstream linkages and the export connection with the rest of the world play an important role.

Topics: Business fluctuations and cycles; International topics
JEL codes: F41, D57

1 Introduction

Fluctuations of commodity prices are frequently associated with the volatility of aggregate output and prices. Their roles in transmitting inflation and inducing macroeconomic adjustments are clearly shown through the recent boom and bust cycles. For a commodity-exporting country, along with the rise and fall in commodity prices, adjustments take place in the economy, ranging from shifts in investment, employment and output to changes in interest rates and exchange rates.

A growing literature examines the effect on the aggregate economy of large movements in global oil prices. While the adjustments to commodity price cycles are already quite complex, one important channel often overlooked in the literature is the input-output linkages. Take Canada, for example: the commodity sector is important not only for exporting but also for providing intermediate inputs to the rest of the domestic economy. The production in the commodity sector also uses intermediate inputs produced in other domestic and foreign sectors. Further, the linkages between the commodity sector and the other sectors are uneven. In this context, ignoring the adjustments taking place through the production network would lead to an inaccurate characterization of the impact of commodity price shocks.

In this paper, we study the propagation of commodity price shocks in a multi-sector general equilibrium model for a small open economy that exports natural commodities. In addition to allowing for resource reallocation, exchange rate movements and monetary policy reaction, we emphasize the roles played by input-output linkages, both domestically and with the rest of the world. In the commodity-exporting small open economy, a shock to commodity prices is both aggregate and sectoral. In the sense of being an aggregate shock, movements of commodity prices lead to a change in the value of domestic currency and inflation, triggering monetary policy responses. As a sectoral shock, changes in commodity prices impact non-commodity sectors through resource reallocation and the production network.

When calibrated to the Canadian production network, our model suggests that, following a negative shock to commodity prices, production and exports in the commodity sector fall. While the net impact on the rest of the economy's exports is positive, the real gross domestic product (GDP) decreases, primarily owing to the decline in investment and lower net exports. Domestic downstream linkages and the export connections with the rest of the world play an important role in the process of adjusting to a commodity price shock. The upstream and import connections also matter, though to a lesser degree.

Using Canadian data on multifactor productivity, national income and expenditure accounts, as well as the World Input-Output Database (WIOD), we first document the Canadian production network, in particular the role of the commodity sector in the network. We find that some portions of all sectors' output are used as intermediate inputs (ranging from 17% to 53%), suggesting the importance of domestic production linkages. Moreover, not only does every sector supply intermediate inputs to other sectors, but also the production in each sector uses intermediate inputs from other sectors. Using network analysis to compute centrality measures, we find that the commodity sector is an important node in the Canadian production network because of its linkages with the rest of the economy both in terms of immediate ties and its influences.

These findings guide the development of a multi-sector model for a small open economy. In the model, each sector's production technology exhibits constant elasticity of substitution (CES) between the capital-labor bundle and the intermediate input bundle. The intermediate input bundle is in turn aggregated by the CES technology using domestic and imported goods. Each sector's output is used towards consumption, investment, export and supplying intermediate inputs to the rest of the economy.

Changes in global commodity prices have impacts on the economy in the following aspects. On one hand, a drop in foreign commodity prices reduces the exports and output in the domestic commodity sector, which in turn decreases the demand of this sector for the upstream goods as intermediate inputs. On the other hand, the lower commodity prices reduce the cost of production in the downstream non-commodity sectors, leading to substitution towards imported inputs. The direction of the net effects crucially depends on the input-output linkages between the commodity sector and other sectors, as well as the elasticity of substitution between the commodities and other inputs, and between domestic commodity goods and foreign counterparts. Calibrated to the Canadian data allowing for these channels through the input-output linkages, our model can account for the slowdown of real GDP in 2015 and a sizeable part of the negative growth of real GDP in 2016 with the decline of commodity prices between 2014 and 2015.

The downstream linkages appear important in driving the adjustments because without them, GDP would increase upon a negative global commodity price shock. In a counterfactual analysis, we allow the negative impacts on the commodity sector but assume away its downward linkages with the rest of the economy. Cheaper imported commodity inputs as a result of the declining global commodity prices lead to increases in production among the non-commodity sectors. Although imports also increase in this scenario, the net impact on real GDP remains positive. If, on top of this, we further shut down the export channel for the commodity sector, the above channels work the same and now the negative impact on the commodity sector is forced to be much smaller. This counterfactual scenario generates even more positive impacts on investment, exports and real GDP.

Domestic and international upstream linkages also matter, though to a much smaller degree, in the process of adjustment. If we shut down the domestic upstream linkages, production in non-commodity sectors becomes higher than in the baseline case, leading to a somewhat improved (though still negative) response of real GDP after the commodity price shock. If further, international upstream linkages are also shut down, since the commodity sector can use only capital and labor for production in this scenario, a reduced return to capital drives a lower investment profile. At the same time, a negative commodity price shock would have a less negative impact on production in some non-commodity sectors, because there are no negative impacts on upstream demand. Overall, the net impact on real GDP is fairly close to what was observed in the baseline.

Introducing heterogeneity in price rigidity to the model leads to less negative responses of real GDP to the global commodity price shock. Heterogeneity in price rigidity impacts the propagation properties of the economy by both delaying the response of the economy to the commodity price shock and distorting the price response of sectors relative to one another.

Our paper is closely linked to the literature that analyzes sector-specific versus aggregate sources of variations in the business cycle; see for example, Foerster et al. (2011). There are three explanations for why sectoral variability does not "average out" in the index of economy-wide production variability. First, aggregate shocks, common to all sectors, do not average out and can be a dominant source of variation in the aggregate economic activity. Second, granular shocks (see, for example, Gabaix (2011)) to a small number of very large sectors or firms in the economy do not average out because of their size. Third, sector-specific shocks that propagate through complementarity in production, such as input-output linkages, can generate substantial aggregate variability. The commodity price shock plays the role of all three of the above in explaining aggregate fluctuations. Our paper provides a structural framework to identify these differential transmission mechanisms.

Our paper is also related to the literature assessing the implications of commodity price on the macro economy. A number of papers have adopted structural vector autoregression (VAR) models to identify relevant shocks to the global crude oil market and assess the implications of these different shocks on the economy.[1] Though much attention has been focused on commodity-importing countries, there are a growing number of studies for countries producing and exporting commodities. In particular, Charnavoki and Dolado (2014) study the dynamic effects of commodity shocks on the Canadian economy using dynamic factor model estimation. Our focus is different from these studies. We explicitly model the structure of the economy, laying out the propagation mechanisms. Allowing for the commodity price shock to act as both an aggregate shock and a sector-specific shock, we examine the role played by the production network, both domestic and with the rest of the world. With this structural framework, we are able to not only reproduce the main stylized features documented in the literature including a Dutch disease effect, but also identify the quantitative significance of the propagation mechanisms.

[1] For example, Kilian (2009), Kilian and Murphy (2012) and Lippi and Nobili (2012).

The paper proceeds as follows. A snapshot of the Canadian production network and the central role of the commodity sector is presented in Section 2. Section 3 develops the multi-sector model with both domestic and foreign input-output linkages. Calibration of the model is discussed in Section 4. Section 5 presents the implied impacts of the 2015 plunge in commodity prices and the counterfactual analysis, and examines how heterogeneity in price rigidity distorts the price response of sectors relative to one another. Finally, Section 6 concludes.

2 Input-output linkages in the Canadian economy

In this section, we study the stylized facts of the Canadian production network and examine the role of the commodity sector in the network. To do so, we use the Canadian data on multifactor productivity, national income and expenditure accounts, and the WIOD 2016 release[2] to construct inputs and outputs for eight sectors: commodity; construction; utility; manufacturing (three sectors); wholesale, retail, transportation, and warehousing; and other services. The commodity sector is constructed such that the basket of goods is the same as that used in the Bank of Canada commodity price index (BCPI), which includes agriculture, forestry, fishery, mining, non-metal minerals, and primary metal. The first manufacturing sector includes manufacturers producing food, textiles, paper, etc. The second manufacturing sector produces petroleum, chemicals and plastic products. The last manufacturing sector produces machinery and equipment. Figures 1 and 2 plot the shares of the eight sectors in gross output, GDP, exports, hours worked, consumption and investment. Over the period of 1981 to 2004, on average, the commodity sector accounts for 13% of Canadian GDP, while being responsible for 35% of Canadian exports. Machinery and equipment manufacturing makes up another 37% of exports. In terms of hours worked, however, these two service sectors constitute 65% of the total, which highlights their labor-intensive nature. The construction sector accounts for over half of domestic investment, while the service sector makes up the majority of domestic consumption. Figure 3 plots the use of output for each sector in terms of consumption, investment, exports and intermediate inputs. Clearly, an important portion of all sectors' output is used as intermediate inputs, which suggests the importance of domestic production linkages. Take the commodity sector, for example. Commodities are used primarily not only for exports (54%) but also as intermediate inputs in the domestic economy (37%).

Not only does each sector supply intermediate inputs to other sectors, but also the production in each sector uses intermediate inputs from other sectors. Table 1 presents the domestic input-output linkages. Each column represents the use of the corresponding sec-

[2] Data from Statistics Canada are from 1981 to 2014. The WIOD data (2016 release) are from 2000 to 2014.

Figure 1: Sector Shares, 1981-2014

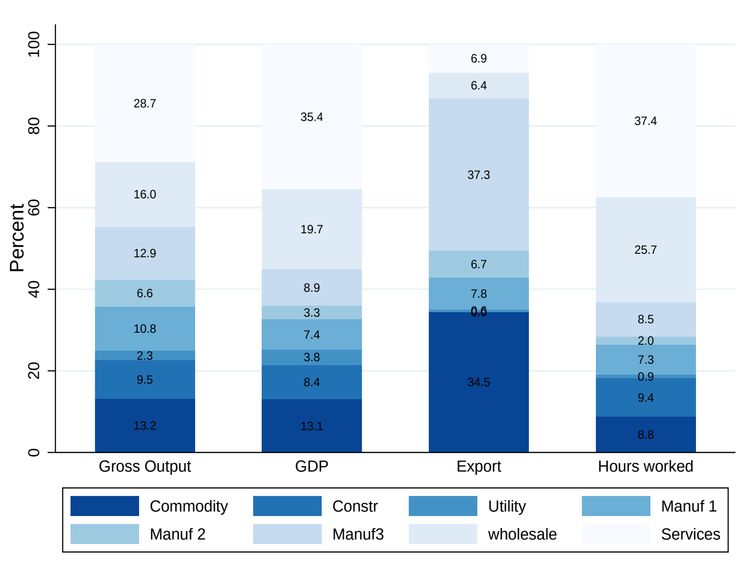
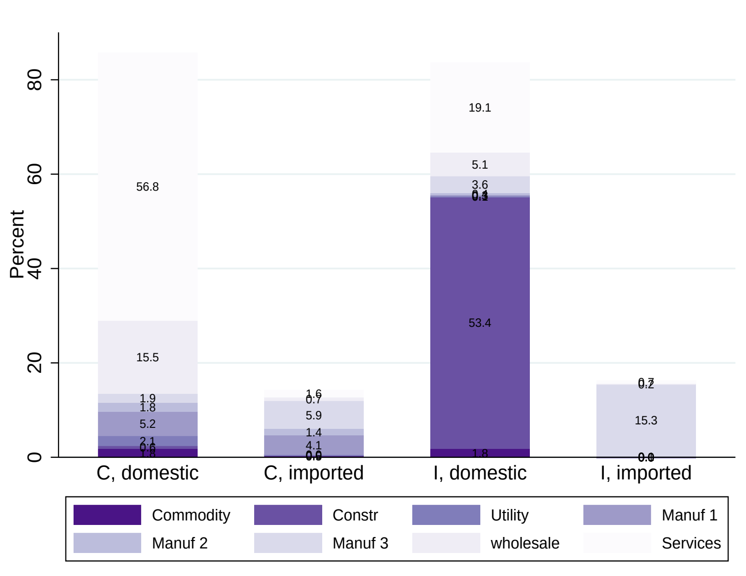


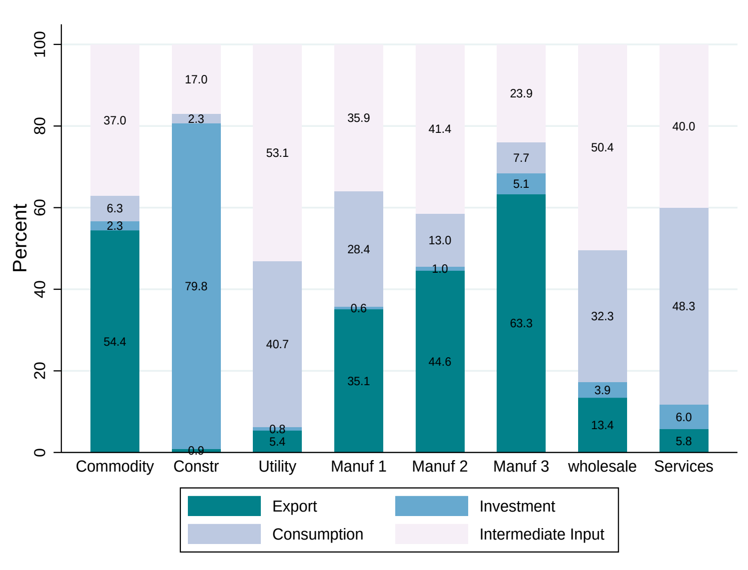
Figure 2: Contribution to Consumption and Investment, WIOD 2000-2014



Note: in the chart, C denotes consumption and I denotes investment.

tor, while each row represents the supply of each sector. For example, column 1 represents the intermediate inputs purchased by the commodity sector from all domestic sectors as a share of total intermediate inputs used to produce the commodity goods. Similar to other sectors, the wholesale and service sectors are the most important upstream sectors for commodity production. The link between the commodity sector and the rest of the economy

Figure 3: The Use of Outputs, WIOD 2000-2014



is also uneven across sectors. In a similar vein, Table 2 shows the input-output linkages of imported intermediate inputs. Overall, imported inputs make up a much smaller share of total inputs. They are more important for machinery, equipment, petroleum and chemical manufacturing, as well as for construction. Breaking it down, it appears that a significant portion of these imported inputs are supplied by the manufacturing sectors of machinery, equipment, petroleum and chemical products.

