Alternative Sources of the Lag Dynamics of Inflation

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Introduction

New Keynesian Phillips curves are widely used in macroeconomic policy models to simulate the inflation consequences of alternative monetary policies. The purely forward-looking inflation specification is appealing, because it is based on a model of optimal pricing behaviour. However, based on empirical evidence, the standard view is that there is considerable persistence in inflation. Consequently, the purely forward-looking specification is controverisal, because it excludes lagged inflation terms and, contrary to empirical evidence, implies that inflation is not sticky. In fact, prior to the implementation of the forward-looking specifications, Phillips curves in policy models assumed that expectations were purely backward-looking. Although such specifications lacked the rational-expectations assumptions preferred when analyzing alternative policies, they captured the strong autocorrelations of actual inflation rates.

This paper discusses four potential sources of lag dynamics in inflation: non-rational behaviour, staggered contracting, frictions on price adjustment, and shifts in the long-run inflation anchor of agent expectations (the

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perceived inflation target). Many attempts to justify hybrid models with both backward- and forward-looking expectations assume non-rational behaviour of some sort. For example, Roberts (1997, 2001) and Ball (2000) assume that a fraction of agents use adaptive expectations; Galí and Gertler (1999) suggest that some firms use rule-of-thumb pricing; and Fuhrer and Moore (1995) use a real-wage-contracting specification where the price base of the real-wage comparison is not the average of prices expected over the life of the contract. Without relaxing the assumption of rational expectations, however, frictions on price adjustment can lead to a hybrid specification (Kozicki and Tinsley 2002).

The lag dynamics of inflation also may be influenced by shifts in the long-run anchor of agents' inflation expectations. Most policy models assume that the inflation target is known by all agents and that it doesn't change. However, learning about shifts in the policy target for inflation may be another source of persistence in the inflation process. Learning can significantly slow aggregate inflation adjustments, particularly after major changes in policy, as shown in Kozicki and Tinsley (2001a).

Section 1 reviews several models of inflation dynamics. The models include purely forward-looking specifications, as well as specifications that admit additional lags or leads of inflation. More complicated dynamic specifications are obtained with the introduction of non-rational agents, staggered contracting, or generalized frictions on price adjustment. Section 2 empirically examines the consistency of Canadian and U.S. inflation with the various sources of lag dynamics. Section 3 discusses monetary policy implications of the empirical results, and concluding comments are offered in the final section.

1 Sources of Lag Dynamics in Structural Models of Inflation

The benchmark for the discussion and analysis of the lag dynamics of inflation is the minimalist, purely forward-looking linear specification for inflation (π_t),

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t + u_t, \tag{1}$$

as derived in closed-economy models of Yun (1996), Woodford (1996), or King and Wolman (1999).[1] In this expression, y_t is the output gap, u_t is a shock, and $E_t \pi_{t+1}$ denotes the expectation of π_{t+1} conditional on

1. Erceg, Henderson, and Levin (2000) develop a parallel description of the evolution of wages.

information available in *t*. McCallum and Nelson (1999, 2000) show how this specification can also apply in open-economy models if imports are treated as raw-material inputs to the home country's productive process. In fact, McCallum and Nelson (2000) and Kara and Nelson (2002) argue that this open-economy treatment implies more realistic inflation dynamics than standard alternatives.

Although it is the basis for many empirical studies, this forward-looking specification is a linearization around a constant long-run anchor for inflation expectations assumed to equal zero, or, in mathematical terminology, around a steady-state inflation rate of zero. However, the assumption that the long-run anchor for inflation expectations is zero, as made in most structural macroeconomic policy models, is empirically unreasonable. Long-run inflation expectations should converge to the perceived inflation target of monetary policy, or the inflation target if it is known and credible, and these targets tend to incorporate small positive inflation rates. Section 1.1 presents an expression for inflation similar to equation (1) that explicitly accounts for a non-zero steady-state inflation rate.

The main criticism offered against purely forward-looking expressions such as equation (1) is that they are inconsistent with empirical evidence of considerable persistence in inflation (Fuhrer 1997; Galí and Gertler 1999; and Roberts 1998). Section 1.2 follows the literature that assumes some form of non-rational behaviour to obtain more general expressions for inflation with additional sources of lag dynamics. Section 1.3 reviews specifications where lag dynamics result from staggered contracting. An alternative source of additional lag dynamics is rational behaviour in the presence of frictions on price adjustment. Such specifications are reviewed in section 1.4.[2]

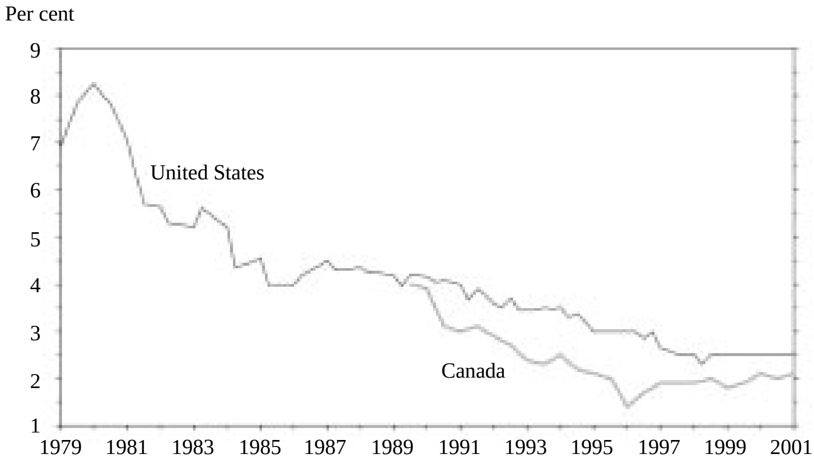
2. Lag dynamics have been introduced using approaches other than those reviewed here. One alternative to introducing inflation stickiness is the recently proposed sticky-information model of price adjustment presented by Mankiw and Reis (2001) and empirically implemented for the United States, Canada, and the United Kingdom by Khan and Zhu (2002). Carroll (2001) derives an expression for inflation expectations identical to the one proposed by Mankiw and Reis, but based on microfoundations with the spread of information likened to the spread of a disease in models from theoretical epidemiology. Another alternative, proposed by Calvo, Celasun, and Kumhof (2000), applies to a world of positive (>0) steady-state inflation and assumes that when firms are allowed to reoptimize they choose both a reset price and the rule-of-thumb rate at which they will update prices until they next reoptimize. This approach gives rise to inflationary inertia.

1.1 Non-zero anchor for inflation expectations

This section presents an expression for inflation that allows for a non-zero anchor for inflation expectations. A non-zero anchor seems more realistic for empirical analysis. Since the end of 1995, the central tendency of the inflation target range of Canadian monetary policy has been 2 per cent. In the United States, through statements suggesting that the Federal Open Market Committee (FOMC) had achieved price stability even though measures of consumer inflation (and GDP price index inflation) were positive, at least some members of the FOMC have suggested that they personally believe the goal of policy is a small positive target based on measured inflation. For instance, in a speech in June 2002, President Broaddus of the Federal Reserve Bank of Richmond commented: ". . . my own view is that a longer-term annual increase in the core personal consumption expenditures index of 1/2 per cent to 1 1/2 per cent is a good working definition of price stability in practice." On 10 June 2002, *The Inner City Reporter's Federal Reserve Beat* quoted Governor Kohn as saying, "I think it's very clear that the current rate of inflation is pretty darned low and we're getting awfully close to some zone of price stability, if we're not already in it." And, Robert Bartley reported in the *Opinion Journal* on 20 May 2002: "Fed officials point to the consumer price increase of only 1.4% in the last year, and similarly slow growth in more sophisticated indexes. New York Fed President William McDonough recently asked, 'If that's not price stability, what is?'"

In addition, evidence from long-horizon survey data suggests that the long-run anchor for inflation expectations has not been constant over the sample periods typically examined in empirical work (Figure 1). The constant-zero assumption on the inflation expectations anchor is likely to lead to particularly misleading empirical results if the steady-state inflation rate changed within a sample over which the specification is estimated. The phrases "steady-state inflation," "(long-run) anchor for inflation expectations," "nominal anchor," and "perceived inflation target" will be used interchangeably to refer to the value of inflation that is expected to obtain in the absence of shocks. Alternatively, since forecasts assume all future shocks will be zero, this will be the value to which long-run forecasts of inflation will converge. In a stable system, this will be what the market perceives the inflation target of monetary policy to be. The market perception of the inflation target is used so that the same model can be applied to countries with stable goals, whether or not their central banks have announced inflation targets. In addition, use of the market perception of the inflation target recognizes that market expectations will be anchored by what the market thinks the inflation target is. Kozicki and Tinsley

Figure 1
Long-horizon inflation expectations



Sources: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia (United States); Consensus Forecasts (Canada).

(2001b) point out that in the presence of imperfect information, the perceived inflation target may differ from the true target of inflation, signalling a form of imperfect policy credibility.

An expression for inflation that allows for a non-zero inflation-expectations anchor is derived in the Appendix. As a starting point, this paper will use an approximation to that expression that closely resembles the benchmark in equation (1),

$$\hat{\pi}_t = bE_t\hat{\pi}_{t+1} + gy_t + \varepsilon_t . \quad (2)$$

In this expression, $\hat{\pi}_t$ represents the percentage deviation of inflation from the nominal anchor. Implications of non-rational behaviour will imply modifications to equation (2) just as they would for equation (1). An example is provided in section 1.2. Models with staggered contracts and frictions imply inflation expectations that are generalizations of equations (1) and (2). These specifications are introduced in sections 1.3 and 1.4.

The expression in equation (2) differs from the benchmark model in three ways. First, inflation appears as the deviation of inflation from the nominal anchor. Second, the functional relationship between the coefficients on expected inflation and the output gap and the structural parameters of the model are different. Third, embedded in the shock in equation (2) is a term with a discounted sum of expected inflation deviations that is correlated with other regressors. As the nominal anchor approaches zero, all three of these differences shrink, and the expression in equation (2) converges to the benchmark model in equation (1).

The replacement of inflation with inflation deviations may lead to important differences in empirical studies of inflation dynamics, including descriptions of the degree of inflation persistence. If the nominal anchor is constant over the sample being examined, then, all else equal, estimates of inflation persistence and the properties of inflation dynamics are unlikely to be affected very much as long as estimated equations contain a constant term. However, if the nominal anchor has changed over the sample period, then empirical results may be considerably different. Stationary series with step changes are often mistaken for I(1) processes, an empirical finding that exaggerates the degree of persistence in the series (Hendry and Neale 1991). The importance of accounting for shifts in steady-state inflation in an analysis of persistence in inflation is empirically examined in section 2.

The fact that the relationship between coefficients and structural parameters is different is less likely to be important for the empirical questions being addressed in this paper. Here, empirical results focus on estimates of coefficients such as b and g, rather than on the underlying structural parameters. While for given estimates of coefficients, estimates of structural parameters may be affected if the inflation expectations anchor is incorrectly assumed to be zero, effects are likely to be small if the anchor is close to zero. For Canada and the United States, the assumption that the nominal anchor is positive, but close to zero, is reasonable—especially for the era of inflation targeting in Canada and the Greenspan policy regime in the United States. Likewise, for a nominal anchor close to zero and plausible values of the structural parameters, the implied difference in coefficients between the benchmark and general specifications is likely to be small.