Table 1: Share of Supply in Use Sector's Total Intermediate Inputs (%)

Supply	Use							
	1	2	3	4	5	6	7	8
Commodity (1)	3.9	2.2	1.9	5.1	7.0	0.7	0.2	0.2
Construction (2)	2.7	1.4	15.8	0.5	0.5	0.5	3.3	5.7
Utility (3)	5.6	0.3	3.2	4.6	3.9	1.2	1.2	2.0
Manuf 1 (4)	6.7	5.8	1.2	20.9	1.7	6.3	1.1	4.4
Manuf 2 (5)	8.4	6.2	5.8	3.1	20.8	2.7	6.1	1.7
Manuf 3 (6)	3.8	9.6	4.3	2.6	1.4	9.8	4.2	4.3
Wholesale (7)	19.5	15.9	11.0	23.8	17.8	19.0	32.4	13.6
Services (8)	31.7	30.0	42.1	22.5	11.3	13.1	35.6	57.2
Total	82.3	71.3	85.3	83.2	64.5	53.2	84.1	89.1

Note: author's calculations, WIOD (2000-2014). Column j shows supply as a share of sector j total intermediate inputs.

In network analysis, indicators of centrality are used to identify the most important nodes within a network. Degree centrality directly measures the number of ties that a node has.

We first ranked the out-degree among 55 sectors in the Canadian economy[3] based on the 2007 and 2014 WIOD data. The ranking is presented in Tables 3 and 4. In-degree measures the number of ties that a node has in purchasing input from others. Mining and quarrying ranks ninth in the list. Out-degree measures the number of ties that a node directs to others, in other words, the number of sectors using a sector output as input. Mining and quarrying ranked number six among all 55 sectors. Manufacturing of coke and refined petroleum products is also among the top sectors in terms of immediate ties of a node in a network.

Table 2: Share of Imported Supply in Use Sector's Total Intermediate Inputs (%)

Imports	Use							
	1	2	3	4	5	6	7	8
Commodity (1)	2.5	1.6	1.4	0.9	5.1	1.7	0.0	0.0
Construction (2)	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.1
Utility (3)	0.1	0.0	0.1	0.1	0.2	0.0	0.0	0.0
Manuf 1 (4)	1.3	2.8	0.1	9.1	1.2	3.9	0.2	1.0
Manuf 2 (5)	5.5	4.7	2.9	4.0	25.3	3.4	3.6	1.0
Manuf 3 (6)	4.7	16.3	6.2	1.1	1.2	36.0	8.3	3.5
Wholesale (7)	0.9	0.7	0.7	0.6	1.6	0.8	1.7	1.0
Services (8)	2.6	2.5	3.2	1.0	0.9	1.0	2.0	4.3
Total	17.7	28.7	14.7	16.8	35.5	46.8	15.9	10.9

Note: author's calculations, WIOD (2000-2014). Column j shows imported supplies as a share of sector j total intermediate inputs.

Table 3: In-degree: Top Buyers of Aggregate Intermediate Inputs

Rank	2007	2014	ISIC Rev.4 name, 2014
1	O84	F	Construction
2	F	O84	Public administration and defence; compulsory social security
3	K64	H49	Land transport and transport via pipelines
4	H49	K64	Financial service activities, except insurance and pension funding
5	C10_C12	C10_C12	Manufacture of food products, beverages and tobacco products
6	L68	L68	Real estate activities
7	G45	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
8	N	C19	Manufacture of coke and refined petroleum products
9	G46	B	Mining and quarrying

We then compute the Katz prestige centrality measure of the sectors. Katz centrality is a form of eigenvector centrality which measures the influence of a node in a network.

[3]The WIOD 2016 release data use ISIC Rev.4 classification of sectors. There are 55 sectors in the Canadian national input-output table.

Table 4: Out-degree: Top Suppliers of Aggregate Intermediate Inputs

Rank	2007	2014	ISIC Rev.4 name, 2014
1	K64	H49	Land transport and transport via pipelines
2	H49	K64	Financial service activities, except insurance and pension funding
3	N	N	Administrative and support service activities
4	G45	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
5	L68	G46	Wholesale trade, except of motor vehicles and motorcycles
6	G46	B	Mining and quarrying
7	Q	L68	Real estate activities
8	F	F	Construction
9	B	C19	Manufacture of coke and refined petroleum products

Specifically, it measures the number of all nodes that can be connected through a path, while the contributions of distant nodes are penalized. The Katz centrality for node i is

$$P_i^K(g) = \sum_{j \neq i} \frac{g_{ij}}{d_j(g)} P_j^K(g),$$

where g_{ij} indicates i and j are linked, and $d_j(g)$ is the number of edges connecting node j. Computing Katz centrality with the 2014 WIOD data shows that the top sectors in terms of centrality are the following:

1. Land transport and transport via pipelines (0.68);
2. Construction (0.28);
3. Real estate activities (0.22);
4. Manufacture of food products, beverages and tobacco products (0.20);
5. Manufacture of coke and refined petroleum products (0.19);
6. Financial service activities, except insurance and pension funding (0.18);
7. Mining and quarrying (0.15).

The commodity sector is again among the top nodes in terms of influences in the Canadian production network. Taken all together, the commodity sector is of importance to the Canadian economy not only through its size and its importance in exports, but also because of its influential linkages with the rest of the economy.

Over the past 20 years, we have observed the commodity price super cycles. A broad-based boom in global commodity markets began in the early 2000s, fuelled by growing demand from emerging-market economies. For a commodity-exporting country like Canada, rising commodity prices boosted economic activity and generated job growth in the natural resource sector. The commodity sector contributed 15% of aggregate output and 40% of total exports for Canada in 2014; business investment in the oil and gas extraction sector peaked at almost \$80 billion. The boom in the oil sector had spill-over impacts on other sectors through resource reallocation. Between 2002 and 2013, more than a quarter of a million people moved from other provinces to the oil-producing regions. In addition, the number of workers commuting to these regions doubled during this period, rising to about 8% of the regions' workforce.[4] On top of the domestic adjustment, the positive link between commodity prices and the value of currency also contributed to shifting resources from non-commodity export sectors to commodity-producing sectors and regions.

Since its historically high levels in mid-2014, the West Texas Intermediate (WTI) crude oil price declined by about 68% between mid-2014 and January 2016. This has triggered a process of restructuring in the oil and gas sector, in the opposite way to what we saw in the previous decade. The value of the Canadian dollar has fallen along with commodity prices, facilitating the adjustment to the new circumstances. Concerned about the impact of lower oil prices and the risks to inflation, the Bank of Canada lowered its policy rate in January 2015 and again in July of the same year. Commodity price fluctuations have macro implications not only through resource reallocations and currency value changes, but also through monetary policy reactions.

3 Model

We consider a small open economy that produces and exports commodities. The economy consists of N sectors and one representative household. Each sector produces one good, which can be used for final demands and as intermediate inputs. The model economy is featured with the input-output linkages, both domestic and with the rest of the world.

3.1 Sector Production Production in sector j requires a capital-labor bundle (K_j and L_j) and intermediate inputs (M_j). We assume that the production technology exhibits constant elasticity of substitution between the capital-labor bundle and the intermediate input, as follows:[5]

$$Q_j = A_j \left[(1-\psi_j)^{\frac{1}{\sigma_q}} \left(\left(\frac{K_j}{\alpha_{kj}} \right)^{\alpha_{kj}} \left(\frac{L_j}{\alpha_{lj}} \right)^{\alpha_{lj}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} + \psi_j^{\frac{1}{\sigma_q}} M_j^{\frac{\sigma_q - 1}{\sigma_q}} \right]^{\frac{\sigma_q}{\sigma_q - 1}} .$$

A_j is the Hicks-neutral productivity, or total factor productivity (TFP), which follows an AR(1) process. We impose that $\alpha_{kj} + \alpha_{lj} = 1$. The elasticity of substitution of capital-labor bundle for intermediate inputs is $\sigma_q > 0$: the larger the elasticity, the higher the degree of substitutability between factors. We impose that the elasticity of substitution is the same

[4] Patterson, Lynn: Adjusting to the Fall in Commodity Prices: One Step at a Time, March 30, 2016.

[5] We omit the time subscript wherever we feel its omission will not create confusion.

across all sectors. The share of intermediate inputs in gross outputs varies across sectors due to differences in ψ_j, productivity, and relative prices.

Cobb-Douglas production is prevalent in the existing studies, under which Hulten's theorem holds: the impact of a productivity shock in one sector on aggregate output is captured by the Domar weight of that sector. That impact quantified by Domar weights is first-order and exact under the Cobb-Douglas production function. Baqaee and Farhi (2017) show, however, under CES production functions, the second-order impact of shocks to sectoral productivity can be quantitatively significant. Such an impact is determined by the network structure, elasticities of substitution, and returns to scale in production.

The intermediate input M_j is aggregated from domestic and imported goods, as follows:

$$M_j = \left[\sum_{i=1}^{N} \left(\omega_{hij}^{\frac{1}{\sigma_m}} M_{hij}^{\frac{\sigma_m - 1}{\sigma_m}} + \omega_{fij}^{\frac{1}{\sigma_m}} M_{fij}^{\frac{\sigma_m - 1}{\sigma_m}} \right) \right]^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sum_{i=1}^{N} (\omega_{hij} + \omega_{fij}) = 1$. M_{hij} is the amount of goods produced by sector i (the second subscript) and supplied to sector j (the third subscript), and M_{fij} is the imported sector i good that is used by sector j for production. Here, we assume that the elasticity of substitution of one input for another is the same across all intermediate inputs, both domestic and imported.

The use of sector j output is

$$Q_j = C_{hj} + V_{hj} + \sum_{i=1}^{N} M_{hji} + X_j,$$

where C_{hj} is a component of aggregate consumption, V_{hj} is the contribution to aggregate investment, and X_j is exports. M_{hji} is the amount of good j supplied to sector i as an intermediate input. We assume that there is no trade cost, so the export price (denominated in domestic currency) P_{xj} and domestic price P_{hj} must then be equal.

Let S be the nominal exchange rate, namely the value of domestic currency per unit of foreign currency. A lower value of S means an appreciation of domestic currency. Let P_j^* be the foreign price denominated in foreign currency.

The foreign demand for sector j good is given by

$$X_j = \alpha_{xj} \left(\frac{P_{hj}^*}{SP^*} \right)^{-\sigma_x} Y^*.$$

P^* and Y^* represent, respectively, aggregate price and aggregate output in the rest of the world, which are exogenous to the domestic economy. The function of demand for exports is consistent with a CES preference in destinations, with σ_x denoting the Armington elasticity in the rest of the world. Current-price exports fall when export prices rise if $\sigma_x > 1$, and increase following a rise in export prices if $\sigma_x < 1$.

We now derive the marginal cost. First, we define the nominal net rate of return to capital as $R_{kt} = r_{kt}P_{vt}$, then the price of capital-labor bundle $P_{yjt} = (R_{kt})^{\alpha_{kj}}(w_{jt})^{\alpha_{lj}}$. $w_{jt} = w_t$ since labor adjustment is costless. The sectoral marginal cost is obtained from the cost minimization problem

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \Gamma_t \Big\{ w_t L_{jt} + R_{kt}K_{jt} + P_{mjt}M_{jt} \Big\},$$

subject to $Q_{jt} \geq \overline{Q}_j$. The Lagrange Multiplier λ_{jt} is interpreted as the marginal cost. The optimal conditions are given by

$$w_{jt} = \lambda_{jt}\frac{\partial Q_{jt}}{\partial L_{jt}}; \quad R_{kt} = \lambda_{jt}\frac{\partial Q_{jt}}{\partial K_{jt}}; \quad P_{mjt} = \lambda_{jt}\frac{\partial Q_{jt}}{\partial M_{jt}}.$$

We write the capital-labor bundle as $Y_{jt} = \left(\frac{K_{jt}}{\alpha_{kj}}\right)^{\alpha_{kj}}\left(\frac{L_{jt}}{\alpha_{lj}}\right)^{\alpha_{lj}}$, then the bundle price is $P_{yjt} = (R_{kt})^{\alpha_{kj}}(w_{jt})^{\alpha_{lj}}$. Optimal conditions are rewritten as

$$\begin{aligned}
\frac{R_{kt}K_{jt}}{\lambda_{jt}Q_j} &= \alpha_{kj}(1-\psi_j)A_{jt}^{\sigma_q-1}\left(\frac{P_{yjt}}{\lambda_{jt}}\right)^{1-\sigma_q};\\
\frac{w_{jt}L_{jt}}{\lambda_{jt}Q_j} &= \alpha_{lj}(1-\psi_j)A_{jt}^{\sigma_q-1}\left(\frac{P_{yjt}}{\lambda_{jt}}\right)^{1-\sigma_q};\\
\frac{P_{mjt}M_{jt}}{\lambda_{jt}Q_j} &= \psi_j A_{jt}^{\sigma_q-1}\left(\frac{P_{mjt}}{\lambda_{jt}}\right)^{1-\sigma_q}.
\end{aligned}$$

Solving for λ_{jt}, we obtain the marginal cost as $MC_{jt} = A_{jt}^{-1}\left[(1-\psi_j)P_{yjt}^{1-\sigma_q} + \psi_j P_{mjt}^{1-\sigma_q}\right]^{\frac{1}{1-\sigma_q}}$.

3.2 Aggregate consumption and investment

Aggregate consumption C_t is bundled from sector-level outputs and imported goods, with the following CES technology:

$$C = \left[\sum_{j=1}^{N}\left(\phi_{hj}^{\frac{1}{\sigma_c}}C_{hj}^{\frac{\sigma_c-1}{\sigma}} + \phi_{fj}^{\frac{1}{\sigma_c}}C_{fj}^{\frac{\sigma_c-1}{\sigma_c}}\right)\right]^{\frac{\sigma_c}{\sigma_c-1}},$$

where $\sum_{j=1}^{N}(\phi_{hj}+\phi_{fj}) = 1$.

Capital is homogeneous across sectors, and its aggregate stock evolves according to

$$K_{t+1} = (1-\delta)K_t + V_t.$$

Aggregate investment V_t is produced using domestic and imported capital goods, as follows:

$$V_t = \left[\sum_{j=1}^{N} \left(\theta_{hj}^{\frac{1}{\sigma_v}} V_{hjt}^{\frac{\sigma_v - 1}{\sigma_v}} + \theta_{fj}^{\frac{1}{\sigma_v}} V_{fjt}^{\frac{\sigma_v - 1}{\sigma_v}} \right) \right]^{\frac{\sigma_v}{\sigma_v - 1}},$$

with $\sum_{j=1}^{N} (\theta_{hj} + \theta_{fj}) = 1$. We do not allow the bundling of investment to be sector-specific, largely because we have no information on flows of investment goods across sectors in the data.

The representative household makes decisions on investment, domestic bond B_t and foreign bond B_t^*. It maximizes utility in an infinite horizon

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \xi L_t],$$

subject to

$$P_{ct} C_t + P_{vt} V_t + \frac{B_t}{R_t} + \frac{S_t B_t^*}{R_t^* r p_t} \leq w_t L_t + r_{kt} P_{vt} K_t + S_t B_{t-1}^* + B_{t-1} + T_t + \Pi_t.$$

Here, R_t and R_t^* are bond rates, T_t is the lump-sum government transfer, and Π_t is the aggregate profit. A quadratic adjustment cost on bond holdings, rp_t, is assumed to ensure the stationarity in the net foreign asset position. We assume a simplistic government budget constraint, $T_t = \frac{B_t}{R_t} - B_{t-1}$. That is, government transfer T_t is exogenous, and is financed by issuing bonds.

The import price of good j, regardless of its use, is given by $P_{fj} = S P_j^*$. At the aggregate level, trade surpluses are saved in the form of foreign bonds. The balance of payment for the economy is given by

$$\sum_{i=1}^{N} P_{xi,t} X_{it} - \sum_{i}^{N} P_{fi,t} \big(V_{fi,t} + C_{fi,t} + \sum_{j}^{N} M_{fi,j,t} \big) = \frac{S_t B_t^*}{R_t^*} - S_t B_{t-1}^*.$$

3.3 Interest rate and exchange rate

The nominal interest rate is determined by central bank with a Taylor-type rule as follows:

$$\ln(R_t/R) = \rho_r \ln(R_{t-1}/R) + (1 - \rho_r)[\alpha_\pi \ln(\pi_t/\pi) + \alpha_z \ln(Z_t/Z)] + \epsilon_{Rt}.$$

Here, π_t is the inflation rate of consumption price P_{ct}, and Z_t is the aggregate real GDP. We define Z_t as the geometric mean of consumption, investment, aggregate exports and imports, as $Z_t = C_t^{\omega_{ct}} V_t^{\omega_{vt}} \cdot \prod_{j=1}^{N} X_{jt}^{\omega_{xjt}} \cdot \prod_{j=1}^{N} I_{mjt}^{-\omega_{mjt}}$, where $\omega_{ct} = \frac{P_{ct} C_t}{P_{zt} Z_t}$ and similarly for other components. I_{mjt} is the total import of sector-j commodities, $I_{mjt} = C_{fjt} + V_{fjt} + \sum_{i=1}^{N} M_{fjit}$. We assume that ω_{ct} and other shares in real GDP are constant and equal to their respective values in the steady state (which are endogenously determined in equilibrium). Given domestic and foreign interest rates, the nominal exchange rate is determined by the interest rate parity condition.

3.4 The shock propagation In this economy, the shocks are those concerning TFP A_{jt}, foreign prices P^*_{jt}, foreign interest rate R^*_t, and foreign output Y^*_{jt} for $j = 1, \cdots, N$. Our focus is on the channels through which commodity price shock affects the domestic economy. We want to emphasize two optimal conditions that are key for the propagation mechanism, one concerning the exchange rate, the other concerning the input-output linkage.

3.4.1 Commodity price and exchange rate The optimal conditions with respect to domestic and foreign bonds imply uncovered interest parity condition,

$$\frac{R_t}{R^*_t rp_t} = \mathbf{E}_t \frac{S_{t+1}}{S_t}.$$

Details of deriving optimal conditions are in Appendix A. A rise in commodity prices can lead to an appreciation of domestic currency. The increased exports and output in the commodity sector raise the demand for capital and labor. The production in the commodity sector is capital-intensive, hence the demand for labor increases, but by a small margin relative to that for capital and investment. As commodity prices rise, the rate of returns to capital tends to rise due to increased demand for investment, and this in turn lowers consumption prices. By the Taylor rule, the domestic nominal interest rate will be higher, leading to domestic currency appreciation.

Output in non-commodity sectors tends to fall under two impacts of a positive shock to commodity prices. The rising real returns to capital makes capital more costly for all sectors, dampening the demand for capital and output in non-commodity sectors. Moreover, for sectors with a large share of exports in gross output such as manufacturing, output can fall due to lower exports following the domestic currency appreciation.

The fall in gross outputs does not necessarily reduce the real value added, which is more relevant for welfare. For sectors where the imported intermediate inputs account for a large share of total inputs, such as manufacturing, appreciated domestic currency leads to lower import prices. Whether the lower import prices increase or reduce value added depends on the value of the elasticity of substitution between individual intermediate inputs, and between the capital-labor bundle and total intermediate input.

3.4.2 Propagation through production network The share of domestic intermediate input i in sector j's production is given by

$$\Omega_{hijt} = \frac{P_{hit}M_{hijt}}{P_{hjt}Q_{jt}} = \omega_{hij}\psi_j P_{hit}^{1-\sigma_m}\Big[\sum_{i=1}^{N}\big(\omega_{hij}P_{hit}^{1-\sigma_m} + \omega_{fij}P_{fit}^{1-\sigma_m}\big)\Big]^{\frac{\sigma_m-\sigma_q}{1-\sigma_m}} P_{hjt}^{\sigma_q-1}A_{jt}^{\sigma_q-1}. \quad (3.1)$$

Let matrix $\Omega_{\mathbf{ht}} = [\Omega_{hijt}]_{N\times N}$, and let $F_t = [F_{jt}]_{N\times 1}$ be the vector of final uses of sector j output. We write $\mathsf{P_{ht}F_t} = [P_{hjt}F_{jt}]_{N\times 1}$ and so on. From the use equation of sector j output, $P_{hjt}Q_{jt} = P_{hjt}F_{jt} + P_{hjt}\sum_{i=1}^{N} M_{hjit}$, we obtain

$$\mathsf{P_{ht}Q_t} = \big[\mathbf{I} - \Omega_{\mathbf{ht}}\big]^{-1}\mathsf{P_{ht}F_t}. \quad (3.2)$$

Here $\mathbf{I}$ is the identity matrix. $\mathbf{H_t} = \big[\mathbf{I} - \Omega_{\mathbf{ht}}\big]^{-1}$ is the Leontief inverse matrix. As shown in Acemoglu et al. (2015), in the case of a Cobb-Douglas production function in which the elements of Leontief inverse matrix are constants, the network effects materialize through the Leontief inverse matrix. When a sector-specific shock hits sector j, the demand and price of that sector change (own effect). The changed demand will impact the upstream suppliers to sector j, and the change in price of output j will impact the downstream sectors that purchase inputs from sector j.

Following a positive shock in commodity prices, the demand for commodity exports rises. This in turn raises the demand for non-commodity goods used as intermediate inputs to produce the commodities. The output of upstream sectors therefore rises, in particular the supplying sectors that account for a large share of intermediate inputs in the commodity sector.

The impact of commodity prices through the production network is first-order, because the Leontief inverse matrix under the Cobb-Douglas production function is invariant to changes in commodity prices or any other prices. It is with the CES production function that the second-order effect presents, and it can be shown that the impact of a change in commodity price is given by

$$\left[\frac{d\ln P_{hj}}{d\ln P_{f1}}\right]_{N\times 1} = \Big[\mathbf{I} - \big[\alpha_{kj}s_{yj}\big]_{N\times 1}\big[s_{hvi}\big]'_{N\times 1} - di\big[s_{mj}\big]_{N\times N}\big[s_{hmij}\big]'_{N\times N}\Big]^{-1}\Big[\alpha_{kj}s_{yj}s_{fv1} + s_{mj}s_{fm1j}\Big]_{N\times 1}.$$

Here $s_{yj} = \frac{P_{yj}Y_j}{P_{hj}Q_j}$, $s_{hvi} = \frac{P_{hi}V_{hi}}{P_v V}$, $s_{fm1j} = \frac{P_{f1}M_{f1j}}{P_{mj}M_j}$, and so on. They are the share of individual inputs in production functions, and are constant if production functions are Cobb-Douglas. The matrix $di\big[s_{mj}\big]_{N\times N}$ is diagonal. Subscript j represents a row, and i a column. Let $\mathbf{S} = \big[\alpha_{kj}s_{yj}\big]_{N\times 1}\big[s_{hvi}\big]'_{N\times 1} + di\big[s_{mj}\big]_{N\times N}\big[s_{hmij}\big]'_{N\times N}$. Then $\big[\mathbf{I} - \mathbf{S}\big]^{-1}$ represents the impact of commodity price change on output prices.

The elasticity of substitution between intermediate inputs and the capital-labor combination and that between imported and domestically produced intermediate inputs are key parameters in determining the magnitude of an impact through the network. An increase in import prices leads to a higher P_{mjt}, and the share of intermediate inputs could become higher or lower depending whether the elasticity of substitution is greater or less than 1, i.e. whether intermediate inputs and capital-labor are gross complements or substitutes.

4 Model Calibration

Elasticity of substitution. We estimate the elasticity of substitution in all production functions and aggregate functions, using the shares of intermediate inputs implied by the model, similar to Atalay (2017).

To estimate σ_q, we use the share of intermediate inputs in sector gross outputs. The optimal condition regarding the composite intermediate input in sector j in any period is given by

$$\frac{P_{mj}M_j}{P_{hj}Q_j} = \psi_j \left(\frac{P_{mj}}{P_{hj}}\right)^{1-\sigma_q} A_j^{\sigma_q - 1}.$$

On the right-hand side are the price of composite intermediate inputs relative to the price of gross output in sector j and the total factor productivity in sector j. The latter is drawn from data by Statistics Canada, which is measured by assuming that production function exhibits constant returns to scale.[6] We take the logarithm of the above equation, and further take first-order differencing over time to get

$$\Delta \ln \frac{P_{mj}M_j}{P_{hj}Q_j} = (1-\sigma_q)\Delta \ln \left(\frac{P_{mj}}{P_{hj}}\right) + (\sigma_q - 1)\Delta \ln A_j.$$

We augment the above equation with a constant term and an error term. The relative price is endogenous and correlated with the exogenous total factor productivity. We thus use the lagged growth of prices of exports and imports as instrument variables, assuming that these latter prices are exogenous to the Canadian economy. Considering that TFP is serially correlated, we also use lagged TFP changes as an instrument variable. The estimation is done with the generalized method of moments for the period 1961-2014, and the estimate $\hat{\sigma}_q = 0.92$ (robust Std.Err. 0.48) is significant at the 5% level.[7]

Next we estimate σ_c, for which we use the optimal condition regarding shares of domestic consumption goods in aggregate consumption. Taking the logarithm and first-order

[6] See Baldwin et al. (2007).

[7] If we use the data sample since 1981, the resulting estimate is $\hat{\sigma}_q = 0.74$ (Std.Err. 0.48), which is statistically insignificant. Overall, there is weak evidence suggesting a declining elasticity of substitution between the capital-labor bundle and the intermediate input. We also tried the estimation using the level of lagged prices of exports and imports, and obtained a smaller estimate $\hat{\sigma}_q = 0.72$.

differencing leads to

$$\Delta \ln \left(\frac{P_{hj} C_{hj}}{P_c C} \right) = (1 - \sigma_c) \Delta \ln \left(\frac{P_{hj}}{P_c} \right).$$

Shares of individual components in aggregate consumption are calculated using the WIOD. We again use changes in prices of exports and imports as instrument variables. The estimate is $\hat{\sigma}_c = 0.89$ (Std.Err. 0.27), which is statistically significant at the 1% level.

A similar approach is used to estimate the elasticity of substitution in the investment aggregation function. The estimate is $\hat{\sigma}_v = 1.01$ (Std.Err. 0.57), and is statistically significant at the 10% level.

Next, we estimate σ_m, for which we use the optimal conditions regarding the domestic shares of intermediate inputs in a sector's total intermediate input,

$$\frac{P_{hi} M_{hij}}{P_{mj} M_j} = \omega_{hij} \left(\frac{P_{hi}}{P_{mj}} \right)^{1-\sigma_m}.$$

The right-hand side is the ratio of domestic output price over the price of intermediate inputs at the sector level. We take the logarithm and first-order differencing of the above equation:

$$\Delta \ln \frac{P_{hi} M_{hij}}{P_{mj} M_j} = (1 - \sigma_m) \Delta \ln \left(\frac{P_{hi}}{P_{mj}} \right).$$

Shares of individual components in composite intermediate inputs in the data are calculated from the 2016 WIOD, spanning from 2000 to 2014. For instrument variables, we use growth rates of export prices corresponding to the supply sectors and import prices corresponding to the use sectors. The estimate is $\hat{\sigma}_m = 1.067$ (Std.Err. 0.19), which is at the 1% significant level.

Weights. The weight parameters in production and aggregation functions are calibrated to match the observed shares of individual components in production and aggregation functions in the steady state of model solution, given the estimated elasticities of substitution. These shares in data are calculated as the average values from the multifactor productivity data by Statistics Canada, and from the 2016 WIOD for intermediate inputs, aggregate consumption and investment. Calibrated weights are reported in Table 5.

Labor shares. We calculate the share of labor in the total cost of the capital-labor bundle from the multifactor productivity data, using the averages over 1981 to 2014. Labor shares in sectors, from commodity to services, are, respectively, 0.39, 0.90, 0.28, 0.71, 0.48, 0.71, 0.72, and 0.62.