Compared with the expression derived in the Appendix, the approximation in equation (2) lumps a term that includes expected inflation deviations into ε_t. In addition to excluding a potentially important explanatory variable, the presence of this term in ε_t introduces a correlation between ε_t and the explanatory variables. However, as shown in the Appendix, for realistic parameterizations, the contribution of this term to movements in $\hat{\pi}_t$ is likely to be negligible, and the size of the bias will likely be very small. Consequently, the simpler approximation that more closely lines up with the standard approach was chosen to be the starting point for the analysis in this paper. More details on the size of the missing term and the relationships between b and β and between g and γ are provided in the Appendix.

1.2 Hybrid models resulting from non-rational behaviour

A standard criticism of purely forward-looking models of inflation such as equation (1) is that because the inflation rate doesn't depend on lagged inflation, it is completely flexible (Chadha, Masson, and Meredith 1992; Fuhrer and Moore 1992; and Fuhrer 1997). This seems at odds with empirical evidence for Canada and the United States, which finds considerable persistence in inflation.

One approach to introducing additional stickiness to inflation is to assume that a fraction of agents are backward-looking and use a simple auto-regressive structure to forecast inflation (Roberts 1997, 2001). Suppose that a fraction ω of agents use a backward-looking proxy for expectations and assume inflation evolves according to

$$\hat{\pi}_t = b\sum_{i=1}^{p} A_i\hat{\pi}_{t-i} + gy_t + g\sum_{i=1}^{p} C_i y_{t-i} + \varepsilon_t, \qquad (3)$$

while the remaining fraction, $1-\omega$, of agents are purely forward-looking, as in equation (2). Aggregating across agents, a hybrid model of inflation with both forward-looking and backward-looking agents is obtained:

$$\hat{\pi}_t = b(1-\omega)E_t\hat{\pi}_{t+1} + b\omega\sum_{i=1}^{p} A_i\hat{\pi}_{t-i} + gy_t + g\omega\sum_{i=1}^{p} C_i y_{t-i} + \varepsilon_t. \qquad (4)$$

This expression resembles the typical hybrid specification, but with inflation replaced by deviations of inflation from the nominal anchor. For instance, if $A_i = 0$ for $i \neq 1$, with $0 < A_1 \leq 1$, and $C_i = 0$, then an expression similar to equation (3) in Roberts (2001) and to those estimated by Fuhrer (1997) is obtained. If $\omega = 1$, then the expression resembles a backward-looking Phillips curve such as implemented by Beaudry and Doyle (2001) and Fuhrer (2001) for Canada and Rudebusch and Svensson (1999) for the United States. Most empirical estimates of backward-looking Phillips curves assume $b\sum_{i=1}^{p} A_i = 1$ to preclude the existence of a permanent trade-off between inflation and unemployment.[3]

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3. Most empirical implementations of backward-looking Phillips curves use the deviation of the unemployment rate from the non-accelerating-inflation rate of unemployment (the NAIRU) rather than the output gap. For more discussion of backward-looking Phillips curves, see Kozicki (2001) and the references therein.

1.3 Models with staggered contracts

Inflation stickiness can also be introduced using variants of the Taylor (1980) staggered-contracting framework. The typical model of inflation dynamics derived from a staggered-contracting specification does not rely on an assumption that the steady-state inflation rate be equal to zero. However, restrictions on the structure of the model imply that the model of inflation dynamics also applies to deviations of inflation from the nominal anchor.

Following the derivations outlined in Fuhrer and Moore (1995), the aggregate log price level, p_t, is a weighted average of the log contract prices, x_t, negotiated in the current and previous quarters that are still in effect. Letting h_i denote the proportions of outstanding contracts negotiated in $t-i$, and using the lag operator L where $L^i x_t \equiv x_{t-i}$, p_t satisfies

$$\begin{aligned} p_t &= \sum_{i=0}^{m-1} h_i x_{t-i} \\ &= h(L)x_t, \end{aligned} \tag{5}$$

assuming no contracts negotiated prior to $t-m+1$ are still in effect, i.e., the longest contract lasts m periods. The distribution of outstanding contracts must satisfy $h(1) = 1$. In the standard contracting specification, the current nominal-wage contract, x_t, depends on the price level expected to prevail over the life of the contract, adjusted for excess demand conditions,

$$\begin{aligned} x_t &= \sum_{i=0}^{m-1} h_i E_t(p_{t+i} + \gamma y_{t+i}) \\ &= E_t h(L^{-1})(p_t + \gamma y_t). \end{aligned} \tag{6}$$

Combining equation (5) with equation (6) and using $h(1) = 1$ results in the price expression

$$p_t = h(L)E_t h(L^{-1})(p_t + \gamma y_t). \tag{7}$$

For two-period staggered contracts with half of all contracts negotiated each period, Fuhrer and Moore (1995) show that equation (7) simplifies to a purely forward-looking expression similar to equation (1) with $\beta = 1$. In other words, for $m = 2$, although wages and prices are sticky, inflation is not. For $m > 2$, staggered Taylor-style contracting implies that inflation depends on additional lags and leads of inflation,

$$\pi_t = E_t\pi_{t+1} + \sum_{i=2}^{m-1}\left(\sum_{j=i}^{m-1} G_j\right)(E_t\pi_{t+i} - \pi_{t-i+1})$$
$$+ \gamma \sum_{i=1}^{m-1} G_i(E_t y_{t+i} + y_{t-i}) + \gamma G_0 y_t + resid_{T,t}, \quad (8)$$

where the coefficients G_j are non-linear functions of the contract proportions h_i, and $G_1 = 1 - \sum_{i=2}^{m-1} G_i$.[4]

The derivation of equation (8) did not require an assumption that the steady-state inflation rate is zero. However, because the sum of coefficients on lags and leads of inflation on the right side of equation (8) is equal to unity, the expression also holds in deviation from steady-state form:

$$\hat{\pi}_t = E_t\hat{\pi}_{t+1} + \sum_{i=2}^{m-1}\left(\sum_{j=i}^{m-1} G_j\right)(E_t\hat{\pi}_{t+i} - \hat{\pi}_{t-i+1})$$
$$+ \gamma \sum_{i=1}^{m-1} G_i(E_t y_{t+i} + y_{t-i}) + \gamma G_0 y_t + resid_{T,t}. \quad (9)$$

To induce additional inflation stickiness, Fuhrer and Moore (1995) and Fuhrer (1997) explored the consequences of a relative-contracting specification developed by Buiter and Jewitt (1981). The Fuhrer-Moore specification assumes that agents set nominal contract prices so that the current real contract index depends on the real contract index expected to prevail over the life of the contract, adjusted for excess demand conditions, with the real contract index defined as a combination of real contract wages negotiated on the contracts currently in effect. With relative contracting, an expression for inflation that resembles the expression for prices in equation (7) is obtained:

$$\pi_t = h(L)E_t h(L^{-1})(\pi_t + \gamma g^{-1}(L) y_t), \quad (10)$$

where $g(L) = \sum_{i=1}^{m-1} g_i L^{i-1}$ with $g_i = \sum_{j=i}^{m-1} h_j$. Fuhrer-Moore contracting implies that for $m > 2$, inflation evolves according to:

4. This expression replaces lagged conditional expectations of lagged variables, $E_{t-k} y_{t-l}$, for $k > l$, with y_{t-l}. The difference between the conditional expectation and the observation is included in the error term and implies that the error term may be serially correlated. Guerrieri (2002) provides a careful derivation for the case of $m = 2$ that doesn't make this substitution.

$$\pi_t = \pi_{t-1} + (E_t\pi_{t+1} - \pi_t) + \sum_{i=2}^{m-1}\left(\sum_{j=i}^{m-1} G_j\right)(E_t\pi_{t+i} - E_t\pi_{t+i-1})$$

$$-\sum_{i=2}^{m-1}\left(\sum_{j=i}^{m-1} G_j\right)(\pi_{t-i+1} - \pi_{t-i}) + \gamma h(L)E_t h(L^{-1})g^{-1}(L)y_t + resid_{F,t}, \quad (11)$$

or, after rearrangement,

$$\pi_t = (1/2)\sum_{i=1}^{m-1} G_i(E_t\pi_{t+i} + \pi_{t-i})$$
$$+ (1/2)\gamma h(L)E_t h(L^{-1})g^{-1}(L)y_t + resid_{F,t}. \quad (12)$$

In these expressions, the coefficients, G_i, are the same non-linear functions of the contract proportions, h_i, as in the Taylor specification, and as before, $G_1 = 1 - \sum_{i=2}^{m-1} G_i$. Once again, because the coefficients on the lags and leads of inflation on the right side of equation (12) sum to one, the expression also holds in deviation from steady-state form:

$$\hat{\pi}_t = (1/2)\sum_{i=1}^{m-1} G_i(E_t\hat{\pi}_{t+i} + \hat{\pi}_{t-i})$$
$$+ (1/2)\gamma h(L)E_t h(L^{-1})g^{-1}(L)y_t + resid_{F,t}. \quad (13)$$

The Fuhrer-Moore specifications imply different coefficients on inflation leads and lags than the Taylor specifications. The most noticeable difference is that in the Fuhrer-Moore specifications, coefficients on all leads and lags are positive, while in the Taylor specifications, coefficients on leads are positive but those on lags are negative. A second difference is that, since the G_is are positive, coefficients in the Taylor specifications decrease in magnitude as the lag/lead order increases, while those in the Fuhrer-Moore specifications are not similarly constrained.

One important difference between inflation specifications that assume a fraction of the population form expectations non-rationally versus those that assume staggered contracts, is the appearance of additional leads of inflation as explanatory variables. For contract lengths greater than two, the Taylor and Fuhrer-Moore specifications imply that inflation will depend on additional leads of expected inflation as well as additional lags. By contrast, as the lag length of the time-series forecasting model used by non-rational agents increases, only extra lags appear in the model with non-rational forecasting agents, equation (4).

A second difference in the specifications is that the Taylor and Fuhrer-Moore inflation expressions include lags and leads of the output gap rather than just the contemporaneous output gap. Although the hybrid model presented in section 1.2 did not include lags of the output gap, if non-rational agents were to use lags of both output and inflation to forecast output, then lags of the output gap would also appear in the hybrid specification. However, the presence of non-rational agents that use a reduced-form time-series model to forecast would not result in the appearance of leads of the output gap in hybrid specifications.

1.4 Generalized frictions on price adjustment

Section 1.2 described how additional sources of lag dynamics can be introduced into models of inflation by relaxing the assumption of rational expectations. Section 1.3 reviewed how staggered-contracting specifications can also result in more complicated lag dynamics. An alternative proposed by Kozicki and Tinsley (1999a), with explicit derivations for inflation in Kozicki and Tinsley (2002), assumes that expectations are formed rationally, but that there are frictions on price adjustment.