Table 5: Intensity Parameters and Productivity Shocks

	Comm (1)	Constr (2)	Util (3)	Mfg 1 (4)	Mfg 2 (5)	Mfg 3 (6)	Whsl (7)	Svc (8)
ψ_j	0.492	0.576	0.180	0.670	0.725	0.640	0.384	0.372
ϕ_{hj}	0.024	0.016	0.011	0.074	0.043	0.033	0.149	0.508
ϕ_{fj}	0.000	0.000	0.000	0.042	0.023	0.039	0.009	0.028
θ_{hj}	0.010	0.539	0.002	0.004	0.005	0.036	0.051	0.188
θ_{fj}	0.001	0.000	0.000	0.000	0.002	0.151	0.003	0.008
ρ_{a_j}	.40	.98	.90	.78	.73	.68	.90	.79
σ_{ε_j}	.22	.22	.22	.23	.24	.22	.22	.22

Data sources: author's calculations, WIOD (2000-2014).

Price elasticity of exports. The price elasticity of demand for exports, σ_x, is hard to pin down. We choose $\sigma_x = 1.5$, which is the Armington elasticity commonly assumed in real business cycle models.[8]

Preference and depreciation rate. We set $\xi = 1$, $\beta = 0.96$, and $\delta = 0.10$. The depreciation rate of capital is calculated using data on capital stocks and flow by Statistics Canada. The implied steady-state real interest rate on capital service is $r = \frac{1}{\beta} - 1 + \delta = 0.142$.

Shocks. We assume that shocks of exogenous variables in the logarithm all follow AR(1) processes. For global commodity prices, we use the prices of imported commodities in U.S. dollars, de-trended with the HP filter. Data span from 1981 to 2017. Using the ARIMA routine in Stata, we estimate that $\widehat{\rho}_{p_{h1}} = 0.779$ (semirobust std.err. is 0.116), and the estimated standard deviation of the noise term is 0.083. For the Canadian monetary policy shock, we use the series created by Champagne and Sekkel (2017). At the yearly frequency, the standard deviation of this shock is 0.079.

We estimate the shocks of foreign price and output using the price and quantity of gross output in the United States for the period of 1981 to 2017.[9] The estimated serial correlation coefficient of output price is 0.77, and the standard deviation of the error term for the price process is 0.022. The serial correlation for gross output is 0.956, and the standard deviation for the error term is 0.021. For shocks to the foreign interest rate, we estimate an AR(1) process using the U.S. 1-year treasury bill constant maturity rate. This gives a serial correlation coefficient of 0.79, and the standard deviation of the error term is 0.39.

Hicks-neutral productivity shocks in our model can be different from those measured in KLEMS data by statistical agency, because our model allows non-productivity shocks

[8]More discussion can be found in Ruhl (2008).

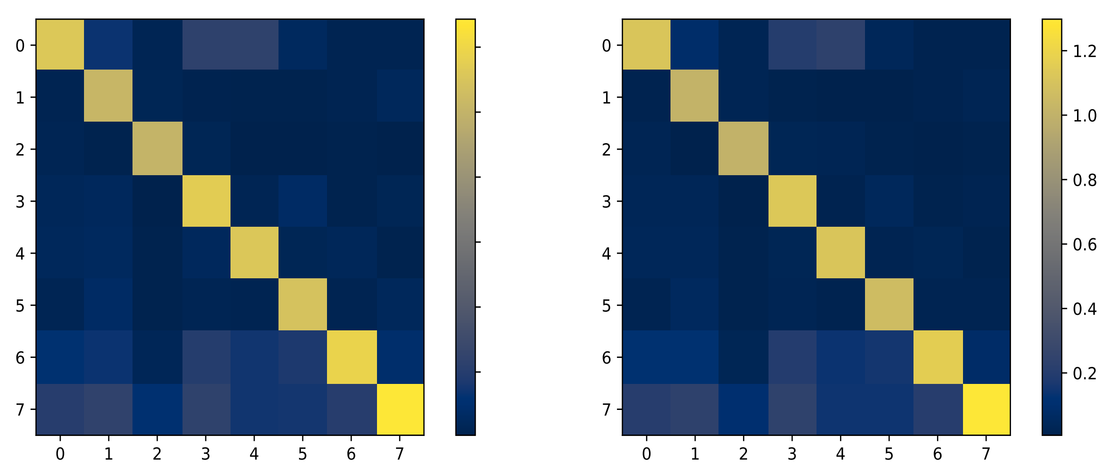
[9]Data are downloaded from www.bea.gov, annual industry statistics.

while in KLEMS, productivity is the only exogenous variable. Thus, we cannot use the measured productivity. Unlike Foerster et al. (2011), our model allows multiple shocks; we cannot back out productivity shocks from the model. We use the model to calibrate the serial correlations of productivity and standard deviations of the AR(1) process white noises, by matching the serial correlation and standard deviation of sector-level gross outputs. The obtained serial correlation coefficients and standard deviations are reported in Table 5. With these parameter values, the serial correlations of gross outputs predicted by the model closely match those in the data for the large sectors (manufacturing and services).[10]

5 Quantitative Analysis

5.1 Some properties in steady state The Leontief inverse matrix in steady state from the model and from the data is shown in Figure 4. Lighter colors suggest a larger total requirement for a sector's gross output, due to a stronger linkage between two sectors. Qualitatively, gross output in the commodity sector is required with the highest weights for final demand for the first two manufacturing sectors, meaning that they are important downstream buyers of commodities. The wholesale sector and the service sector are the largest suppliers to the commodity sector other than the sector itself. The Leontief inverse matrix in the steady state matches well that in the Canadian data.

Figure 4: Leontief Inverse Matrix: Model vs. Data



Model steady state

Data

5.2 Impact of the 2015 plunge in commodity prices We use the calibrated model to quantify the impact of the sharp fall in commodity prices that started in the second half

[10]The model overfits the standard deviation of gross outputs though, at a scale much larger than measured in the KLEMS data by a factor of 10.

of 2014. This allows us to assess the conformity of the model to the Canadian data. In 2015, the price of imported commodities in U.S. dollars dropped 13.7% from its trend, and BCPI dropped by 27.5%. Real GDP and its main components also declined. Table 6 reports the magnitude of the decline in the data. Specifically, real GDP dropped 1.3% in 2015 and 2.3% in 2016, both from its trend.

Table 6: Percentage Deviation from Trends

Year	Commodity import price	BCPI	Real GDP	Consumption	Investment	Exports	Imports
2014	8.4	18.2	0.03	-0.6	5.0	-3.2	-1.1
2015	-13.7	-27.5	-1.3	-1.1	-2.9	-1.5	-3.6
2016	-	-38.4	-2.3	-1.5	-9.8	-1.8	-6.7
2017	-	-24.3	-1.3	-0.9	-9.4	-2.4	-5.6

Note: author's calculations. Commodity import price is in U.S. dollars. Data sources: Statistics Canada, Bank of Canada.

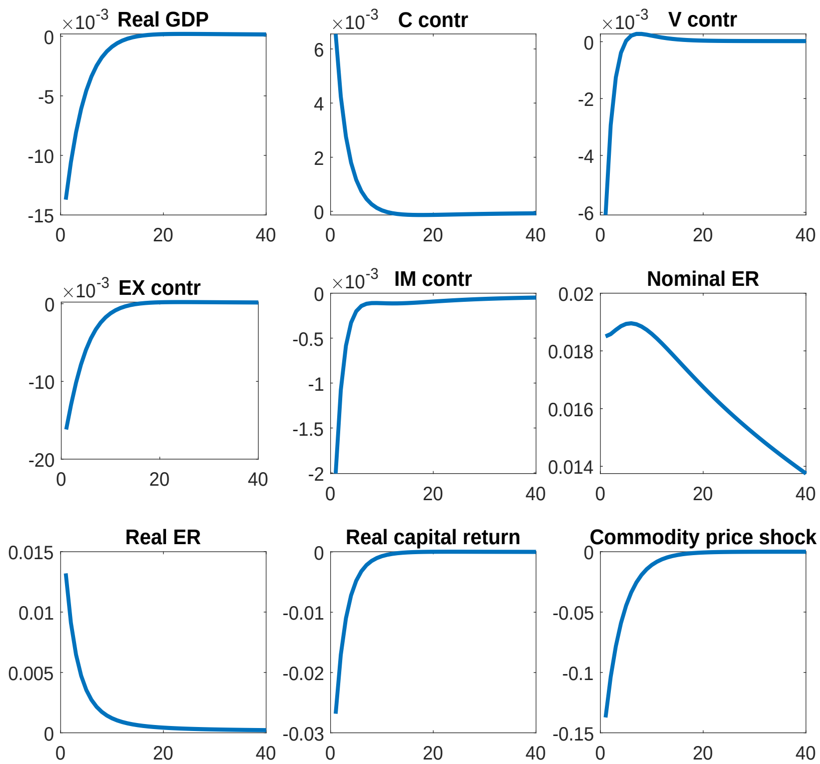
To quantify the impact of the commodity price plunge in the model, we impose a one-time negative shock of the size of 13.7%, the same magnitude as the 2015 commodity price drop. We feed this one-time shock to the system, together with the total factor productivity shocks at the sector level. We obtain the shocks to sectoral TFP from the multifactor productivity data by Statistics Canada. From 2014 to 2015, TFP is above the trends in three sectors: commodities (sector 1), manufacturing 1 (sector 4), and services (sector 8). In other sectors, TFP is below the trends. Simulating the model economy with both a shock to global commodity prices and shocks to measured TFP in all sectors shows that these shocks together led to a decline in real GDP of 0.8%, compared with the 1.3% drop in the data. This suggests that our model is able to explain a large part of the decline in GDP in the year after.

5.3 Baseline impacts: Macro variables Next, we examine the impacts of the large drop in commodity prices and plot the impulse responses of real GDP and contribution from its components. Shown in Figure 5, the negative shock to commodity prices led to a drop of real GDP by 1.37% in 2015 and 1.06% in 2016. In our calibration, the commodity price shock primarily explains the negative growth in real GDP in 2015 and 2016.[11] The negative impact on real GDP is mainly accounted for by the worsened net exports. The contribution of lowered investment is offset by a positive response of aggregate consumption. Aggregate exports drop by 1.6%, entirely due to the drop of exports in the commodity sector. Aggregate imports edge down slightly.

[11] Commodity prices continued to fall in 2016. This would certainly contribute to the more negative growth in 2016, though this is not reflected in our exercise as we only impose a one-time 2015 shock.

The domestic nominal interest rate increases slightly following a rise in the consumer price index. By the uncovered interest rate parity condition, the nominal exchange rate rises by about 1.8%, suggesting a depreciation of domestic currency. Since the consumer price index rises by a smaller margin than the nominal exchange rate, the economy experiences a real depreciation.

Figure 5: Impacts of the Commodity Price Shock: Baseline



Note: Figures of C contr, V contr, EX contr and IM contr display the contribution of consumption, investment, exports and imports respectively to the response of real GDP.

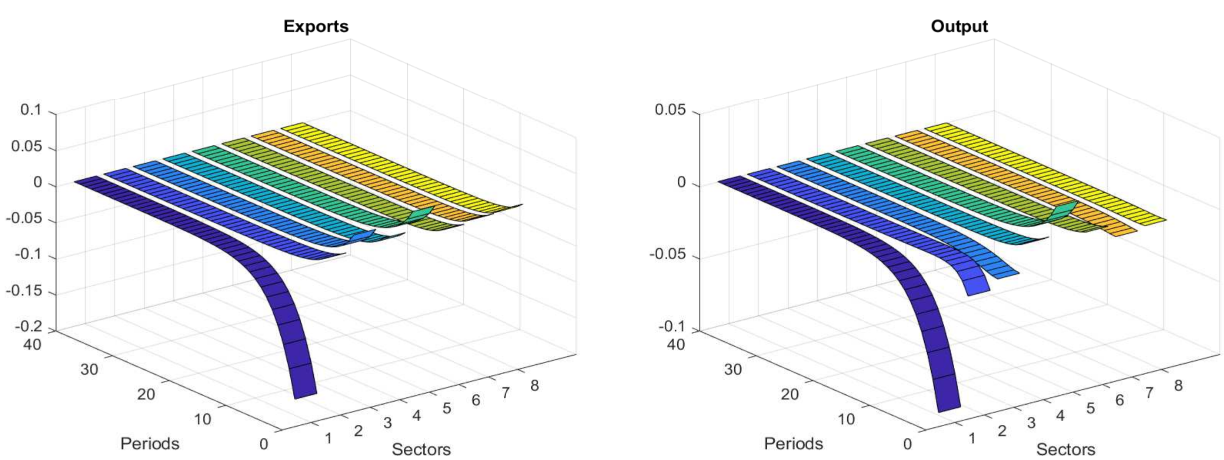
The general equilibrium effect on the real exchange rate and interest rates feeds back to consumption, investment, and production. Real depreciation leads to increases in exports in all sectors other than the commodity sector, and part of the impact is offset by increased domestic output prices. On net, exports in non-commodity sectors rise. Aggregate imports increase as commodity prices fall, resulting from two offsetting forces. On the one hand, depreciated domestic currency encourages non-commodity exports, raising demand for imported intermediate inputs and imported investment. On the other hand, depreciated currency raises the relative price of imported goods and tends to lower the demand for imported goods. The calibrated model suggests that the substitution effect dominates and imports in non-commodity investment and consumption goods all fall. The aggregate consumption increases though, as consumers spend less of their income on commodities and increase their demand for other goods and services.

There are some important issues to keep in mind when assessing the model's goodness of fit to the data. First, prices are flexible in the baseline setup. Second, we assume a standard Taylor rule to capture the monetary policy reaction to the global commodity price drop. In the real world, a commitment to a simple instrument rule may not be adequate as a description of forward-looking monetary policy where judgment and extra-model information are also used.[12] In summary, adding more features to the model could improve its performance in terms of reproducing the features of the data. Nevertheless, the current model does a reasonably good job in terms of both capturing important properties of the data and providing a simple framework for intuitive economic interpretation of the results.

5.4 Baseline impacts: sectoral aspects

Figure 6 illustrates the impulse responses of sector exports and production to the commodity price shock. Upon a negative commodity price shock, the home country's commodity goods become relatively more expensive, thus both the production and exports in the commodity sector (sector 1) drop significantly. A depreciated Canadian dollar makes other sectors' (particularly the manufacturing sectors) exports more attractive, thus increasing other sectors' exports.

Figure 6: Baseline Impacts of the Commodity Price Shock: Exports and Production



It should be noted that all sectors not only export to the rest of the world, but also import intermediate inputs for their own production. In particular, manufacturing sectors producing chemicals, plastics, and machinery and equipment have both significant exports (as a share of output) and important imports (as a share of intermediate inputs). Take

[12] Poloz, Stephen: Monetary Policy as Risk Management, December 12, 2013.

machinery and equipment manufacturers, for example; they accounted for 37% of Canadian exports over the period of 1981-2014; while 47% of intermediate inputs used by this sector were imported over the period of 2000-2014. The depreciation of the Canadian dollar driven by the declining commodity prices would make the imported inputs relatively more expensive compared with domestic inputs, further dampening domestic production.

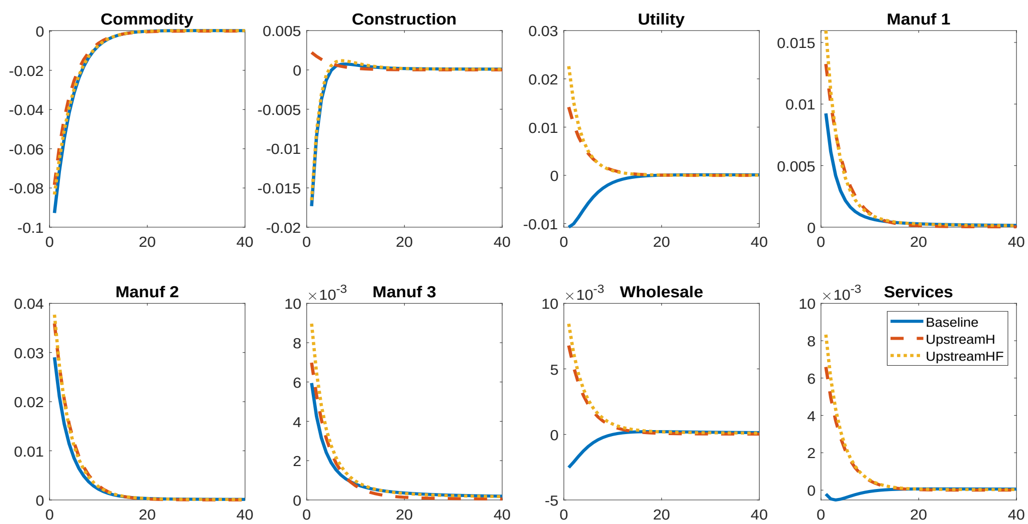
Through the production network, a shrinking commodity sector demands fewer intermediate inputs from other sectors (thus reducing other sectors' production). At the same time, a declining commodity price causes other sectors to use imported input less intensively. Demand for intermediate input from the other non-commodity sectors in the home country is likely to increase. This would increase domestic non-commodity sectors' production. The net impact, in the end, would depend on the magnitude of edges between nodes in the network. Figure 6 shows that sectoral production declines significantly in the commodity sector, somewhat in the construction and utility sector and slightly in the two service sectors. The three manufacturing sectors saw increases in production, thanks to improved exports as well as cheaper imported commodities.

Finally, our model predicts that labor inputs drop in all sectors in responding to the declining commodity prices, which is also consistent with the data, except for the construction sector.

5.5 Shock Propagation and Input-Output Linkages To disentangle the different forces behind adjustments in the multi-sector open economy, in this section, we run four counterfactual simulations to understand the scale of impacts of the following channels. We do this by modifying the input-output linkages concerning the commodity sector. First, we shut down the domestic upstream supply to the commodity sector from other sectors in the domestic economy and label this case "UpstreamH". In other words, we assume that all other sectors use intermediate inputs from the commodity sector for their own production, and that the commodity sector does not use domestic inputs for its production (it could, however, still import intermediate inputs). Secondly, in addition to the domestic upstream channel, we also shut down the foreign supply of intermediate inputs to the home commodity sector, and label this case "UpstreamHF". The upstream counterfactual analysis results are presented in Figures 7 and 8. Next, we turn to investigate the downstream impacts and shut down the supply of commodity inputs to the domestic economy. This case is labeled "DownstreamH". Finally, on top of ceasing the domestic downstream, we shut down the exports of commodity products from the home economy and label this scenario "DownstreamHX". The downstream counterfactual analysis results are presented in Figures 9 and 10.

Shutting down domestic upstream channels means that other sectors no longer supply to the commodity sector. Rather, the commodity sector needs to import these intermediate inputs from abroad. A drop in commodity prices is still negative on commodity sector

Figure 7: Counterfactual Analysis (Upstream): Sectoral Production

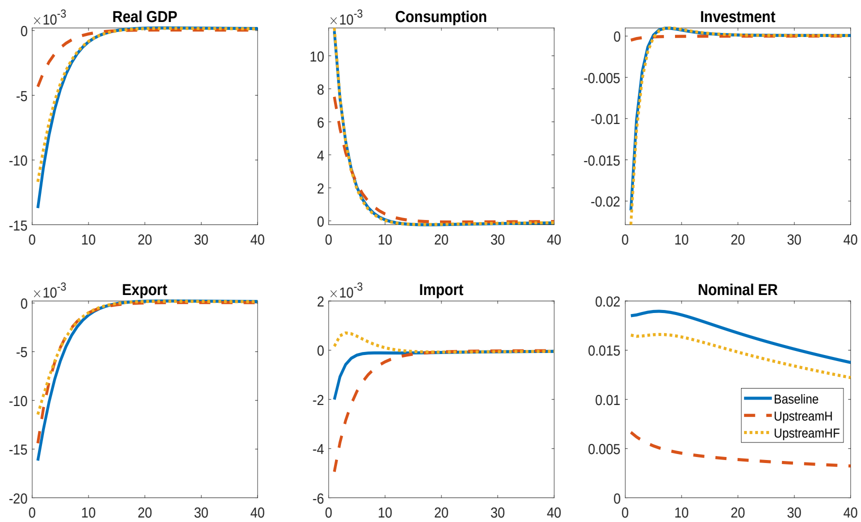


production and exports. On one hand, demand for other sectors' output would be reduced, thus other sectors' output prices are lower. On the other hand, within the non-commodity sectors, demand for each other's products would increase due to the lower prices. A drop in commodity prices leads to a depreciated CAD, further adding to the switch away from imported inputs to domestically produced inputs. In the end, shutting down the domestic upstream supplies drives the sectoral production to be higher than the baseline case for all non-commodity sectors. In terms of the overall impact on GDP, the more softened import profile and the improved investment prospects appear to drive a somewhat improved (though still negative) response in GDP upon the shock compared with the baseline case.

Further, if the commodity sector cannot import intermediate inputs, it can use only capital and labor for production. A negative commodity price shock would have less negative impact on some non-commodity sector production because there are no longer negative upstream demand impacts. On the other hand, as return to capital further declined, the investment profile also saw a decline. Contributing to over 50% of investment, the construction sector also saw a decline in production. Overall, the net impact on GDP is very similar to what was observed in the baseline.

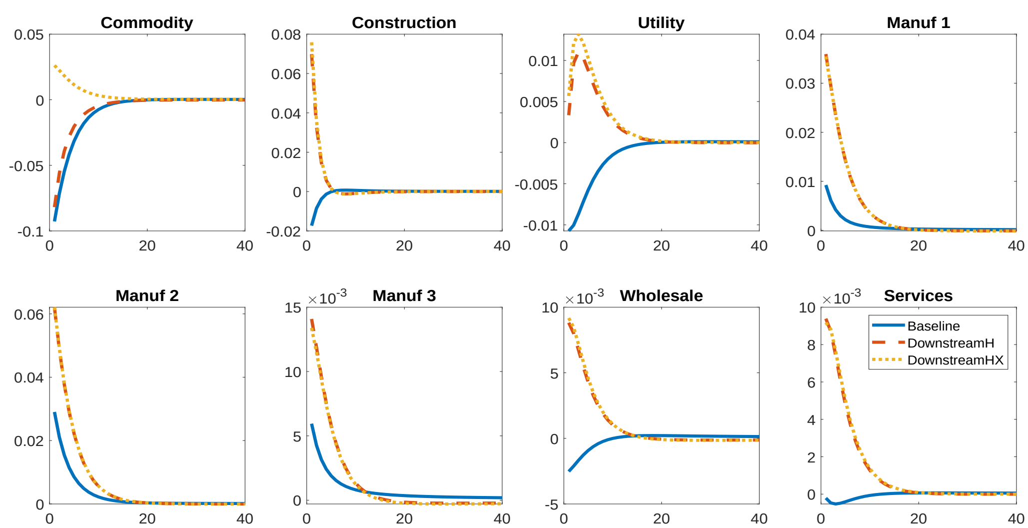
Turning to the downstream impacts of the open economy's production network, shutting down downstream channels means the domestic commodity sector can no longer supply to the rest of the economy. The intensity parameters do not change though, so other sectors import commodity inputs instead. Ceasing the domestic downstream propagation channel causes sectoral production to increase from the baseline benchmark. In most cases, the increase

Figure 8: Counterfactual Analysis (Upstream): Macro Variables



is large such that the impulse responses to commodity price shocks turn from negative to positive. The only exception would be the commodity sector, as a drop in commodity prices is still negative on commodity sector production and exports.

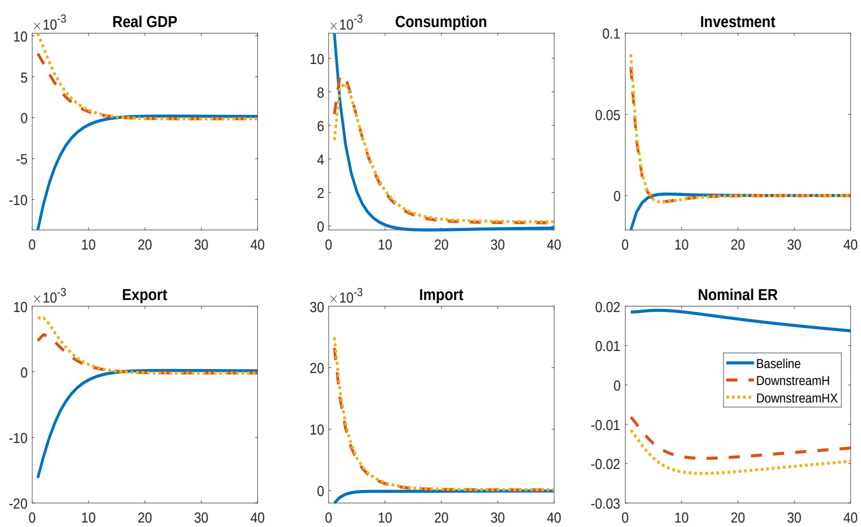
Figure 9: Counterfactual Analysis (Downstream): Sectoral Production



A declining commodity price means cheaper imported commodity inputs. Demand for intermediate inputs from the rest of the non-commodity sectors is likely to increase. This would increase non-commodity sectors' production. It follows that exports increase and demand for capital and labor increases. Although imports also increase in this scenario, the net impact on GDP is still positive. It is important to note that in the "DownstreamH" scenario, where the negative impacts on the commodity sector persist but its downward linkages with the rest of the economy are assumed away, the baseline impacts change signs in many places. In other words, it is important to note that downstream linkages play an important role in driving the baseline results. This is consistent with mining and quarrying ranked among the top in terms of out-degree measure, which counts the number of ties that a node directs to others. Without these downstream linkages in a hypothetical exercise, we won't observe what we have seen in reality.

If, on top of that, we further shut down the export path for commodity inputs so the commodity sector is not allowed to export, the negative impact on commodity exports is forced to be zero. The commodity price shock leads to even a small positive response of production in the domestic commodity sector. The rest of the mechanisms work the same way, leading to a slightly even more positive impact on investment, exports and real GDP.

Figure 10: Counterfactual Analysis (Downstream): Macro Variables



In the current model setup, we do not build in any frictions to the adjustment process and allow all potential channels to adjust simultaneously. But it would be reasonable to assume that, upon the shock, the domestic production network impacts would happen faster while the trade part of the network takes time to adjust due to factors such as the delayed impacts of monetary policy. Under this assumption, it is not hard to understand why in 2014, when the commodity industry in Canada took a hit, the export-oriented manufacturing industries did not take over the engine of growth immediately. This is because the impacts coming from the domestic input-output linkages came before the open economy network adjustments.