This section's approach may be better viewed as a general approach rather than one with a different motivation from some of the models already discussed. In particular, in the Taylor staggered-contracts model and in the Calvo (1983) assumptions behind the purely forward-looking specifications in equations (1) and (2), it is also true that expectations are formed rationally, and price stickiness derives as a result of assumptions that there are frictions associated with price adjustment. Furthermore, as will be discussed, New Keynesian Phillips curves implied by the Calvo, Taylor, and Fuhrer-Moore formulations can be derived as special cases of the generalized-frictions approach (although the coefficients may have different structural interpretations in the various formulations).

Frictions on price adjustment may include factors that lead to lags between cost changes and price adjustment, the deterrent effect of concerns that competitors will not also adopt price increases, and the reluctance of firms to antagonize customers. These three factors were identified as important potential explanations of price stickiness in a report by Blinder et al. (1998) on the results of a survey of heads of small companies and appropriate officers of large corporations. Alternatively, frictions may be due to managerial and customer costs associated with price adjustment, as examined by Zbaracki et al. (2003). They provide explicit estimates of costs of price adjustment and determine that managerial and customer costs are substantial (more than an order of magnitude larger than menu costs) and, importantly, that these costs appear to be convex functions of the size of the price change.

The frictions approach (or polynominal adjustment-cost approach) does not require an assumption that the nominal anchor be equal to zero. The review below provides examples of conditions and modifications that result in models of the dynamics of inflation and in models of the deviation of inflation from the nominal anchor.

In Kozicki and Tinsley (2002), optimal intertemporal planning is captured by assuming that agents choose their relative price to minimize percentage deviations from the optimal relative price path subject to frictions on price adjustment. The planning problem can be stated as:

$$\min_{p} E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[(1/2)(p_{t+i}-p^*_{t+i})^2+(1/2)(v(L)p_{t+i})^2\right]\right\}, \quad (14)$$

where $v(L) = v_0 + v_1 L + ... + v_{m-1}L^{m-1}$ is an $(m-1)$ order frictions polynomial in the lag operator, L, and $v(1) = 0$ to capture that frictions are binding only in disequilibrium. Use of a quadratic loss function to model frictions or adjustment costs associated with price changes may be justified by, for example, the convex managerial and customer costs identified by Zbaracki et al. (2003) and their claim that customer-antagonism costs can arise through any price change—either a decrease or an increase. The implied expression for inflation under agent optimization is:

$$\pi_t = \sum_{i=1}^{m-1}\left(\sum_{j=i}^{m-1}\beta^j G_j\right)E_t\pi_{t+i} - \sum_{i=2}^{m-1}\left(\sum_{j=i}^{m-1}G_j\right)\pi_{t-i+1} + \gamma^* y_t + resid_{P,t}, \quad (15)$$

where $p^*_t - p_t = \gamma^* y_t$ has been assumed as in Kozicki and Tinsley (2002) and the coefficients are functions of the coefficients in the frictions polynomial, $G_i = f_i/(\sum_{j=1}^{m-1} f_j)$ with $f_i \equiv -\sum_{j=0}^{m-i-1} v_j v_{j+i}\beta^j$.

As in the staggered-contracting specifications, the frictions approach results in an expression with additional leads as well as lags of inflation. This expression closely resembles that obtained under Taylor contracting in equation (8). In particular, if $\beta = 1$, then equations (15) and (8) differ only in the way the output gap enters the expression, with the frictions-based expression corresponding to a Taylor-contracting specification derived using a slightly modified version of equation (7):

$$p_t = E_t h(L) h(L^{-1}) p_t + \gamma y_t . \tag{16}$$

For the special case when $m = 2$, the planning problem in equation (14) corresponds to the quadratic adjustment-cost model of Rotemberg (1982), and, as noted by McCallum and Nelson (1999), implies the purely forward-looking expression for inflation in equation (1). If $\beta = 1$, then equation (15) also holds with inflation replaced by the percentage deviation of inflation from the nominal anchor:

$$\hat{\pi}_t = E_t \hat{\pi}_{t+1} + \sum_{i=2}^{m-1} \left(\sum_{j=i}^{m-1} G_j \right) (E_t \hat{\pi}_{t+i} - E_t \hat{\pi}_{t-i+1}) + \gamma^* y_t + resid_{P,t} . \tag{17}$$

This expression closely resembles equation (9), although only the contemporaneous output gap appears here. However, if $\beta \neq 1$, then this expression will only hold approximately.

The adjustment-cost formulation can also support a structure similar to that obtained by Fuhrer and Moore. In particular, to maintain the assumption that frictions are binding only in disequilibrium when the nominal anchor is non-zero, it may be more appropriate to assume that the frictions polynominal applies to inflation deviations, i.e., that frictions are binding only when inflation deviates from the nominal anchor. In this case, the planning problem can be restated as

$$\min_p E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[(1/2)(p_{t+i} - p^*_{t+i})^2 + (1/2)(v(L)\hat{\pi}_{t+i})^2 \right] \right\} . \tag{18}$$

This formulation may be motivated, for instance, by a view that consumers may be antagonized by price changes that they do not regard as "fair," as in Rotemberg (2002). He suggests that "If recent inflation has been relatively low, [customers] are likely to believe that cost increases have been modest and [they] are likely to be less tolerant of price increases." Zbaracki et al. (2003) cite a pricing manager in the 1970s as observing: "The [cost] increases we experienced during that [inflationary] time were very much largely driven by cost and our average costs were going up and we were trying to recoup that. . . . [During the] high-inflation period you could get away with the high price increases. I think there [were] expectations in the market place; our customers [were] saying 'I am able to inflate my prices to the end user so I shouldn't be surprised when my vendor raises their prices. . . .'" If the perceived inflation target is used by consumers to estimate cost increases and gauge whether price changes are reasonable, then frictions would apply to deviations of inflation from the perceived inflation target.

Optimization of equation (18) implies an expression for the change in inflation that is similar to equation (15). After manipulation, inflation can be shown to evolve according to:

$$\hat{\pi}_t = \sum_{i=1}^{m-1} G_i(\beta^j E_t \hat{\pi}_{t+i} + \hat{\pi}_{t-i}) \Big/ \left(1 + \sum_{j=1}^{m-1} G_j \beta^j\right) + \gamma^{**} y_t + resid_{D,t}. \quad (19)$$

For $\beta = 1$, the frictions-based expression corresponds to a Fuhrer-Moore contracting specification based on this modified version of equation (10),

$$\pi_t = E_t h(L) h(L^{-1}) \pi_t + \gamma y_t, \quad (20)$$

and equation (19) simplifies to equation (13) but with leads and lags of the output gap replaced by the contemporaneous output gap.

The next section will examine whether empirical evidence favours specifications with non-rational expectations formations as in equation (4), Taylor-type staggered contracting as in equation (8) or (9), Fuhrer and Moore staggered contracting as in equation (12) or (13), or rational adjustment in the presence of generalized frictions as in equations (15), (17), or (19).

2 An Empirical Analysis of the Sources of Inflation Persistence

This section examines the persistence properties of Canadian and U.S. inflation and assesses which of the various models of lag dynamics seem to be most consistent with the data. The first subsection estimates time series for the nominal anchor in Canada and the United States. The second subsection summarizes the persistence properties of inflation and examines to what extent shifts in the nominal anchor may explain observed persistence. The third subsection contrasts results from estimates of structural models of inflation dynamics, including purely forward-looking expressions, hybrid models that assume partially non-rational expectations formation, expressions based on staggered-wage contracting, and models with rational expectations and generalized frictions on price adjustment.

2.1 Historical estimates of the perceived inflation target

Kozicki and Tinsley (2001c) argue that there have been shifts in the perceived long-run inflation target of monetary policy in the United States.

Such shifts explain low-frequency movements in long-horizon inflation expectations evident in survey data (Figure 1) and help resolve empirical puzzles in the U.S. Treasury term structure (Kozicki and Tinsley 2001a, 2001b, 2001c).[5] Estimates of a perceived inflation-target series based on breakpoint tests are provided in Kozicki and Tinsley (2001a, 2001c). Cogley and Sargent (2001) estimate long-horizon forecasts of inflation that shift considerably over 1965–2000.

Empirical evidence suggests that there have been shifts in the conditional mean of the inflation process in Canada as well. Laxton, Ricketts, and Rose (1994) estimate a three-state model of Canadian inflation; and Perron (1994) provides evidence suggesting that the mean of Canadian inflation has shifted. Hostland (1995) documents that the time-series properties of Canadian inflation were quite different from the mid-1950s to the early 1970s than before and after this period. And, although only available for a relatively short sample, the long-horizon survey data shown in Figure 1 support the view that there have been shifts in the nominal anchor in Canada.

While using directly observable information on shifts in the anchor to long-horizon inflation expectations would be preferable, insufficient data are available. Long-horizon survey data might provide one proxy for the nominal anchor, but such data are available only since 1979 for the United States and since 1990 for Canada. Another alternative for Canada is to use the midpoint of the inflation-control target range. However, such a range is available only since 1991 and if the policy wasn't regarded as credible, the anchor for long-run inflation expectations could possibly have differed from the midpoint of the range. Consequently, the approach taken in this paper is to estimate a series to proxy for the nominal anchor. Survey data are then used as a check on whether or not the estimated series for the nominal anchor is reasonable.

A reduced-form procedure similar to that described in Kozicki and Tinsley (1999b) is used to estimate the anchor of long-horizon inflation expectations. For each country, a four-variable vector autoregression (VAR) with shifting endpoints is used to proxy for agent expectations.[6] The variables included in the VAR are quarterly data on the output gap, y_t, inflation, π_t, a 10-year nominal government yield, $R_{10,t}$, and a short-term real interest rate, r_t (constructed as the difference between an observed nominal short-term

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5. Missing observations for long-horizon survey data are linearly interpolated from observations for surrounding quarters.