5.6 Sticky prices In this section, we show that nominal price rigidity at the sector level spills over to other sectors through input-output linkages. A small degree of price rigidity can have a large impact, particularly on sectors that strongly link with others where prices are sticky. Sectoral-good prices set in both the domestic market and the foreign market are assumed to be sticky. As in Pasten et al. (2017), we assume Calvo pricing, the probability that sectoral-good firms change prices in each period is $1-\mu_j$, the degree of price indexation is τ_j, and that both are sector-specific. Consider a producer s in sector j who is randomly selected to set new prices at time t. Let $\overline{P}_{hj,t}(s)$ and $\overline{P}^*_{hj,t}(s)$ denote the prices chosen by the firm in the home and foreign markets, respectively. If the price is still in effect at time $t+k$, then the firm's sales in the domestic market and the foreign market, respectively, are given by:

$$D_{hj,t+k}(s) = D_{hj,t+k}\left(\frac{\overline{P}_{hj,t}(s)(P_{cj,t+k-1}/P_{cj,t-1})^{\tau_j}}{P_{hj,t+k}}\right)^{-\varepsilon} \tag{5.1}$$

$$X_{j,t+k}(s) = X_{j,t+k}\left(\frac{\overline{P}^*_{hj,t}(s)(P_{cj,t+k-1}/P_{cj,t-1})^{\tau_j}}{S_{t+k}P^*_{hj,t+k}}\right)^{-\varepsilon}. \tag{5.2}$$

Since the probability that $\overline{P}_{hj,t}(s)$ and $\overline{P}^*_{hj,t}(s)$ are still in effect at date $t+k$ is μ_j^k, the firm chooses $\overline{P}_{hj,t}(s)$ and $\overline{P}^*_{hj,t}(s)$ to maximize the present discounted value of profits:

$$E_t\Sigma_{k=0}^{\infty}\mu_j^k\Gamma_{t,t+k}\Bigg\{\overline{P}_{hj,t}(s)\left(\frac{P_{cj,t+k-1}}{P_{cj,t-1}}\right)^{\tau_j}D_{hj,t+k}(s)+\overline{P}^*_{hj,t}(s)\left(\frac{P_{cj,t+k-1}}{P_{cj,t-1}}\right)^{\tau_j}X_{j,t+k}(s)$$
$$-MC_{t+k}\big[D_{hj,t+k}(s)+X_{j,t+k}(s)\big]\Bigg\},$$

where MC_{t+k} is the nominal marginal cost. $\Gamma_{t,t+k}$ is the stochastic discount factor that is expressed in units of the consumption good

$$\Gamma_{t,t+k} = \beta^k\frac{U_{c,t+k}/P_{c,t+k}}{U_{c,t}/P_{ct}}.$$

Substitute (5.1) and (5.2) into the profit function and obtain the first-order conditions. The solution to this problem is:

$$\overline{P}_{hj,t}(s) = \frac{E_t\Sigma_{k=0}^{\infty}\mu_j^k\Gamma_{t,t+k}\varepsilon P^{\varepsilon}_{hj,t+k}Y_{hj,t+k}MC_{t+k}(P_{cj,t+k-1}/P_{cj,t-1})^{-\tau_j\varepsilon}}{E_t\Sigma_{k=0}^{\infty}\mu_j^k\Gamma_{t,t+k}(\varepsilon-1)P^{\varepsilon}_{hj,t+k}Y_{hj,t+k}(P_{cj,t+k-1}/P_{cj,t-1})^{-\tau_j(\varepsilon-1)}} = \overline{P}_{hj,t}$$

$$\overline{P}^*_{hj,t}(s) = \frac{E_t\Sigma_{k=0}^{\infty}\mu_j^k\Gamma_{t,t+k}\varepsilon(P^*_{hj,t+k}S_{t+k})^{\varepsilon}Y^*_{hj,t+k}MC_{t+k}(P_{cj,t+k-1}/P_{cj,t-1})^{-\tau_j\varepsilon}}{E_t\Sigma_{k=0}^{\infty}\mu_j^k\Gamma_{t,t+k}(\varepsilon-1)(P^*_{hj,t+k}S_{t+k})^{\varepsilon}Y^*_{hj,t+k}(P_{cj,t+k-1}/P_{cj,t-1})^{-\tau_j(\varepsilon-1)}} = \overline{P}^*_{hj,t}.$$

The optimal price choices for intermediate good producers are contingent only on aggregate prices and quantities, and thus are not dependent on the intermediate good variety s. The price index for intermediate goods sold domestically, $P_{hj,t}$, and the export price index, $P^*_{hj,t}$, can then be expressed as:

$$P_{hj,t} = \left\{ \mu_j \left[P_{hj,t-1} \left(\frac{P_{cj,t-1}}{P_{cj,t-2}} \right)^{\tau_j} \right]^{1-\varepsilon} + (1-\mu_j) \overline{P}_{hj,t}^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} \tag{5.3}$$

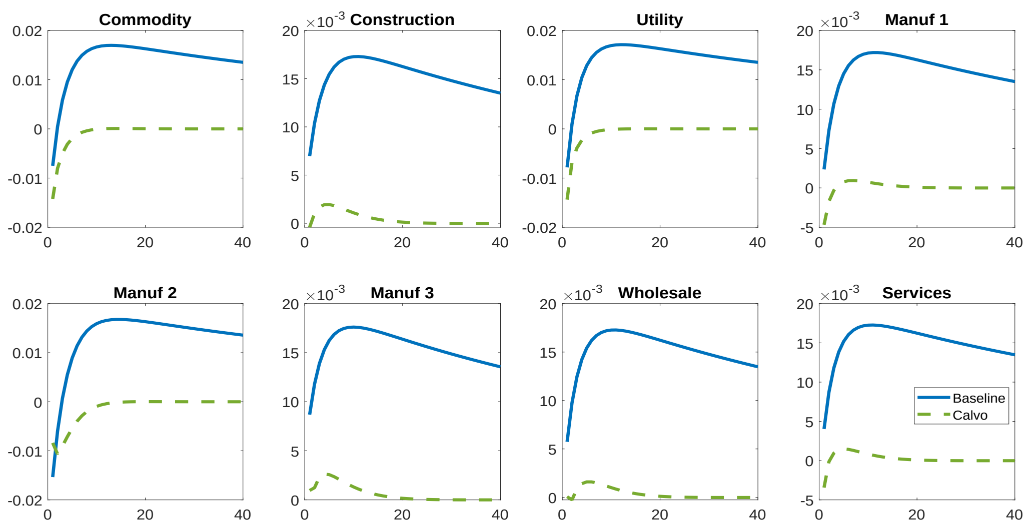
$$P^*_{hj,t} = \left\{ \mu_j \left[P^*_{hj,t-1} \left(\frac{P^*_{cj,t-1}}{P^*_{cj,t-2}} \right)^{\tau_j} \right]^{1-\varepsilon} + (1-\mu_j) \left(\frac{\overline{P}^*_{j,t}}{S_t} \right)^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}} . \tag{5.4}$$

Calibration of Calvo parameters While the facts about producer price setting at the disaggregated level have been documented for the U.S. (see, for example, Gopinath and Itskhoki (2010), Hafedh Bouakez and Ruge-Murcia (2014)), little work has been done for Canada. Cao et al. (2015) used unpublished monthly data from Statistics Canada's Price Report Survey (PRS) over the period of January 2006 to March 2010 and documented the frequency of price adjustment (in the currency of pricing) for the manufacturing sector and its sub-groups. The prices collected in the PRS are for goods sold at the factory gate and exclude all direct and indirect taxes (such as sales taxes and tariffs), as well as transportation and distribution costs. Mapping into sectors in our model, the mean monthly price change frequencies for the three Canadian manufacturing sectors (sectors 3, 4 and 5) are 25.4, 29.3 and 20.4%, respectively.

For the calibration on other sectors' price stickiness, we rely on the empirical evidence from the U.S. disaggregated data as proxies. Specifically, Hafedh Bouakez and Ruge-Murcia (2014) estimated a multi-sector model and reported frequencies of price changes for 30 sectors. They found that sectors such as agriculture, oil and gas extraction, fabricated metal, some machinery and instruments, as well as FIRE are flexible price sectors, while sectors such as construction, apparel, transport and utilities and TRADE have implied price duration for over a year. These macro estimates of price rigidities are consistent with those computed using micro data by Bils and Klenow (2004) and Nakamura and Steinsson (2008). Mapping these macro estimates with the rest of the sectors in our model, we assign Calvo parameters of 0 for the commodity sector, 0.5 for construction, 0.4 for utility, 0.63 for wholesale and finally 0 for other services to match the implied price durations, respectively.

Implications of price rigidity Figure 11 plots the impulse responses of sectoral prices upon a one-time negative shock to the foreign commodity price, in the baseline case as well as under the sticky price setup. In response to a negative foreign commodity price shock, demand for domestic commodity goods drops, substituted by intermediate inputs supplied by other sectors and imported commodity inputs. Without price stickiness, domestic prices for the other sectors would increase. When prices are rigid, however, they tend not to increase as much. Interestingly, even though only the construction sector, the utility sector and the wholesale sector have embedded price stickiness while the rest of the sectors can adjust their prices in a flexible manner, it is optimal for producers in all non-commodity sectors to set a lower price compared with the baseline case. The impact of a commodity price shock in the presence of price rigidities may be related to the importance of the sectors displaying price rigidity in the production network. In the calibrated steady-state model, the wholesale sector ranks the second largest among all eight sectors. The Domar weight of the wholesale sector is 0.3. The construction sector has a Domar weight of 0.2. The impact from the utility sector into the production network is likely to be small, as its Domar weight is only 0.02.

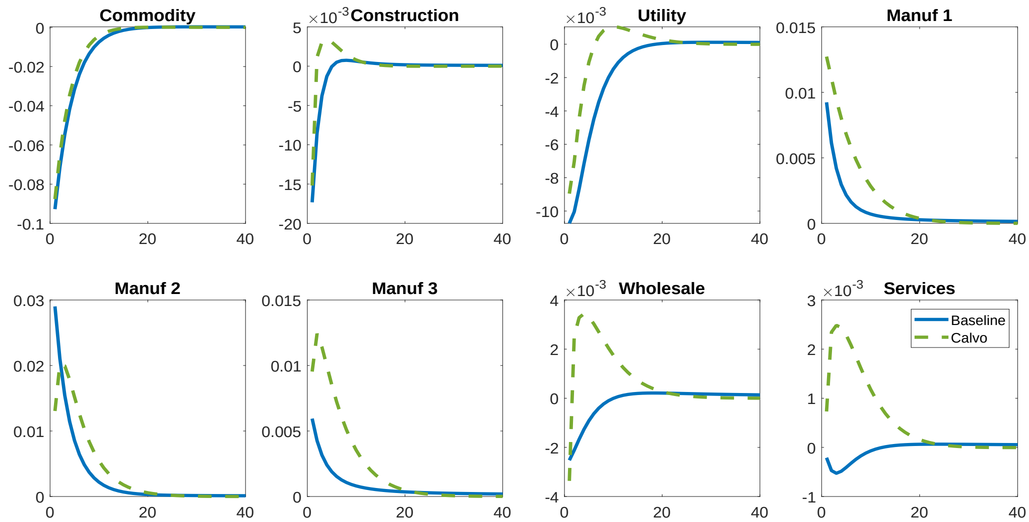
Figure 11: Sticky Prices: Sectoral Prices



With domestic prices at a lower level compared with the base case, production in non-commodity sectors is generally higher in the sticky price case (Figure 12). As more production implies more demand for intermediate inputs, the positive feedback impacts reinforce the positive non-commodity sectoral output responses. Not surprisingly, the feedback impacts are particularly strong for the two service sectors, as they are major intermediate inputs into all the other sectors and not many services are imported.

Turning to the macro and open economy aspects of the small country, Figure 13 plots the impulse responses of the macro aggregates. When prices are sticky, the nominal exchange rate rises less than the base case, which suggests less depreciation of domestic currency. Following the commodity price shock, the real exchange rate rises 1.2% in the baseline model and slightly less than 1.0% under the price rigidity scenario. When prices are rigid,

Figure 12: Sticky Prices: Sectoral Production



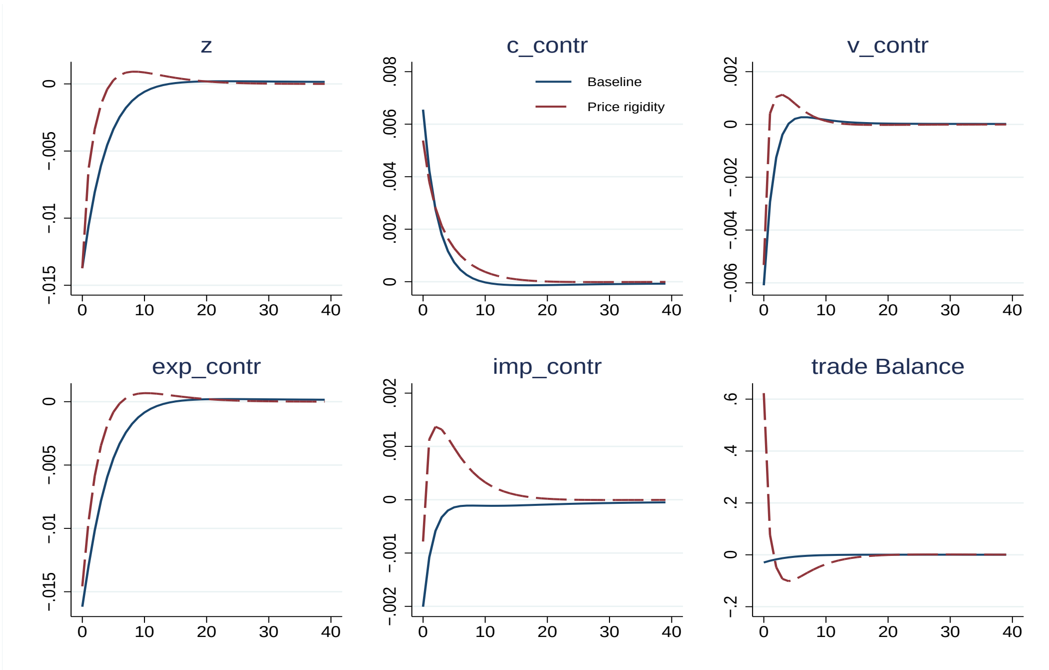
the responses of domestic prices to the commodity price shock are smaller, thus inflation is lower, leading to a smaller change in the domestic nominal interest rate. This results in less currency depreciation. On the one hand, the less-depreciated domestic currency helps to amplify the already more positive response of imports. On the other hand, it would dampen the positive response of exports that is demand driven. Since the scale of the export response is much larger than that of the import response, the net impact on GDP is still less negative than the baseline case.

The role of frictions (price stickiness in particular) has been at the core of the discussions on the driver of aggregate fluctuations. More recently, Pasten et al. (2017) studied how heterogeneity in price stickiness by itself can have an impact on GDP volatility by distorting the "granular" effect of large sectors and the "network" effect of central sectors on aggregate volatility. Despite a different focus, our findings are similar to those of Pasten et al. (2017): that heterogeneity in sectoral price stickiness have macro impacts through the production network linkages.

6 Conclusions

A commodity price shock can go a long way in a commodity-exporting small open economy. Complex adjustments to commodity price changes take place not only through resource reallocation, currency depreciation/appreciation and monetary policy response, but also through the production network. In this paper, we account for the production linkages both domestically and with the rest of the world and explore the role of upstream and downstream

Figure 13: Sticky Prices: Macro Variables



linkages in driving the adjustments. We find that domestic downstream linkages and the export connection with the rest of the world are important in adjusting to a commodity price shock. The upstream and import connections also play a role, but to a lesser degree.

Our paper contributes to the stream of research that analyzes sector-specific versus aggregate sources of variations in the business cycle. Aggregate fluctuations can come from macro shocks (aggregate shocks that are common across sectors), the granular effect of large sectors and the network effect of central sectors. The commodity price shock plays the role of all three in the above in explaining the variability in the aggregate. Our paper provides a structural framework to identify the importance of these transmission channels.