6. The use of shifting, or moving, endpoints in AR and VAR time-series models is discussed in Kozicki and Tinsley (1998, 2001a, 2001b, 2001c).

interest rate and inflation over the previous quarter).[7] Each variable appears in the VAR in deviation from steady-state form, and each deviation variable is assumed to be stationary. Thus, any source of non-stationarity derives from shifts in the steady state. Four steady-state variables are included in the model: the equilibrium real short-term interest rate, μ, the 10-year term premium, ϕ, the steady-state output gap, $\bar{y}$, and the long-run anchor of inflation expectations, $\bar{\pi}^P$ (i.e., the perceived inflation target). The equilibrium real rate, the term premium, and the steady-state output gap are assumed to be constant, while the data are allowed to determine whether and how the inflation steady state varies.[8]

The reduced-form model assumes that the dynamics of the deviations of the variables from their steady states are well described by a four-lag VAR. In each quarter, updates to agents' perceptions about the perceived inflation target are assumed to be independent normal innovations. The reduced-form model is:

$$\begin{bmatrix} y_t \\ \pi_t \\ R_{10,t} \\ r_t \end{bmatrix} = A(L)\begin{bmatrix} y_t \\ \pi_t \\ R_{10,t} \\ r_t \end{bmatrix} + (I - A(1))\begin{bmatrix} \bar{y} \\ \bar{\pi}_t^p \\ \mu + \bar{\pi}_t^p + \phi \\ \mu \end{bmatrix} + u_t, \tag{21}$$

where $A(L) = A_1L + A_2L^2 + A_3L^3 + A_4L^4$ and updates to the perceived inflation target follow

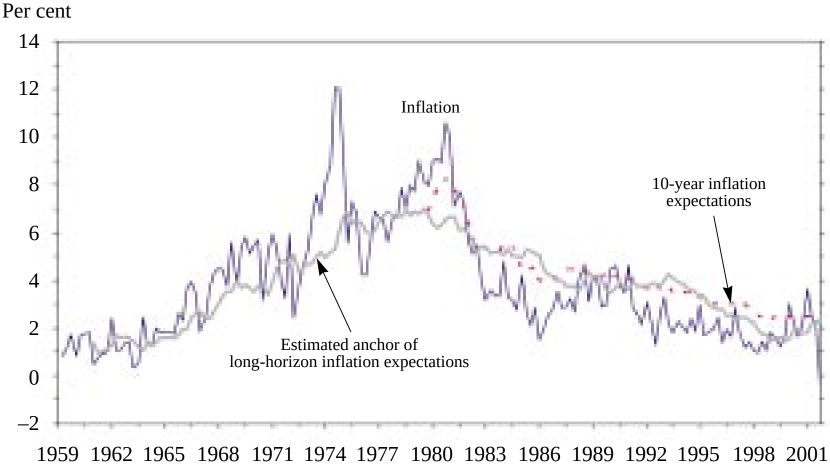
$$\bar{\pi}_{t+1}^p = \bar{\pi}_t^p + v_t. \tag{22}$$

The innovations u_t and v_t are assumed to be uncorrelated across time and with each other. The model was estimated using maximum likelihood with Kalman filtering techniques to deal with the unobserved state variable, $\bar{\pi}_t^p$.

7. For the United States, Congressional Budget Office estimates are used for the output gap, the federal funds rate is used as a short-term interest rate, and inflation is measured using the GDP price index. For Canada, the output gap is estimated using a Hodrick-Prescott filter with smoothing parameter equal to 1,600, a three-month government bill rate is used as a short-term interest rate, and inflation is measured using the CPI. The price index choices were made to match series for which near-term quarterly survey forecasts are available.

8. The steady-state output gap is allowed to be non-zero for empirical reasons. The theoretical steady-state output gap is zero. However, the average output gap may be non-zero over some samples, and the econometric procedure maps the sample average into the steady-state estimate.

Figure 2
U.S. inflation and the estimated nominal anchor



Sources: Bureau of Economic Analysis, U.S. Department of Commerce; Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia; authors' calculations.

The variance of innovations to the state variable was chosen to match the variance of innovations to available long-horizon survey data.

Figure 2 shows inflation, the estimated anchor of long-horizon inflation expectations, and survey data on 10-year inflation expectations for the United States.[9] The estimated nominal anchor is the unsmoothed estimate of the state variable from the Kalman filter $(E_{t-1}\pi_t^p)$. The estimated nominal anchor follows the trajectory of the survey data quite well. After a period of elevated inflation in the mid- to late-1970s, the nominal anchor was quite high. However, the nominal anchor gradually declined through the 1980s as the lower inflation rate obtained under Volker was not reversed. An interesting feature of both the survey data and the estimated nominal anchor is that the series remained above the actual inflation rate for most of the 1980s and 1990s. One possible explanation of this gap is that the FOMC did not have full credibility in its efforts to achieve price stability.

Figure 3 shows results for Canada. The estimated nominal anchor for Canada is closer to actual Canadian inflation than was the case for U.S. data. Since 1994, the estimated nominal anchor and the survey data are close to the central tendency of Canadian inflation and to the central tendency (2 per

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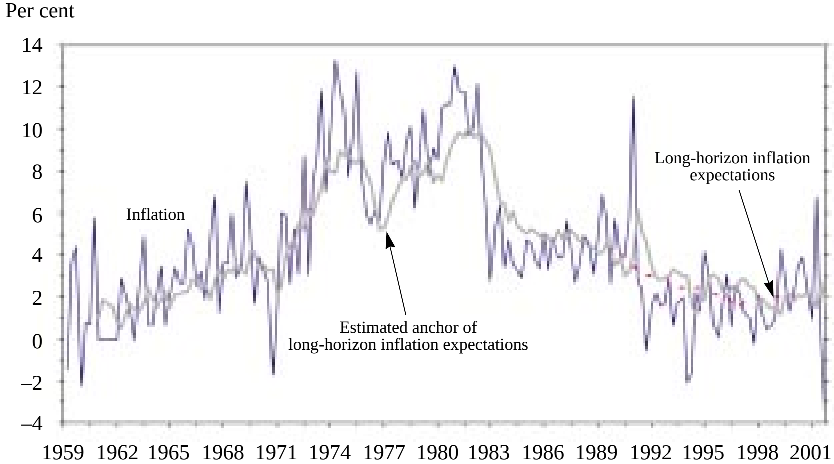
9. The survey series is missing observations for some quarters. Missing observations were linearly interpolated.

Figure 3
Canadian inflation and the estimated nominal anchor



Sources: Bank of Canada; Consensus Forecasts; authors' calculations.

cent) of the inflation-control target range (not shown in Figure 3). In fact, the survey data lie inside the inflation-control target range from the inception of the new policy regime. These results suggest that the Bank of Canada's inflation-control targeting regime has been credible.

2.2 Reduced-form estimates of inflation persistence

This section investigates to what extent persistence in inflation may be linked to shifts in the nominal anchor for inflation expectations rather than to sluggishness of inflation dynamics in the presence of a constant steady state. Inflation persistence is measured as the sum of coefficients from an estimated AR(4) model of inflation. Time-series models are estimated over 1962–2001 and various subsamples. Models are estimated using raw inflation data, deviations of inflation from the estimated nominal anchor, deviations of inflation from long-horizon survey data (over those subsamples for which survey data are available), and deviations of inflation from a spliced nominal-anchor series that uses Kalman estimates only when survey data are unavailable. If some of the sluggishness of inflation is associated with shifts in the nominal anchor, then estimates of persistence should be smaller for inflation deviations and over subsamples where shifts in the nominal anchor were less likely to have occurred (the 1990s).

Table 1
Estimates of inflation persistence

Estimation sample	Inflation	Inflation deviations	Survey deviations	Spliced deviations
United States				
1962Q1–2001Q4	0.94	0.80		0.81
	(0.04)	(0.07)		(0.07)
1962Q1–1970Q4	0.92	0.54		0.54
	(0.10)	(0.24)		(0.24)
1971Q1–1984Q4	0.85	0.75		0.76
	(0.10)	(0.12)		(0.13)
1985Q1–2001Q4	0.83	0.74	0.53	0.53
	(0.12)	(0.13)	(0.18)	(0.18)
1992Q1–2001Q4	0.39	0.78	0.53	0.53
	(0.26)	(0.18)	(0.28)	(0.28)
Canada				
1962Q1–2001Q4	0.90	0.46		0.47
	(0.05)	(0.13)		(0.13)
1962Q1–1970Q4	0.54	0.03		0.03
	(0.22)	(0.41)		(0.41)
1971Q1–1984Q4	0.77	0.56		0.56
	(0.11)	(0.20)		(0.20)
1985Q1–2001Q4	0.68	0.16		0.23
	(0.16)	(0.22)		(0.22)
1992Q1–2001Q4	–0.01	0.14	0.21	0.21
	(0.33)	(0.27)	(0.29)	(0.29)

Note: Entries are sum of coefficients in an AR(4) model of inflation with standard errors provided in parentheses.

Results are presented in Table 1. For U.S. data, inflation persistence is much lower in the 1990s than for any other subsample examined or for the full sample. These results are consistent with evidence reported by Cogley and Sargent (2001) and Willis (2003). Survey data on inflation expectations and movements in actual inflation were, on average, much flatter over this period than over any other. Thus, this result supports the view that some of the persistence in U.S. inflation may be due to shifts in the nominal anchor. Also supporting this view, for all other subperiods, persistence of inflation exceeds persistence of deviations of inflation from the estimated nominal anchor, from the survey data, and from the spliced series. This result is consistent with Levin and Piger (2002), who find that conditional on a statistically detected break in the intercept and innovation variance, inflation exhibits less persistence.

Results for Canada are stronger than those for the United States. Estimates of inflation persistence are lower in every subsample than over the full 1962–2001 sample. In fact, inflation persistence disappears entirely during the inflation-targeting regime. Results based on inflation deviations offer further support that in Canada, most of the persistence in inflation is due to shifts in the nominal anchor. Very little persistence remains after such shifts have been removed.

2.3 Estimates of forward-looking models of inflation

The empirical results from the previous section suggest that shifts in the long-run anchor of inflation expectations help explain the observed persistence of inflation. Inflation persistence was estimated to be considerably lower since 1992, a period with relatively small movements in long-run inflation expectations, than in earlier periods. In addition, after accounting for shifts in the anchor for long-run inflation expectations, inflation persistence declines for both countries (although considerably more for Canada). However, these results were based on estimation of reduced-form autoregressive models of inflation. This section provides more structure to the analysis, and takes the models described in the previous section to the data.

All of the models outlined in section 1 include conditional expectations of future inflation (and perhaps future output gaps) as explanatory variables. Following Roberts (1997), survey data are used to proxy for the conditional expectations of inflation.[10] U.S. survey data are taken from the Survey of Professional Forecasters, published by the Federal Reserve Bank of Philadelphia. Forecasts of U.S. inflation are constructed from forecasts of the implicit GDP price deflator starting in 1992, and from forecasts of the GNP price deflator prior to 1992. Quarterly forecasts of one-quarter through four-quarter-ahead inflation are available starting from the fourth quarter of 1969. Canadian survey data are taken from Consensus Forecasts. Quarterly forecasts of one-quarter through four-quarter-ahead inflation are available from the second quarter of 1990. A few missing observations were encountered early in the sample. Missing observations for $E_t\pi_{t+k}$ were interpolated as the average of $E_{t-1}\pi_{t+k+1}$ and $E_{t+1}\pi_{t+k-1}$. In estimations that use inflation deviations instead of inflation, the nominal anchor is approximated using the spliced series that approximates the nominal anchor with long-horizon survey expectations when available, and using the Kalman-estimated nominal anchor prior to the availability of the survey data.

Some of the models include expectations of future values of the output gap. For both countries, ex post values of the output gap are used for future values. To account for potential correlation between contemporaneous (and

10. Roberts (1997) provides a discussion of the properties of surveys of inflation expectations.

future) values of the output gap and equation errors, estimation proceeds using instrumental variables with the relevant survey data for that estimation, four lags of the output gap, and four lags of inflation used as instruments.