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A Optimal conditions

1.1 Household Let λ_t be the Lagrangian multiplier for the budget constraint; the household's optimal conditions are given by

$$\begin{aligned}
C_t &: \quad P_{ct}C_t = \frac{w_t}{\xi} = \frac{1}{\lambda_t};\\
K_{t+1} &: \quad \lambda_t P_{v,t} = \beta \mathbf{E}_t\big\{\lambda_{t+1}P_{vt+1}\big[r_{kt+1}+1-\delta\big\};\\
B_t &: \quad \frac{1}{\beta R_t} = \mathbf{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\right];\\
B_t^* &: \quad \frac{1}{\beta R_t^*} = \mathbf{E}_t\frac{S_{t+1}}{S_t}\cdot\frac{\lambda_{t+1}}{\lambda_t}.
\end{aligned}$$

The first two Euler equations also imply that $R_t = \mathbf{E}_t\frac{P_{vt+1}}{P_{vt}}(r_{kt+1}+1-\delta)$. The last two Euler equations imply that $\frac{R_t}{R_t^*} = E_t\frac{S_{t+1}}{S_t}$. The budget constraint is given by

$$P_{ct}C_t + P_{vt}V_t + \frac{B_t}{R_t} + \frac{S_tB_t^*}{R_t^*} = w_tL_t + r_{kt}P_{vt}K_t + S_tB_{t-1}^* + B_{t-1} + T_t + \Pi_t.$$

The optimal aggregation consumption suggests that the optimal consumption goods are given by

$$C_{hjt} = \phi_{hj}\left(\frac{P_{hjt}}{P_{ct}}\right)^{-\sigma_c} C_t, \quad j = 1,\cdots,N; \tag{A.1}$$

and

$$C_{fjt} = \phi_{fj}\left(\frac{P_{fjt}}{P_{ct}}\right)^{-\sigma_c} C_t, \quad j = 1,\cdots,N. \tag{A.2}$$

Optimal aggregation also gives the aggregate consumption price index as follows:

$$P_{ct} = \left[\sum_{j=1}^{N}\left(\phi_{hj}P_{hjt}^{1-\sigma_c} + \phi_{fj}P_{fjt}^{1-\sigma_c}\right)\right]^{\frac{1}{1-\sigma_c}}.$$

This price index can be obtained from the dual cost minimization problem, as the shadow price of aggregate consumption. Cost minimization also shows that profit is zero if sector j takes P_{hj} as given. Other price indexes below are obtained in the same way.

1.2 Producers The state vector of sector j is (A_{jt}, L_{jt-1}). Producers in sector j solve the following problem:

$$\max_{\{K_{jt},L_{jt},M_{jt}\}} E_0\sum_{t=0}^{\infty}\beta^t\lambda_t\big\{P_{hjt}Q_{jt} - r_{kt}P_{vt}K_{jt} - w_tL_{jt} - P_{mjt}M_{jt}\big\}.$$

Here, $\beta^t\lambda_t$ is the stochastic discount factor, and $\lambda_t = (P_{ct}C_t)^{-1}$ is technically the Lagrange multiplier for the household's budget constraint.

Define $Y_{jt} = \left(\frac{K_{jt}}{\alpha_{kj}}\right)^{\alpha_{kj}} \left(\frac{L_{jt}}{\alpha_{lj}}\right)^{\alpha_{lj}}$, and let P_{yj} be the price of capital-labor combination. The producer's first-order necessary conditions are given by

$$\begin{aligned}
K_{jt} &: \quad r_{kt}P_{vt}K_{jt} = \alpha_{kj}(1-\psi_j)^{\frac{1}{\sigma_q}} Y_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}. \\
M_{jt} &: \quad P_{mjt}M_{jt} = \psi_j^{\frac{1}{\sigma_q}} M_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}; \\
L_{jt} &: \quad P_{hjt}\frac{\partial Q_{jt}}{\partial L_{jt}} = w_t.
\end{aligned}$$

The optimal condition with respect to labor input can be re-written as $w_{jt} = P_{hjt}\frac{\partial Q_{jt}}{\partial L_{jt}}$, or

$$w_{jt}L_{jt} = \alpha_{lj}(1-\psi_j)^{\frac{1}{\sigma_q}} Y_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}.$$

Summing optimal conditions with respect to K_{jt} and L_{jt}, we obtain

$$r_{kt}P_{vt}K_{jt} + w_{jt}L_{jt} = (1-\psi_j)^{\frac{1}{\sigma_q}} Y_{jt}^{\frac{\sigma_q-1}{\sigma_q}} A_{jt}^{\frac{\sigma_q-1}{\sigma_q}} P_{hj,t} Q_{jt}^{\frac{1}{\sigma_q}}.$$

The right-hand side equals $P_{hjt}\frac{\partial Q_{jt}}{\partial Y_{jt}}Y_{jt}$. By the optimal condition with respect to Y_{jt}, we have $P_{yjt} = P_{hjt}\frac{\partial Q_{jt}}{\partial Y_{jt}}$, we then have

$$r_{kt}P_{vt}K_{jt} + w_{jt}L_{jt} = P_{yjt}Y_{jt}.$$

Using the optimal relation, $w_{jt}L_{jt} = \frac{\alpha_{lj}}{\alpha_{kj}} \cdot r_{kt}P_{vt}K_{jt}$, we obtain $P_{yjt} = (r_tP_{vt})^{\alpha_{kj}} (w_{jt})^{\alpha_{lj}}$, where w_{jt} is the marginal cost of labor input in sector j.

The optimal factor inputs satisfy

$$\begin{aligned}
Y_{jt} &= (1-\psi_j)A_{jt}^{\sigma_q-1}\left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
K_{jt} &= \alpha_{kj}(1-\psi_j)A_{jt}^{\sigma_q-1}\left(\frac{r_{kt}P_{vt}}{P_{yjt}}\right)^{-1}\left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
L_{jt} &= \alpha_{lj}(1-\psi_j)A_{jt}^{\sigma_q-1}\left(\frac{w_{jt}}{P_{yjt}}\right)^{-1}\left(\frac{P_{yjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}; \\
M_{jt} &= \psi_j A_{jt}^{\sigma_q-1}\left(\frac{P_{mjt}}{P_{hjt}}\right)^{-\sigma_q} Q_{jt}.
\end{aligned}$$

The share of intermediate input in total production cost (or equivalently total revenue) is given by

$$\frac{P_{mjt}M_{jt}}{P_{hjt}Q_{jt}} = \psi_j \left(\frac{P_{mjt}}{P_{hjt}}\right)^{1-\sigma_q} A_{jt}^{\sigma_q - 1}.$$

The optimal capital-labor aggregate satisfies

$$\frac{P_{yjt}Y_{jt}}{P_{hjt}Q_{jt}} = (1-\psi_j) \left(\frac{P_{yjt}}{P_{hjt}}\right)^{1-\sigma_q} A_{jt}^{\sigma_q - 1}. \tag{A.3}$$

The optimal conditions with respect to capital and labor imply that

$$\frac{r_{kt}P_{vt}K_{jt}}{w_{jt}L_j} = \frac{\alpha_{kj}}{\alpha_{lj}}. \tag{A.4}$$

From the first-order conditions, we obtain the ratio of intermediate inputs over capital-labor bundle as

$$\frac{M_{jt}}{Y_{jt}} = \frac{\psi_j}{1-\psi_j} \left(\frac{P_{mjt}}{P_{yjt}}\right)^{-\sigma_q}.$$

For $\sigma_q \in (0,1)$, the above ratio decreases as P_{mj} rises. But the cost of intermediate inputs over the cost of capital-labor bundle increases as P_{mj} rises, suggesting that the share of intermediate inputs in total production cost rises with P_{mj}.

In a steady state, the zero-profit condition holds for sector j gross output production, and we obtain the price of gross output

$$P_{hjt} = A_{jt}^{-1} \left[(1-\psi_j)P_{yjt}^{1-\sigma_q} + \psi_j P_{mjt}^{1-\sigma_q}\right]^{\frac{1}{1-\sigma_q}}.$$

Off the steady state, the zero-profit condition does not necessarily hold, and price P_{hjt} then no longer has a closed form.

Profits in sector j are given by $\Pi_{jt} = P_{hjt}Q_{jt} - r_{kt}P_{vt}K_{jt} - w_t L_{jt} - P_{mt}M_{jt}$. In the steady state, profit is zero.

1.3 Intermediate inputs Using the properties of the CES aggregation, for example as in the case of consumption, we can obtain the optimal intermediate input produced by sector i and used in the production in sector j, as

$$M_{hijt} = \omega_{hij} \left(\frac{P_{hit}}{P_{mjt}}\right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N.$$

Similarly, the optimal imported intermediate input i for the production in sector j is

$$M_{fijt} = \omega_{fij} \left(\frac{P_{fit}}{P_{mjt}}\right)^{-\sigma_m} M_{jt}, \quad \text{for } i = 1, \cdots, N.$$

The price index of sector j intermediate input satisfies

$$P_{mjt} = \left[\sum_{i=1}^{N} \left(\omega_{hij} P_{hit}^{1-\sigma_m} + \omega_{fij} P_{fit}^{1-\sigma_m} \right) \right]^{\frac{1}{1-\sigma_m}}.$$

The share of M_{hij} in total intermediate input used in sector j production is given by

$$\frac{P_{hit} M_{hijt}}{P_{mjt} M_{jt}} = \omega_{hij} \left(\frac{P_{hit}}{P_{mjt}} \right)^{1-\sigma_m}, \quad \text{for } i = 1, \cdots, N.$$

Similarly, for sector-i type of imported intermediate input, we have

$$\frac{P_{fit} M_{fijt}}{P_{mjt} M_{jt}} = \omega_{fij} \left(\frac{P_{fit}}{P_{mjt}} \right)^{1-\sigma_m}, \quad \text{for } i = 1, \cdots, N.$$

The shares of these intermediate inputs in the gross output of sector j are respectively given by

$$\begin{aligned} \frac{P_{hit} M_{hijt}}{P_{hjt} Q_{jt}} &= \omega_{hij} \psi_j P_{hit}^{1-\sigma_m} P_{mjt}^{\sigma_m - \sigma_q} P_{hjt}^{\sigma_q - 1} A_{jt}^{\sigma_q - 1}, \quad \text{for } i = 1, \cdots, N; \\ \frac{P_{fit} M_{fijt}}{P_{hjt} Q_{jt}} &= \omega_{fij} \psi_j P_{fit}^{1-\sigma_m} P_{mjt}^{\sigma_m - \sigma_q} P_{hjt}^{\sigma_q - 1} A_{jt}^{\sigma_q - 1}, \quad \text{for } i = 1, \cdots, N. \end{aligned}$$

Importantly, the impact of import prices on the share of intermediate input in total production cost depends upon the elasticity of substitution between intermediate input and the capital-labor combination and that between imported and domestically produced intermediate inputs. A reduction in import price leads to a lower P_{mjt}, and the share of intermediate input could become higher or lower depending whether the elasticity of substitution is greater or less than 1, i.e. whether intermediate inputs and capital and labor are complements or substitutes.

1.4 Investment

In a similar way, the optimal investment choices are given by

$$\begin{aligned} V_{hjt} &= \theta_{hj} \left(\frac{P_{hjt}}{P_{vt}} \right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N; \\ V_{fjt} &= \theta_{fj} \left(\frac{P_{fjt}}{P_{vt}} \right)^{-\sigma_v} V_t, \quad \text{for } j = 1, \cdots, N. \end{aligned}$$

The investment price index is given by

$$P_{vt} = \left[\sum_{j=1}^{N} \left(\theta_{hj} P_{hjt}^{1-\sigma_v} + \theta_{fj} P_{fjt}^{1-\sigma_v} \right) \right]^{\frac{1}{1-\sigma_v}}.$$

B Steady State

In steady state, $V = \delta K$. Also it is straightforward that $r = \frac{1}{\beta} - 1 + \delta$. The system of optimal conditions is given by the following, for all $j = 1, \cdots, N$

$$\begin{aligned}
V: &\quad V = \delta K; \\
C: &\quad \frac{1}{C} = P_c; \\
L_j: &\quad P_{hj} \frac{\partial Q_j}{\partial L_j} = \xi; \\
K_j: &\quad P_{hj} \frac{\partial Q_j}{\partial K_j} = \big[\frac{1}{\beta} - 1 + \delta\big] P_v; \\
M_j: &\quad P_{hj} \frac{\partial Q_j}{\partial M_j} = P_{mj}; \\
C_{hj}: &\quad C_{hj} = \phi_{hj} \Big(\frac{P_{hj}}{P_c}\Big)^{-\sigma_c} C; \\
C_{fj}: &\quad C_{fj} = \phi_{fj} \Big(\frac{P_{fj}}{P_c}\Big)^{-\sigma_c} C; \\
V_{hj}: &\quad V_{hj} = \theta_{hj} \Big(\frac{P_{hj}}{P_v}\Big)^{-\sigma_v} V; \\
V_{fj}: &\quad V_{fj} = \theta_{fj} \Big(\frac{P_{fj}}{P_v}\Big)^{-\sigma_v} V; \\
M_{hij}: &\quad M_{hij} = \psi_j A_j^{\sigma_q - 1} \omega_{hij} \left(\frac{P_{hi}}{P_{mj}}\right)^{-\sigma_m} \left(\frac{P_{mj}}{P_{hj}}\right)^{-\sigma_q} Q_j, \ i = 1, \cdots, N; \\
M_{fij}: &\quad M_{fij} = \psi_j A_j^{\sigma_q - 1} \omega_{fij} \left(\frac{P_{fi}}{P_{mj}}\right)^{-\sigma_m} \left(\frac{P_{mj}}{P_{hj}}\right)^{-\sigma_q} Q_j, \ i = 1, \cdots, N; \\
P_{mj}: &\quad P_{mj} = \left[\sum_{i=1}^{N} \big(\omega_{hij} P_{hi}^{1-\sigma_m} + \omega_{fij} P_{fi}^{1-\sigma_m} \big) \right]^{\frac{1}{1-\sigma_m}}; \\
P_{yj}: &\quad P_{yj} = (r P_v)^{\alpha_{kj}} w^{\alpha_{lj}}; \\
P_{hj}: &\quad P_{hj} = A_j^{-1} \left[(1 - \psi_j) P_{yj}^{1-\sigma_q} + \psi_j P_{mj}^{1-\sigma_q} \right]^{\frac{1}{1-\sigma_q}};
\end{aligned}$$

In addition:

$$P_c: \quad P_c = \left[\sum_{j=1}^{N} \left(\phi_{hj} P_{hj}^{1-\sigma_c} + \phi_{fj} P_{fj}^{1-\sigma_c}\right)\right]^{\frac{1}{1-\sigma_c}};$$

$$P_v: \quad P_v = \left[\sum_{j=1}^{N} \left(\theta_{hj} P_{hj}^{1-\sigma_v} + \theta_{fj} P_{fj}^{1-\sigma_v}\right)\right]^{\frac{1}{1-\sigma_v}};$$

$$Q_j: \quad Q_j = C_{hj} + V_{hj} + X_j + M_{hj\cdot};$$

$$X_j: \quad X_j = \alpha_f \left(\frac{P_{hj}}{SP^*}\right)^{-\sigma_x} Y^*;$$

$$\text{A.R.C.}: \quad \sum_{j=1}^{N} P_{hj} Q_j - \sum_{j=1}^{N} P_{mj} M_j = P_c C + P_v V + \sum_{j=1}^{N} P_{xj} X_j - \sum_{j} P_{fj} \left(C_{fj} + V_{fj} + M_{fj\cdot}\right);$$

$$K: \quad K = \sum_{j=1}^{N} K_j;$$

$$L: \quad L = \sum_{j=1}^{N} L_j.$$

First, we solve for $\mathsf{P_h} = [P_{hj}]_{N\times 1}$. Noticing that $P_{yj} = r^{\alpha_{kj}} w^{\alpha_{lj}} P_v^{\alpha_{kj}}$, then

$$P_{yj} = r^{\alpha_{kj}} w^{\alpha_{lj}} \left[\sum_{i=1}^{N} \left(\theta_{hi} P_{hi}^{1-\sigma_v} + \theta_{fi} P_{fi}^{1-\sigma_v}\right)\right]^{\frac{\alpha_{kj}}{1-\sigma_v}}.$$

Substituting for P_{mj} and P_{yj} in the equation for P_{hj}, we obtain

$$\begin{aligned}(A_j P_{hj})^{1-\sigma_q} &= (1-\psi_j)(r^{\alpha_{kj}} w^{\alpha_{lj}})^{1-\sigma_q} \left[\sum_{i=1}^{N} \left(\theta_{hi} P_{hi}^{1-\sigma_v} + \theta_{fi} P_{fi}^{1-\sigma_v}\right)\right]^{\frac{\alpha_{kj}(1-\sigma_q)}{1-\sigma_v}} \\ &+ \psi_j \left[\sum_{i=1}^{N} \left(\omega_{hij} P_{hi}^{1-\sigma_m} + \omega_{fij} P_{fi}^{1-\sigma_m}\right)\right]^{\frac{1-\sigma_q}{1-\sigma_m}}.\end{aligned}$$

Let $\alpha_{\mathsf{k}} = [\alpha_{kj}]_{N\times 1}$ be a column vector, similarly for $\alpha_{\mathsf{l}}, \theta_{\mathsf{h}}, \theta_{\mathsf{f}}$, etc. We define the square diagonal matrix $\mathrm{di}(\tilde{\mathsf{r}})$ with $r^{\alpha_{kj}} w^{\alpha_{lj}}$ along the diagonal. The above system of equations, in vector form, is given by

$$[\mathrm{di}(\mathsf{A})\mathsf{P_h}]^{1-\sigma_q} = \mathrm{di}(\mathbf{1}-\psi)\left[[\mathrm{di}(\tilde{\mathsf{r}})]^{1-\sigma_q}\left[\theta_{\mathsf{h}}'\mathsf{P_h}^{1-\sigma_v} + \theta_{\mathsf{f}}'\mathsf{P_f}^{1-\sigma_v}\right]^{\alpha_{\mathsf{k}}\frac{1-\sigma_q}{1-\sigma_v}}\right] + \mathrm{di}(\psi)\left[\omega_{\mathbf{h}}'\mathsf{P_h}^{1-\sigma_m} + \omega_{\mathbf{f}}'\mathsf{P_f}^{1-\sigma_m}\right]^{\frac{1-\sigma_q}{1-\sigma_m}}. \tag{B.1}$$

Here, exponents are element-wise. The above vector equation has N unknowns; they cannot be solved with closed forms. We numerically solve for $\mathsf{P_h}$, which are functions of sector productivity and import prices. Note that, in a closed economy ($\theta_{fj} = \omega_{fij} = 0$), there is a closed form for the steady-state prices. Prices of the capital-labor bundle and intermediate inputs can also be expressed with vectors, as follows:

$$\mathsf{P_y} = \mathrm{di}(\tilde{\mathsf{r}}) \left[\theta_{\mathsf{h}}' \mathsf{P_h}^{1-\sigma_v} + \theta_{\mathsf{f}}' \mathsf{P_f}^{1-\sigma_v} \right]^{\frac{\alpha_{\mathsf{k}}}{1-\sigma_v}}.$$

The intermediate input prices in vector form are given by

$$\mathsf{P_m}^{1-\sigma_m} = \omega_{\mathbf{h}}' \mathsf{P_h}^{1-\sigma_m} + \omega_{\mathbf{f}}' \mathsf{P_f}^{1-\sigma_m}.$$

It is noted that $(\theta_{\mathsf{h}}' \mathsf{P_h}^{1-\sigma_v} + \theta_{\mathsf{f}}' \mathsf{P_f}^{1-\sigma_v})$ is a scalar, while its exponent is a vector because α_{k} is a vector.

Given $\mathsf{P_h}$, we obtain the Lagrangian multiplier (marginal value of consumption) P_c and investment price

$$P_c = \left[\phi_{\mathsf{h}}' \mathsf{P_h}^{1-\sigma_c} + \phi_{\mathsf{f}}' \mathsf{P_f}^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}};$$
$$P_v = \left[\theta_{\mathsf{h}}' \mathsf{P_h}^{1-\sigma_v} + \theta_{\mathsf{f}}' \mathsf{P_f}^{1-\sigma_v} \right]^{\frac{1}{1-\sigma_v}}.$$

Other prices, $\mathsf{P_m}$ and $\mathsf{P_y}$ are also obtained from their expressions above.

Second, we solve for sector-level gross output Q_j by using the market clearing condition. Note that $C = 1/P_c$. Consumption C_{hj} is given by

$$C_{hj} = \phi_{hj} \left(\frac{P_{hj}}{P_c} \right)^{-\sigma_c} P_c^{-1}. \tag{B.2}$$

Share of sector j domestic consumption in aggregate consumption is $s_{hcj} = \frac{P_{hj} C_{hj}}{P_c C} = \phi_{hj} \left(\frac{P_{hj}}{P_c} \right)^{1-\sigma_c}$.

Investment V_{hj} is a share of $V = \delta K$, and $K = \sum_{i=1}^{N} K_i$. Then

$$V_{hj} = \theta_{hj} \left(\frac{P_{hj}}{P_v} \right)^{-\sigma_v} \cdot \delta \sum_{i=1}^{N} \left[\alpha_{ki} (1-\psi_i) A_i^{\sigma_q - 1} \left(\frac{r P_v}{P_{yi}} \right)^{-1} \left(\frac{P_{yi}}{P_{hi}} \right)^{-\sigma_q} Q_i \right]. \tag{B.3}$$

Share of sector j domestic investment in aggregate investment is $s_{hvj} = \frac{P_{hj} V_{hj}}{P_v V} = \theta_{hj} \left(\frac{P_{hj}}{P_v} \right)^{1-\sigma_v}$. Share of the capital-bundle in total cost is $s_{yj} = \frac{P_{yjt} Y_{jt}}{P_{hjt} Q_{jt}} = (1-\psi_j) \left(\frac{P_{yjt}}{P_{hjt}} \right)^{1-\sigma_q} A_{jt}^{\sigma_q - 1}$. The share of capital service in total cost is $\alpha_{kj} s_{yj}$. Domestic investment good produced in sector

j is then given by

$$P_{hj}V_{hj} = s_{hvj}\delta P_v \sum_{i=1}^{N} K_i = s_{hvj}(\delta/r_k) \sum_{i=1}^{N} \alpha_{ki} s_{yi} P_{hi} Q_i.$$

Intermediate input $M_{hj\cdot}$ produced in sector j is given by

$$M_{hj\cdot} = \sum_{i=1}^{N} M_{hji} = \sum_{i=1}^{N} \left[\psi_i A_i^{\sigma_q - 1} \omega_{hji} \left(\frac{P_{hj}}{P_{mi}} \right)^{-\sigma_m} \left(\frac{P_{mi}}{P_{hi}} \right)^{-\sigma_q} Q_i \right]. \tag{B.4}$$

The sector j output used as an intermediate input for sector i production, as a share of sector i' total intermediate input, is

$$s_{hmji} = \frac{P_{hj}M_{hji}}{P_{mi}M_i} = \omega_{hji} \left(\frac{P_{hj}}{P_{mi}} \right)^{1-\sigma_m},$$

and the share of intermediate inputs in total production cost in sector i is

$$s_{mi} = \frac{P_{mi}M_i}{P_{hi}Q_i} = \psi_i \left(\frac{P_{mi}}{P_{hi}} \right)^{1-\sigma_q} A_i^{\sigma_q - 1}.$$

Then, the share of m_{hji} in sector i's total production cost is $s_{hmji} = s_{hmji} \cdot s_{mi}$.

Export by sector j is given by

$$X_j = \alpha_x \left(\frac{P_{hj}}{SP^*} \right)^{-\sigma_x} Y^*. \tag{B.5}$$

Substituting equations (B.2) to (B.5) for $C_{hj}, V_{hj}, M_{hj\cdot}$ and X_j in the market clearing condition for sector j, we obtain

$$\begin{aligned} Q_j = \phi_{hj} \left(\frac{P_{hj}}{P_c} \right)^{-\sigma_c} P_c^{-1} + \theta_{hj} \left(\frac{P_{hj}}{P_v} \right)^{-\sigma_v} \cdot \delta \sum_{i=1}^{N} \left[\alpha_{ki}(1-\psi_i) A_i^{\sigma_q - 1} \left(\frac{rP_v}{P_{yi}} \right)^{-1} \left(\frac{P_{yi}}{P_{hi}} \right)^{-\sigma_q} Q_i \right] \\ + \sum_{i=1}^{N} \left[\psi_i A_i^{\sigma_q - 1} \omega_{hji} \left(\frac{P_{hj}}{P_{mi}} \right)^{-\sigma_m} \left(\frac{P_{mi}}{P_{hi}} \right)^{-\sigma_q} Q_i \right] + \alpha_f \left(\frac{P_{hj}}{SP^*} \right)^{-\sigma_x} Y^*. \end{aligned} \tag{B.6}$$

In terms of shares, the market clearing condition for sector j output in current price is then

$$P_{hj}Q_j = s_{hcj}P_cC + P_{hj}x_j + s_{hvj}(\delta/r_k) \sum_{i=1}^{N} \alpha_{ki} s_{yi} P_{hi} Q_i + \sum_{i=1}^{N} s_{hmji} P_{hi} Q_i. \tag{B.7}$$

Let $\mathsf{C_h}$ be a column vector of C_{hj} and let X be a column vector of exports. Define the following column vectors and matrices

$$\Omega_{\mathsf{ch}} = \left[s_{hcj}\right]_{N\times 1};$$
$$\Omega_{\mathsf{vh}} = \left[s_{hvj}\right]_{N\times 1};$$
$$\Omega_{\mathsf{k}} = [\alpha_{ki}s_{yi}]_{N\times 1};$$
$$\mathbf{\Omega_{mh}} = \left[s_{hmji}\right]_{j,i=1,\cdots,N}.$$

Row j in $\mathbf{\Omega_{mh}}$ are amounts of output by sector j supplied to all sectors including sector j itself, as a proportion of gross output in the use sector. Equation (B.7) in vector form is given by

$$\mathsf{P_h}\cdot\mathsf{Q} = (\delta/r_k)\Omega_{\mathsf{vh}}\cdot(\Omega_{\mathsf{k}}'(\mathsf{P_h}\cdot\mathsf{Q})) + \mathbf{\Omega_{mh}}(\mathsf{P_h}\cdot\mathsf{Q}) + \Omega_{\mathsf{ch}} + \mathsf{X}, \tag{B.8}$$

where the dot represents element-wise multiplication. In the equation, we note that $\Omega_{\mathsf{k}}'\mathsf{Q}$ is a scalar, representing the amount of aggregate capital. Also, $P_cC = 1$. Then

$$\mathsf{P_h}\cdot\mathsf{Q} = \left[\mathbf{I} - (\delta/r_k)\Omega_{\mathsf{vh}}\Omega_{\mathsf{k}}' - \mathbf{\Omega_{mh}}\right]^{-1}[\Omega_{\mathsf{ch}} + \mathsf{X}]. \tag{B.9}$$

This system of N equations consists of prices, productivity, as well as elasticity of substitution. It can be solved for gross outputs.

We can obtain shares of all kinds, as well as relative prices, given that we have solved for $\mathsf{P_h}$ and Q.

We obtain the Leontief inverse matrix. Let the final demand be $\mathsf{D_h} = [d_{hj}]_{N\times 1}$ with $d_{hj} = c_{hj} + v_{hj} + x_j$. Then

$$\mathsf{P_h}\cdot\mathsf{Q} = \mathbf{\Omega_{mh}}(\mathsf{P_h}\cdot\mathsf{Q}) + \mathsf{P_h}\cdot\mathsf{D_h},$$

or

$$\mathsf{P_h}\cdot\mathsf{Q} = \left[\mathbf{I} - \mathbf{\Omega_{mh}}\right]^{-1}(\mathsf{P_h}\cdot\mathsf{D_h}).$$

Matrix $\left[\mathbf{I} - \mathbf{\Omega_{mh}}\right]^{-1}$ is the Leontief inverse.