Unfortunately, the sample over which the models can be estimated is constrained because of limited survey data. For the United States, estimation is based on data from the fourth quarter of 1969 through the fourth quarter of 2001 and, for Canada, estimation is over data from the second quarter of 1990 through the fourth quarter of 2001.[11]

Estimates of the benchmark model (equation 1) and the approximating model (equation 2) for the non-zero steady-state inflation case are provided in Table 2. Results are presented for estimated β and for β constrained to equal one. The purely forward-looking model of inflation does not fit U.S. data well. *Q*-statistics strongly reject the null hypothesis of no serial correlation up to lags 4 and 8. This is true for both inflation and inflation deviations. The inability of the model to explain inflation should not be surprising given the evidence, presented in Table 1, of considerable persistence in both inflation and inflation deviations (in all cases but the 1990s for inflation).[12]

Results for Canada are less pessimistic for the purely forward-looking model. Although there is weak evidence of residual serial correlation in the estimates for inflation with β restricted to equal one, the model tends to fare better when β and b are estimated, and when the model is applied to inflation deviations. Estimates of β and b seem somewhat small, however. One possible reason for this is that estimates are biased owing to omitted variables. Although *Q*-statistics do not detect significant serial correlation in residuals, these tests may have low power given the limited sample. The possibility that low estimates of β and b are due to omitted variables will be explored as additional sources of lag dynamics are added to the structure.

Empirical results from estimation of hybrid models that assume that a fraction of agents form expectations non-rationally are provided in Table 3.

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11. Robustness of results for the United States was examined by also estimating the models over 1990Q2 to 2001Q4 for U.S. data. Differences between the qualitative results for the sample examined in the paper and those for the shorter sample are discussed in footnotes. One empirical complication not addressed in the paper is whether the results are sensitive to the use of 2002 vintage price data in combination with real-time survey data. However, as differences between real-time and latest available data are likely to be smaller for more recent samples, results from analysis of the shorter sample are less likely to be driven by data vintage mismatches.

12. When estimated over 1990Q2 through 2001Q4, the U.S. data no longer rejected the null hypothesis of no residual serial correlation.

Table 2
Estimates of purely forward-looking models of inflation

$$infl_t = c_1 + (b \text{ or } \beta)E_t infl_{t+1} + \gamma y_t + resid_t$$

Inflation variable	γ	β	b	Q(4)	Q(8)	S.E.
United States						
$infl_t \equiv \pi_t$	0.144	1.00		0.000	0.000	1.15
	(0.047)					
$infl_t \equiv \pi_t$	0.198	1.180		0.000	0.000	1.11
	(0.047)	(0.050)				
$infl_t \equiv \hat{\pi}_t$	0.146		1.00	0.000	0.000	1.15
	(0.046)					
$infl_t \equiv \hat{\pi}_t$	0.159		1.419	0.000	0.000	1.11
	(0.045)		(0.124)			
Canada						
$infl_t \equiv \pi_t$	0.631	1.00		0.076	0.275	2.26
	(0.228)					
$infl_t \equiv \pi_t$	0.613	0.402		0.259	0.570	2.12
	(0.214)	(0.223)				
$infl_t \equiv \hat{\pi}_t$	0.621		1.00	0.075	0.272	2.26
	(0.228)					
$infl_t \equiv \hat{\pi}_t$	0.602		0.025	0.451	0.816	2.02
	(0.204)		(0.280)			

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

Results are presented for both inflation and inflation deviations, even though the model for inflation is misspecified in the presence of a non-zero nominal anchor. During estimation, non-rational agents were assumed to use only lags of inflation and the contemporaneous output gap to forecast inflation, i.e., $C_i = 0$ for $i = 1, \ldots, p$ in equation (3). No constraints on the sum of coefficients on expected inflation and lags of inflation were imposed during estimation. For U.S. data, estimates of ω and ωA_i imply that the non-rational agents use a model with considerable inflation persistence. The sums of coefficients on lags of inflation in the implied forecasting model used by rational agents equal $\sum_{i=1}^{m} \omega A_i / \omega$ and exceed one for all values of m and both inflation and inflation deviations.[13] For Canadian data, empirical results suggest that non-rational agents use a model with less

13. For the shorter sample, $\sum_{i=1}^{m} \omega A_i / \omega$ is close to one for inflation, but somewhat smaller than one for inflation deviations. Obtaining a smaller sum for inflation deviations is consistent with the earlier result that inflation persistence declines after controlling for shifts in the nominal anchor.

Table 3
Estimates of hybrid models of inflation persistence

$$infl_t = c_1 + gy_t + (1-\omega)E_t infl_{t+1} + \omega(A_1 infl_{t-1} + A_2 infl_{t-2} + A_3 infl_{t-3} + A_4 infl_{t-4}) + resid_t$$

Lags	g	ω	ωA_1	ωA_2	ωA_3	ωA_4	**Q(4)**	**Q(8)**	**S.E.**
United States—Inflation $(infl_t \equiv \pi_t)$									
1	0.155	0.358	0.470				0.033	0.156	0.98
	(0.042)	(0.102)	(0.080)						
2	0.160	0.375	0.444	0.043			0.030	0.152	0.99
	(0.043)	(0.108)	(0.096)	(0.087)					
3	0.184	0.428	0.438	–0.066	0.178		0.106	0.284	0.97
	(0.044)	(0.109)	(0.095)	(0.100)	(0.085)				
4	0.208	0.436	0.403	–0.060	0.097	0.130	0.774	0.858	0.97
	(0.045)	(0.109)	(0.097)	(0.100)	(0.099)	(0.085)			
United States—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$									
1	0.128	0.123	0.421				0.005	0.017	0.99
	(0.040)	(0.143)	(0.072)						
2	0.133	0.145	0.378	0.068			0.005	0.019	0.99
	(0.041)	(0.146)	(0.088)	(0.080)					
3	0.153	0.163	0.356	–0.039	0.183		0.007	0.026	0.97
	(0.041)	(0.144)	(0.087)	(0.092)	(0.079)				
4	0.181	0.123	0.294	–0.042	0.063	0.217	0.737	0.926	0.95
	(0.041)	(0.141)	(0.088)	(0.090)	(0.089)	(0.079)			
Canada—Inflation $(infl_t \equiv \pi_t)$									
1	0.542	0.748	0.194				0.654	0.893	2.11
	(0.221)	(0.256)	(0.165)						
2	0.558	0.569	0.195	–0.183			0.651	0.849	2.11
	(0.222)	(0.310)	(0.165)	(0.179)					
3	0.608	0.299	0.156	–0.212	–0.185		0.710	0.853	2.12
	(0.231)	(0.458)	(0.173)	(0.184)	(0.229)				
4	0.655	–0.156	0.091	–0.299	–0.269	–0.184	0.833	0.901	2.13
	(0.245)	(0.858)	(0.203)	(0.231)	(0.267)	(0.292)			
Canada—Inflation deviations $infl_t \equiv \hat{\pi}_t$									
1	0.542	1.078	0.151				0.712	0.947	2.02
	(0.214)	(0.302)	(0.162)						
2	0.564	0.904	0.145	–0.162			0.812	0.955	2.03
	(0.216)	(0.356)	(0.163)	(0.175)					
3	0.606	0.699	0.116	–0.188	–0.132		0.883	0.964	2.05
	(0.229)	(0.499)	(0.172)	(0.182)	(0.224)				
4	0.618	0.608	0.102	–0.207	–0.149	–0.039	0.910	0.970	2.07
	(0.243)	(0.760)	(0.193)	(0.217)	(0.250)	(0.243)			

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

persistence than was the case for U.S. data. The sums of coefficients on lags of inflation in the implied forecasting model are generally considerably less than one.

For the two countries and for both inflation and inflation deviations, the hybrid models generate considerable improvement in fit compared with the purely forward-looking models with plausible values of β or b. The hybrid model that obtains the lowest standard error of U.S. inflation residuals models the evolution of inflation deviations using four lags of inflation deviations.[14] Consistent with the interpretation that shifts in the nominal anchor help explain U.S. inflation persistence, estimates of ω are lower when the model is estimated using inflation deviations rather than raw inflation. Interpretation of results is difficult, however. The null hypothesis that no agents are non-rational $(H_0:\omega = 0)$ cannot be rejected; but, if this is the case, then contrary to the empirical results, no additional lags of inflation deviations should be significant![15] Perplexing results are also obtained for Canadian data. Both of the one-lag models find that a statistically significant fraction of agents are backward-looking (as does the two-lag model for inflation deviations), but point estimates of the coefficients on lagged inflation are small in magnitude and insignificantly different from zero.

These results highlight some of the difficulties associated with interpreting results from estimated hybrid models. In addition to the contradictory results discussed above, with the introduction of non-rational agents, the form of the forecasting model assumed to be used by these agents alters the interpretation of the structure. For instance, a large collection of reduced-form forecasting models could be used to represent the expectations formation of the non-rational agents. The hybrid expression for inflation provided in section 1.2 allowed for the possibility that non-rational agents used lags of inflation and the output gap to forecast inflation. However, VAR forecasting systems with more variables could also have been used. Alternatively, if non-rational agents use only lags of inflation to forecast

14. A model with three (or more) lags of inflation, $m = 3$, appears to fit the U.S. raw inflation data quite well, with no evidence of residual serial correlation. For this specification, the empirical results suggest that about 43 per cent of agents form expectations non-rationally, and the presence of these non-rational agents explains the significance of lags. Structural interpretation of this specification is hampered by the fact that it derives from an aggregation of non-rational agents that use a time-series model to forecast inflation with rational agents that adopt equation (1), even though it is misspecified under positive steady-state inflation.

15. For the shorter sample, a model with four lags of inflation fits U.S. inflation and inflation deviations best. Estimates of ω are 0.32 for inflation and 0.41 for inflation deviations, but neither estimate is statistically significant. However, estimates of ωA_4 are statistically significant.

inflation (and not the contemporaneous output gap), then the coefficient on the output gap in equation (4) becomes $(1-\omega)g$ instead of g. For estimates of ω greater than zero, this would imply a larger value for the structural parameter g.

Table 4 presents results from estimation of the inflation expression that was obtained from the Taylor staggered-contracting framework. For $m > 2$, the number of free parameters to be estimated is m. However, free estimation resulted in very large standard errors on estimates of the coefficient G_0 that multiplies the contemporaneous output gap. Consequently, G_0 was restricted during estimation to equal the theoretical value that would obtain with equal distribution of contracts across m periods.[16]

The Taylor contracting specification results in comparable residual standard errors to the hybrid specifications. However, estimated coefficients are inconsistent with the staggered-contracts formulation for U.S. data. In particular, under the staggered-contracts formulation, G_i should be positive, reflecting that the proportions of outstanding contracts negotiated in $t-i$ should fall between zero and one (and sum to one over i). However, empirical estimates of G_2 and G_4 are statistically significant and negative. A consequence of the negative coefficient estimates is that the coefficients on a given lag/lead of inflation increase with the lag/lead order, rather than decrease, as predicted by the theory.

Empirical results for Canada are slightly more favourable towards the Taylor contracting specification. The specification with $m = 5$ fits the data best, both for inflation and inflation deviations. For this specification, estimates of G_4 are positive and significant, and estimates of G_2 and G_3 are insignificantly different from zero. *P*-values for tests of residual serial correlation are much higher and standard errors are more than 5 per cent smaller for $m = 5$ than for $m = 4$.

Table 5 contains results for estimates of a variant of the Fuhrer-Moore contracting specification. The difference between the specification estimated and the one described in section 1.3 is the treatment of the output gap. The specification that was estimated includes only the contemporaneous output gap. For U.S. inflation, standard errors are somewhat larger than those obtained for the Taylor contracting specification, and estimates of G_i are now insignificantly different from zero. For the case of $m = 5$, *Q*-statistics reject the presence of residual serial correlation for inflation deviations. Although estimates of coefficients on additional lags and leads of inflation

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16. In other words, G_0 is restricted to be the theoretical value that would obtain if the outstanding contract proportions satisfy $h_i = 1/m$.

Table 4
Estimates under Taylor contracting

$$infl_t = c_1 + E_t infl_{t+1} + (G_2 + G_3 + G_4) infl_{t+2} + (G_3 + G_4) infl_{t+3} + G_4 infl_{t+4} - (G_2 + G_3 + G_4) infl_{t-1} - (G_3 + G_4) infl_{t-2} - G_4 infl_{t-3} + \gamma G_0 y_t + \gamma\left[\left(1 - \sum_{i=2}^{4} G_i\right)(y_{t+1} + y_{t-1}) + G_2(y_{t+2} + y_{t-2}) + G_3(y_{t+3} + y_{t-3}) + G_4(y_{t+4} + y_{t-4})\right] + resid_t$$

m	g	G_2	G_3	G_4	G_0	Q(4)	Q(8)	S.E.
United States—Inflation $(infl_t \equiv \pi_t)$								
3	0.041	–0.480			1	0.016	0.073	0.96
	(0.013)	(0.067)						
4	0.048	–0.360	–0.073		2/3	0.014	0.076	0.97
	(0.014)	(0.142)	(0.075)					
5	0.058	–0.414	0.257	–0.220	1/2	0.020	0.049	0.94
	(0.015)	(0.139)	(0.139)	(0.080)				
United States—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
3	0.041	–0.416			1	0.007	0.034	0.98
	(0.013)	(0.061)						
4	0.047	–0.291	–0.074		2/3	0.007	0.041	0.98
	(0.015)	(0.136)	(0.071)					
5	0.055	–0.330	0.213	–0.188	1/2	0.008	0.027	0.96
	(0.015)	(0.133)	(0.134)	(0.075)				
Canada—Inflation $(infl_t \equiv \pi_t)$								
3	0.169	–0.131			1	0.094	0.305	2.23
	(0.090)	(0.210)						
4	0.178	–0.490	0.315		2/3	0.117	0.286	2.09
	(0.104)	(0.251)	(0.174)					
5	0.262	–0.224	–0.223	0.446	1/2	0.551	0.608	1.96
	(0.124)	(0.252)	(0.254)	(0.167)				
Canada—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
3	0.169	–0.108			1	0.074	0.255	2.23
	(0.091)	(0.215)						
4	0.186	–0.452	0.316		2/3	0.099	0.243	2.10
	(0.108)	(0.249)	(0.177)					
5	0.277	–0.218	–0.177	0.426	1/2	0.717	0.672	1.98
	(0.122)	(0.251)	(0.252)	(0.169)				

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

Table 5
Estimates under Fuhrer-Moore contracting

$$infl_t = c_1 + (1/2)\sum_{i=1}^{m-1} G_i(E_t infl_{t+i} + infl_{t-i}) + \gamma y_t + resid_t$$

$$G_1 \equiv 1 - \sum_{i=2}^{m-1} G_i$$

m	γ	G_2	G_3	G_4	**Q(4)**	**Q(8)**	**S.E.**
United States—Inflation $(infl_t \equiv \pi_t)$							
2	0.121				0.016	0.055	1.00
	(0.040)						
3	0.115	–0.086			0.019	0.064	1.00
	(0.041)	(0.139)					
4	0.125	–0.196	0.143		0.019	0.056	1.00
	(0.042)	(0.179)	(0.146)				
5	0.122	–0.197	0.135	0.007	0.020	0.059	1.00
	(0.043)	(0.180)	(0.189)	(0.142)			
United States—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$							
2	0.113				0.012	0.050	1.01
	(0.040)						
3	0.115	0.025			0.013	0.050	1.01
	(0.041)	(0.136)					
4	0.125	–0.105	0.188		0.018	0.061	1.01
	(0.042)	(0.168)	(0.143)				
5	0.131	–0.125	0.099	0.118	0.062	0.188	1.01
	(0.039)	(0.169)	(0.176)	(0.135)			
Canada—Inflation $(infl_t \equiv \pi_t)$							
2	0.444				0.610	0.891	2.24
	(0.226)						
3	0.504	0.241			0.470	0.831	2.25
	(0.229)	(0.261)					
4	0.499	0.050	0.396		0.311	0.687	2.23
	(0.227)	(0.297)	(0.305)				
5	0.540	0.023	0.149	0.424	0.278	0.590	2.21
	(0.226)	(0.294)	(0.347)	(0.293)			
Canada—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$							
2	0.420				0.584	0.881	2.26
	(0.228)						
3	0.492	0.263			0.420	0.801	2.26
	(0.230)	(0.259)					
4	0.484	0.054	0.446		0.255	0.616	2.23
	(0.227)	(0.292)	(0.304)				
5	0.526	0.014	0.183	0.476	0.174	0.426	2.19
	(0.224)	(0.288)	(0.339)	(0.291)			

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

are generally insignificant, regression standard errors are 10–15 per cent smaller than for the purely forward-looking specifications.[17]

The Fuhrer-Moore specification is less successful at explaining the behaviour of Canadian inflation and inflation deviations than the Taylor specification. Although estimates of coefficients on leads and lags of inflation are positive, they are not statistically significant, and for $m = 4$ and $m = 5$, standard errors are larger than for the Taylor specification, respectively, by 5 and 10 per cent.

Results from estimation of inflation dynamics from models with generalized frictions on price adjustment are provided in Table 6. These results may also be interpreted as a variant on the Taylor contracting specification. The Taylor specification can be obtained from the price-frictions model with $\beta = 1$ imposed and different constraints on how leads and lags of the output gap enter the expression.

For U.S. data, the standard errors from the price-frictions model are comparable with those obtained from the hybrid and Taylor specifications. As in the case of the Taylor specification, evidence of residual serial correlation remains. The slight improvement in fit obtained with the Taylor specification over the price-frictions specification is likely due to the inclusion of the additional leads and lags of the output gap (with the theoretical cross-coefficient restrictions imposed). Although not tabulated in this paper, when the Taylor specification was estimated with only the contemporaneous gap included, the fit was similar to that obtained from the price-frictions model.[18]

The advantage of the price-frictions approach over the Taylor contracting motivation is that the generalized-frictions formulation does not require the G_is to be positive. For instance, if the frictions polynomial punishes changes in inflation, then $v(L)$ is proportional to $(1 - L)^2 = 1 - 2L + L^2$, and the theoretical value for G_2 is $-1/3$ for $\beta = 1$. Given the negative estimates of the G_i coefficients for U.S. data, the price-frictions motivation seems to be a more appropriate interpretation of the results.

The Canadian data favour the price-frictions specification for $m = 3$, but the Taylor specification for $m = 5$. For $m = 3$, the Canadian data reject the implicit unit restriction on β in the Taylor specification, but for $m = 5$,

17. For the shorter sample, the Fuhrer-Moore specification with $m = 5$ obtained positive significant estimates of G_4, smaller standard errors than the purely forward-looking specification, and large rejection probabilities for Q-statistics.
18. For the shorter sample, fit was comparable to the purely forward-looking specification, and coefficients on additional lags and leads of inflation (G_2, G_3, G_4) were insignificantly different from zero.

Table 6
Estimates under rational expectations with frictions on price adjustment

$$infl_t = c_1 + \gamma y_1 + \left(1 - G_2(1-\beta) - G_3(1-\beta^2) - G_4(1-\beta^3)\right)\beta E_t infl_{t+1}$$
$$+ (G_2 + G_3\beta + G_4\beta^2)\beta^2 E_t infl_{t+2} + (G_3 + G_4\beta)\beta^3 E_t infl_{t+3} + G_4\beta^4 E_t infl_{t+4}$$
$$- (G_2 + G_3 + G_4) infl_{t-1} - (G_3 + G_4) infl_{t-2} - G_4 infl_{t-3} + resid_t$$

m	γ	β	G_2	G_3	G_4	**Q(4)**	**Q(8)**	**S.E.**
United States—Inflation $(infl_t \equiv \pi_t)$								
3	0.141	0.720	–0.500			0.028	0.127	0.98
	(0.040)	(0.262)	(0.056)					
4	0.152	0.726	–0.384	–0.058		0.025	0.120	0.98
	(0.042)	(0.266)	(0.155)	(0.074)				
5	0.175	0.938	–0.412	0.253	–0.206	0.014	0.027	0.96
	(0.044)	(0.069)	(0.146)	(0.147)	(0.082)			
United States—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
3	0.120	0.622	–0.465			0.011	0.045	1.00
	(0.040)	(0.278)	(0.067)					
4	0.128	0.602	–0.317	–0.088		0.008	0.041	1.00
	(0.041)	(0.270)	(0.143)	(0.074)				
5	0.154	0.601	–0.406	0.250	–0.212	0.011	0.038	0.98
	(0.041)	(0.246)	(0.150)	(0.147)	(0.082)			
Canada—Inflation $(infl_t \equiv \pi_t)$								
3	0.573	0.215	–0.191			0.690	0.911	2.11
	(0.221)	(0.257)	(0.165)					
4	0.589	0.393	–0.345	0.161		0.706	0.885	2.11
	(0.222)	(0.371)	(0.244)	(0.175)				
5	0.707	0.932	–0.269	–0.043	0.368	0.386	0.441	2.06
	(0.217)	(0.060)	(0.231)	(0.249)	(0.173)			
Canada—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
3	0.586	–0.057	–0.141			0.695	0.943	2.03
	(0.214)	(0.255)	(0.162)					
4	0.601	0.079	–0.299	0.159		0.812	0.958	2.03
	(0.215)	(0.339)	(0.234)	(0.174)				
5	0.734	0.859	–0.252	–0.007	0.325	0.586	0.608	2.06
	(0.219)	(0.137)	(0.228)	(0.246)	(0.182)			

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

the estimate of β is insignificantly different from one and the information in the additional lags and leads of the output gap reduces the standard error of the Taylor specification relative to the price-frictions specification, although the latter specification also fits the data reasonably well.

The last specification considered is an expression for inflation under frictions on inflation adjustment. The inflation-frictions specification resembles the Fuhrer-Moore contracting specification, but with β unconstrained. Results are reported in Table 7. Standard errors are very similar to those in Table 5. This is not surprising since in most cases, the null hypothesis that $\beta = 1$ is not rejected by the data. One notable difference for the United States is that evidence of residual serial correlation is largely gone for $m = 4$ and $m = 5$.[19] For Canadian data, as estimation routines were not converging for $m > 2$, results are presented only for the case $m = 2$.

Overall, inflation dynamics seem to be better captured by models that include lags and leads of inflation rather than by specifications that are only forward-looking. In addition, lags and leads of the output gap in the Taylor specification appeared to help explain inflation dynamics. However, the data did not strongly support any one model. Estimates of backward-looking behaviour by a fraction of agents in the hybrid model were contradictory. For U.S. data, coefficient estimates in the Taylor model were inconsistent with positive contract-distribution coefficients, and residual serial correlation was evident for Taylor and price-frictions specifications. Overall, the evidence appeared most favourable for the Fuhrer-Moore and inflation-frictions specifications. For larger m, these specifications had cleaner residuals and coefficient estimates that were not inconsistent or contradictory with the theory, although standard errors were slightly larger for these specifications than for some of the others.[20]

For Canadian data, the Taylor and price-frictions specifications for $m = 5$ seemed to be favoured by the data with the smallest standard errors obtained for the Taylor specification. With these specifications, Q-statistics provided no evidence of residual serial correlation. Restrictions imposed by the Fuhrer-Moore contracting specification and the inflation-frictions specification appeared inconsistent with Canadian data, since standard errors for these specifications were roughly 5 per cent larger than for hybrid models and 10 per cent larger than for Taylor and price-frictions specifications.

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19. For the shorter sample, all aspects of the estimation of the inflation deviations specification with $m = 5$ were similar to those obtained for the Fuhrer-Moore specification with $m = 5$.

20. Similar arguments favoured the Fuhrer-Moore and inflation-frictions specifications for the shorter sample.

Table 7
Estimates under rational expectations with frictions on inflation adjustment

$$infl_t = c_1 + \gamma y_t + resid_t$$

$$+[G_1(\beta E_t infl_{t+1} + infl_{t-1}) + G_2(\beta^2 E_t infl_{t+2} + infl_{t-2})]/[1 + G_1\beta + G_2\beta^2 + G_3\beta^3 + G_4\beta^4]$$

$$+[G_3(\beta^3 E_t infl_{t+3} + infl_{t-3}) + G_4(\beta^4 E_t infl_{t+4} + infl_{t-4})]/[1 + G_1\beta + G_2\beta^2 + G_3\beta^3 + G_4\beta^4]$$

$$G_1 \equiv 1 - \sum_{i=2}^{m-1} G_i$$

m	γ	β	G_2	G_3	G_4	**Q(4)**	**Q(8)**	**S.E.**
United States—Inflation $(infl_t \equiv \pi_t)$								
2	0.120	0.928				0.020	0.076	1.00
	(0.041)	(0.294)						
3	0.118	1.887	–0.221			0.008	0.021	1.00
	(0.041)	(0.645)	(0.087)					
4	0.137	0.688	–0.142	0.249		0.103	0.312	1.00
	(0.042)	(0.210)	(0.162)	(0.130)				
5	0.145	0.650	–0.133	0.173	0.109	0.330	0.683	1.00
	(0.044)	(0.201)	(0.157)	(0.157)	(0.130)			
United States—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
2	0.116	1.192				0.009	0.026	1.01
	(0.041)	(0.346)						
3	0.119	2.070	–0.166			0.004	0.009	1.01
	(0.041)	(0.735)	(0.074)					
4	0.130	0.823	–0.077	0.248		0.041	0.149	1.01
	(0.042)	(0.191)	(0.159)	(0.134)				
5	0.144	0.741	–0.074	0.122	0.207	0.604	0.876	1.00
	(0.043)	(0.160)	(0.149)	(0.150)	(0.124)			
Canada—Inflation $(infl_t \equiv \pi_t)$								
2	0.533	2.819				0.468	0.803	2.22
	(0.233)	(2.511)						
Canada—Inflation deviations $(infl_t \equiv \hat{\pi}_t)$								
2	0.521	3.042				0.415	0.760	2.23
	(0.236)	(2.873)						

Notes: Standard errors of coefficient estimates are provided in parentheses. Entries in columns Q(4) and Q(8) are *p*-values for *Q*-statistics for the null hypothesis of no serial correlation up to lags 4 and 8, respectively. The column labelled S.E. contains the regression standard error.

The additional information attributable to the restricted lags and leads of the output gap in the Taylor specifications suggests that alternative specifications of the frictions polynomial may improve the performance of this specification. One possibility is to generalize the adjustment-cost polynomial to account for costs to a producer associated with altering the output rate. Such a generalization fits within the vector rational error-correction (VREC) framework developed by Kozicki and Tinsley (1999a). A model of price-setting described by McCallum and Nelson (1999) may be taken as an example of a restricted version of the VREC framework. In their model, deviations of prices from their no-friction optimal level are due to quadratic costs associated with adjusting output rates, but not prices.

3 Implications for Monetary Policy

The results of the previous section suggest several lessons for monetary policy-makers. First, historical shifts in the nominal anchor for expectations provide evidence against the theory that a long-run trade-off exists between inflation and economic activity. Second, the increased use of structural macroeconomic models should reduce the likelihood that low inflation persistence is misinterpreted as signalling that a long-run trade-off exists between inflation and economic activity. Third, the results suggest that the introduction of an explicit inflation-targeting regime in Canada resulted in increased credibility for low-inflation goals.

Some of the persistence of Canadian and U.S. inflation can be associated with historical shifts in the anchor of long-run inflation expectations. In both countries, inflation persistence was lower after accounting for shifts in the nominal anchor, and inflation persistence in the 1990s, a period of relatively low and stable inflation, was almost non-existent. Results suggesting that the nominal anchor has not been constant provide evidence supporting the Friedman (1968) and Phelps (1967) critiques of Phillips curve representations that embody a long-run trade-off between inflation and economic activity. The fact that the nominal anchor has shifted historically should be taken as a warning to policy-makers that it may shift again if policy actions do not continue to support low and stable inflation. In other words, as noted by Cogley and Sargent (2001) and Taylor (1998), while persistence has declined, it would not be appropriate to revert to the view that a long-run trade-off exists between inflation and economic activity.

One factor that should help reduce the likelihood that beliefs in a long-run inflation-output trade-off will re-emerge, is the increased emphasis on structural macroeconomic models in policy evaluation. Of course, not all models are structural and some structural models incorporate unrealistic assumptions. As noted in this paper, purely forward-looking models generally fail at explaining the lag dynamics of inflation. Modifications to price-setting behaviour that result in inflation expressions with additional lags and leads of inflation improve the ability of models to explain the lag dynamics of inflation. Recognizing this potential improvement, some policy models already incorporate such modified inflation expressions.

Few models, however, admit a shifting anchor for long-run inflation expectations.[21] Policy models that exclude nominal anchor shifts are taking a strong view that the goals of policy are fully known, fully credible, and do not change. By contrast, the empirical results of this paper suggest that the nominal anchor has shifted and that there have been episodes when policy was less than fully credible. Thus, introducing the potential for nominal anchor shifts and imperfect policy credibility would be an important improvement for models to be used for evaluating monetary policy alternatives.

While the empirical results suggest that both Canada and the United States experienced historical shifts in their nominal anchors, the experience of the two countries differed somewhat in the 1990s. Since 1995, long-horizon inflation expectations for Canada have fallen close to the midpoint of the inflation-control target range, which is also close to the central tendency of inflation. By contrast, in the United States, although inflation was relatively low and stable for most of the 1990s, long-horizon inflation expectations tended to be higher than measured inflation. The announcement of an inflation target range in Canada helped reduce public uncertainty about the inflation goal of policy. Credibility of the inflation target likely increased relatively quickly as subsequent policy actions were taken to be consistent with the announced inflation-targeting regime.

Conclusions

This paper examined four potential sources of lag dynamics in inflation: non-rational behaviour, staggered contracting, frictions on price adjustment, and shifts in the long-run inflation anchor of agent expectations. The empirical evidence suggests that shifts in the long-run inflation anchor of agent expectations have contributed importantly to observed persistence in U.S. and Canadian inflation. Such shifts, however, don’t appear to explain all of the historical persistence in inflation. Models of inflation and deviations of inflation from the nominal anchor, which admit additional lags and leads, explain the historical behaviour of inflation better than purely forward-looking models. Interestingly, structural models derived from

21. The FRB/US model of the Federal Reserve Board of Governors is one exception (Brayton, Levin, Tryon, and Williams 1997).

assumptions about staggered contracts or frictions associated with price adjustment are better able to explain inflation dynamics than hybrid models that assume a fraction of agents form expectations non-rationally.

The empirical evidence suggests that shifts in the nominal anchor, less than full policy credibility, and inflation stickiness have all been important features of the historical behaviour of inflation. Although many policy models incorporate some form of inflation stickiness, few currently allow shifts in the nominal anchor and accommodate the potential for less than full policy credibility. These are important features, particularly for models to be used for monetary policy analysis, that one hopes will be incorporated into the next generation of policy models.

Appendix

This Appendix provides a derivation for a New Keynesian Phillips curve with a non-zero steady-state inflation rate. Similar derivations are provided in Ascari (2002) and Bakhshi, Burriel-Llombart, Khan, and Rudolf (2002). The expression for inflation derived here is consistent with that in Bakhshi et al. under a common-factor market assumption.

A retail distributor combines the differentiated output of a continuum of monopolistically competitive firms, $Y_{i,t}$, into a composite product, Y_t, with price elasticity of demand, θ:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{A1}$$

The retailer sells this composite product directly to households. Maximizing retailer profits implies that the retailer's demand for the *i*th firm's output is:

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t, \tag{A2}$$

where $P_{i,t}$ is the price of firm i output, and P_t is an index of goods prices at date t. The aggregate price index is defined as:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{A3}$$

In the absence of constraints on price adjustment, each firm chooses the optimal price level, $P^*_{i,t}$, to maximize current period real profits, $\Pi_{i,t}$, where

$$\Pi_{i,t} = \left[\frac{P_{i,t}}{P_t} - \frac{W_t}{P_t Z_t}\right]\left[\frac{P_{i,t}}{P_t}\right]^{-\theta} Y_t, \tag{A4}$$

W_t are nominal wages paid to workers, and Z_t is the labour-augmenting productivity process. Profit maximization implies the optimal relative price, $(P_{i,t}/P_t)^*$, satisfies

$$\left(\frac{P_{i,t}}{P_t}\right)^* = \mu \frac{W_t}{P_t Z_t}, \tag{A5}$$

where $\mu = (1 - 1/\theta)^{-1}$ is a markup and $W_t/(P_t Z_t)$ is real marginal cost.

Under Calvo (1983) pricing, price adjustment is constrained. Each period, a firm is allowed to change its price with probability $(1 - \lambda)$. Firms choose their optimal reset price according to

$$P_{r,t} = \underset{\{P_{i,t}\}}{arg\,max} \left\{ E_t(1-\lambda) \sum_{j=0}^{\infty} \lambda^j R_{t,t+j} \left[\frac{P_{i,t}}{P_{t+j}} - \frac{W_{t+j}}{P_{t+j} Z_{t+j}} \right] \left[\frac{P_{i,t}}{P_{t+j}} \right]^{-\theta} Y_{t+j} \right\}, \tag{A6}$$

where $R_{t,t+j}$ is a j-period discount rate. After algebraic manipulation, it is easy to show that the optimal relative reset price, $P_{r,t}/P_t$, satisfies

$$\frac{P_{r,t}}{P_t} = \sum_{j=0}^{\infty} w_{jt} \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^* \prod_{k=1}^{j} \Pi_{t+k}, \tag{A7}$$

where $\Pi_{t+k} = P_{t+k}/P_{t+k-1}$ is the gross inflation rate from $t + k - 1$ to $t + k$, and the expression for the weights can be simplified to

$$w_{jt} = \frac{(\lambda\beta)^j \left(\prod_{k=1}^{j} \Pi_{t+k} \right)^{\theta-1}}{\sum_{j=0}^{\infty} (\lambda\beta)^j \left(\prod_{k=1}^{j} \Pi_{t+k} \right)^{\theta-1}} \tag{A8}$$

under additional assumptions.[1]

With Calvo-type constraints on price adjustment, the aggregate price index evolves according to

$$P_t = \left[\lambda P_{t-1}^{1-\theta} + (1-\lambda) P_{r,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{A9}$$

1. One set of assumptions that generates this convenient simplification is that the discount rate, $R_{t,t+j}$, equals $\beta u'(C_{t+j})/u'(C_t)$ for marginal utility of consumption $u'(C)$ and utility discount factor β, households maximize discounted utility with $u(C) = \log(C)$, and consumption equals the composite product in equilibrium.

Rearranging this expression, the optimal relative reset price is related to aggregate inflation,

$$\frac{P_{r,t}}{P_t} = \left[\frac{1-\lambda\Pi_t^{\theta-1}}{1-\lambda}\right]^{\frac{1}{1-\theta}}. \tag{A10}$$

The higher is aggregate inflation, the higher is the optimal relative reset price. Intuitively, with higher aggregate inflation, the optimal reset price is set higher relative to the aggregate price level in t because a firm's reset price may not be adjusted for several periods even though the aggregate price level will continue to increase.

To account for a non-zero steady-state inflation rate, linearize equations (A10) and (A7) in terms of per cent deviations of $(P_{r,t}/P_t)$, $(P_{i,t}/P_t)^*$, and Π_t from their steady-state values. In this notation, let $\bar{\pi}$ represent the steady-state inflation rate, so that $1+\bar{\pi}$ is the steady-state gross inflation rate. An expression similar to equation (1) also involves relating the percentage deviation of $(P_{i,t}/P_t)^*$ from its steady state to the percentage deviation of Y_t from its steady state, as in McCallum and Nelson (1999) and Kozicki and Tinsley (2002). Let $\hat{\pi}_t$ denote the percentage deviation of gross inflation from $1+\bar{\pi}$, y_t denote the percentage deviation of output, Y_t, from potential, and γ^* represent the factor of proportionality between percentage deviations of $(P_{i,t}/P_t)^*$ from steady state and y_t. The resultant expression for inflation is:

$$\hat{\pi}_t = bE_t\hat{\pi}_{t+1} + gy_t + c(1-\lambda\beta(1+\bar{\pi})^{\theta-1})\sum_{j=1}^{\infty}\left(\lambda\beta(1+\bar{\pi})^{\theta-1}\right)^j \sum_{k=1}^{j} E_t\hat{\pi}_{t+1+k} + \varepsilon_t^*, \tag{A11}$$

where

$$b = \beta\left[\lambda(1+\bar{\pi})^{\theta} + \left(1-\lambda(1+\bar{\pi})^{\theta-1}\right)\left(\theta(1+\bar{\pi}) + (1-\theta)\right)\right]$$

$$c = \beta\bar{\pi}(\theta-1)\left(1-\lambda(1+\bar{\pi})^{\theta-1}\right)$$

$$g = \left[\gamma^*\left(1-\lambda(1+\bar{\pi})^{\theta-1}\right)\left(1-\beta\lambda(1+\bar{\pi})^{\theta}\right)\right]/\left[\lambda(1+\bar{\pi})^{\theta-1}\right] \tag{A12}$$

are non-linear functions of the structural parameters $\beta, \bar{\pi}, \theta$, and λ; and ε_t^* is an error term. For a positive steady-state inflation rate, $\bar{\pi}$ will be greater than zero. When steady-state inflation is equal to zero, the model simplifies to the benchmark model in equation (1) with $b = \beta$, $c = 0$, and $g = \gamma^*(1-\lambda)(1-\beta\lambda)/\lambda \equiv \gamma$.[2]

Table A1 shows ranges of coefficient estimates for different values of structural parameters. Values for β and λ are in the range of estimates obtained by Galí and Gertler (1999). Values of $\bar{\pi}$ equal to .01 and .005 correspond to steady-state inflation rates of about 4 per cent and 2 per cent, respectively, expressed at an annual rate. Values for θ are within the range suggested by the literature, with θ equal to 11, 5, and 2.67 corresponding to markups (μ) of 1.10, 1.25, and 1.60, respectively. Basu and Fernald (1997) estimated the average value of the markup to be 1.16 for the entire private economy. Cooper and Haltiwanger (2000) estimated the markup for manufacturing to be 1.27. Domowitz, Hubbard, and Petersen (1988) estimated the markup to be 1.58. In work on magazine prices, Willis (2000) estimated the markup to be 1.75. Hall (1988) estimated the markup to be over 2. As the markup increases, the implied value of θ decreases (and the differences between b and β, g/γ^* and γ/γ^*, and c and zero shrink). As noted earlier, as the anchor of inflation expectations approaches zero, the coefficients approach those for the benchmark expression. The limiting results are summarized in the top row of the table.

The expression in equation (2) is an approximation to the expression in equation (A11). Compared with the approximation, equation (A11) has an additional term that includes discounted expected inflation deviations. The analysis in the main body of this paper uses the simpler approximating expression, because the empirical relevance of the discounted sum of expected inflation deviations is likely to be small. However, the relevance of the term also depends on the degree of inflation persistence. The intuition follows by substituting for expected inflation in the discounted sum, forecasts based on the following simple reduced-form representation of inflation, $E_t\hat{\pi}_{t+k} = \rho^{k+1}\hat{\pi}_{t-1}$. In this case, the discounted sum simplifies to:

2. When $\bar{\pi} = 0$, the expression for g/γ^* matches that derived, for example, by Galí and Gertler (1999).

Table A1
Coefficient estimates in benchmark and approximating models

θ	β	λ	$\bar{\pi}$	b	g/γ^*	γ/γ^*	c	**coef. on $\hat{\pi}_{t-1}$** $\rho = 0.9$	$\rho = 0.5$
			0	β	γ/γ^*		0	0	0
11	1	0.8	0.01	1.022	0.014	0.050	0.0116	0.041	0.005
11	1	0.8	0.005	1.013	0.029	0.050	0.0080	0.022	0.003
11	1	0.75	0.01	1.027	0.034	0.083	0.0172	0.045	0.006
11	1	0.75	0.005	1.016	0.056	0.083	0.0106	0.023	0.003
11	0.8	0.8	0.01	0.817	0.038	0.090	0.0093	0.015	0.003
11	0.8	0.8	0.005	0.810	0.061	0.090	0.0064	0.009	0.002
11	0.8	0.75	0.01	0.822	0.069	0.133	0.0137	0.018	0.003
11	0.8	0.75	0.005	0.813	0.098	0.133	0.0085	0.010	0.002
5	1	0.8	0.01	1.017	0.032	0.050	0.0067	0.018	0.002
5	1	0.8	0.005	1.009	0.041	0.050	0.0037	0.009	0.001
5	1	0.75	0.01	1.019	0.060	0.083	0.0088	0.019	0.003
5	1	0.75	0.005	1.010	0.071	0.083	0.0047	0.009	0.002
5	0.8	0.8	0.01	0.813	0.066	0.090	0.0054	0.007	0.001
5	0.8	0.8	0.005	0.807	0.078	0.090	0.0029	0.004	0.001
5	0.8	0.75	0.01	0.815	0.104	0.133	0.0070	0.008	0.002
5	0.8	0.75	0.005	0.808	0.118	0.133	0.0038	0.004	0.001
2.67	1	0.8	0.01	1.013	0.041	0.050	0.0031	0.008	0.001
2.67	1	0.8	0.005	1.007	0.045	0.050	0.0016	0.004	0.001
2.67	1	0.75	0.01	1.014	0.072	0.083	0.0040	0.008	0.001
2.67	1	0.75	0.005	1.007	0.077	0.083	0.0020	0.004	0.001
2.67	0.8	0.8	0.01	0.811	0.079	0.090	0.0025	0.003	0.001
2.67	0.8	0.8	0.005	0.805	0.084	0.090	0.0013	0.002	0.000
2.67	0.8	0.75	0.01	0.811	0.120	0.133	0.0032	0.004	0.001
2.67	0.8	0.75	0.005	0.806	0.126	0.133	0.0016	0.002	0.000

$$\left(1-\lambda\beta(1+\pi)^{\theta-1}\right)\sum_{j=1}^{\infty}\left(\lambda\beta(1+\bar{\pi})^{\theta-1}\right)^{j}\sum_{k=1}^{j}E_t\hat{\pi}_{t+1+k}$$

$$=\left[\left(1-\lambda\beta(1+\bar{\pi})^{\theta-1}\right)\sum_{j=1}^{\infty}\left(\lambda\beta(1+\bar{\pi})^{\theta-1}\right)^{j}\rho^{2}\frac{(1-\rho^{j})}{(1-\rho)}\right]\hat{\pi}_{t-1}$$

$$=\lambda\beta(1+\bar{\pi})^{\theta-1}\frac{\rho^{2}}{(1-\rho)}\left[1-\frac{\rho\left(1-\lambda\beta(1+\bar{\pi})^{\theta-1}\right)}{1-\lambda\beta\rho(1+\bar{\pi})^{\theta-1}}\right]\hat{\pi}_{t-1}. \qquad \text{(A13)}$$

As shown in Table A1, for various degrees of inflation persistence, the coefficient on this term (in the column labelled "coef. on $\hat{\pi}_{t-1}$") is two to three orders of magnitude smaller than the coefficient on expected inflation (in the column labelled "*b*"). Consequently, the discounted sum is merged into the error term in equation (2),

$$\varepsilon_t = \varepsilon_t^* + c\left(1 - \lambda\beta(1+\bar{\pi})^{\theta - 1}\right)\sum_{j=1}^{\infty}\left(\lambda\beta(1+\bar{\pi})^{\theta-1}\right)^j \sum_{k=1}^{j} E_t\hat{\pi}_{t+1+k}. \qquad \text{(A14)}$$

With this approximation, the error term ε_t in equation (2) will be correlated with the explanatory variables, and estimation procedures that do not account for this correlation will induce bias into coefficient estimates. However, the size of the bias will generally be small as long as c is small.

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