Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel
by
Auclert, Rognlie, Souchier and Straub

Discussion by Sushant Acharya

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question

how do changes in exchange rates affect aggregate demand?

this paper argues:

- ☐ devaluations generally expansionary in RANK
- ☐ but can be contractionary in HANK via real income channel

what is the "**real income**" channel?

$$\max_{c_H, c_F} \left[(1-\alpha)^{\frac{1}{\eta}} \left(c_H\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(c_F\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad \text{s.t} \qquad p c_H + c_F = p\omega$$

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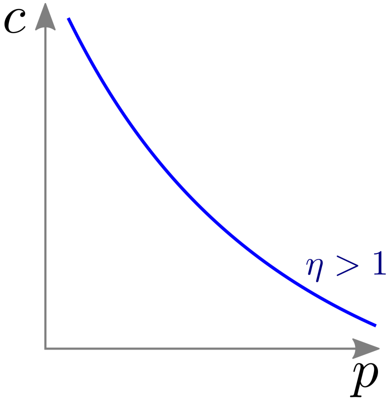
- ☐ ... but slope of **Walrasian** demand depends on η relative to 1

$$\boxed{-\frac{d \ln c_H}{d \ln p} = \underbrace{\frac{(1-\alpha)\, p + \alpha p^{\eta} \eta}{(1-\alpha)\, p + \alpha p^{\eta}}}_{\substack{\text{substitution} \\ \text{due to } p\downarrow}} - \overbrace{1}^{\substack{\text{income effect} \\ \text{due to } p\downarrow}}}$$

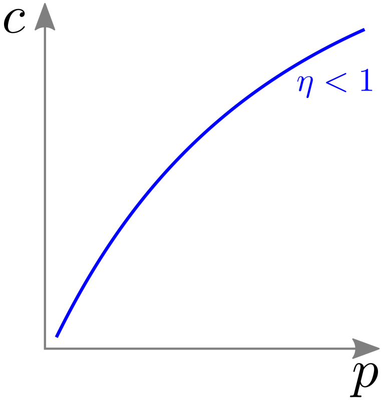
real income channel



$\eta > 1$: substitution dominates income effect

decrease in price **increases** demand

real income channel



$\eta < 1$**: income effect overwhelms substitution**

decrease in price **decreases** demand

$\boxed{\textbf{Need small } \eta \textbf{ for real income channel to have bite}}$

why is this effect small in RANK SOE models?

- infinite horizon **permanent income hypothesis** consumers
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- if instead **borrowing constrained** (htm) $\qquad dc_t = dy_t = -1\%$

a bare bones heterogeneous agent SOE

- fixed PPI $\pi_{H,t} = \pi^*_{F,t} = 0$ but not CPI

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- Demand for Home goods

$$\hat{y}_t = (1-\alpha)\left[(1-\theta)\hat{c}_t^{\mathsf{pih}} + \theta\hat{c}_t^{\mathsf{htm}}\right] + \underbrace{\frac{\alpha}{1-\alpha}\chi\hat{q}_t}_{\text{substitution}}$$

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- aggregate euler equation

$$\Delta \hat{c}_{t+1} = (1-\theta)\underbrace{\gamma\left(\hat{i}_t - \frac{\alpha}{1-\alpha}\Delta \hat{q}_{t+1}\right)}_{\text{consumption growth of pih}} + \theta \underbrace{\left(\overbrace{-\frac{\alpha}{1-\alpha}\Delta \hat{q}_{t+1}}^{\text{real-income channel}} + \Delta \hat{y}_{t+1}\right)}_{\text{consumption growth of htm}}$$

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- **uip**

$$i_t = i_t^* + \frac{1}{1-\alpha}\Delta q_{t+1}$$

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  - $\hat{i}_t = 0$: monetary policy lets ex-rate depreciate $\Delta \hat{q}_{t+1} = -(1-\alpha)\hat{i}_t^*$
  - Auclert et al: **real rate unchanged**:

$$\hat{r}_t = 0 \qquad \Rightarrow \qquad \hat{i}_t = -\frac{\alpha}{1-\alpha}\hat{i}_t^*$$

effect of exchange rates on Home output

- RANK with $\hat{i}_t = 0$:

$$\Delta \hat{y}_{t+1} = \Big(\underbrace{\alpha\gamma}_{\substack{\text{intertemporal}\\ \text{substitution}}} - \underbrace{\frac{\alpha\chi}{1-\alpha}}_{\substack{\text{expenditure}\\ \text{switching}}} \Big) \hat{i}_t^*$$

  - **contractionary depreciation in RANK:** $\hat{y}_t < 0$ and $\hat{q}_t > 0$ if $\gamma > \frac{\chi}{1-\alpha}$

effect of exchange rates on Home output

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$$\Delta \hat{y}_{t+1} = \left(\alpha\gamma - \frac{\alpha\chi}{1-\alpha} \right) \hat{i}_t^*$$

- RANK with $\widehat{r}_t = 0$

$$\Delta \hat{y}_{t+1} = -\frac{\alpha\chi}{1-\alpha} \hat{i}_t^*$$

no contractionary depreciation: $\hat{y}_t > 0$ and $\hat{q}_t > 0$

effect of exchange rates on Home output

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- HANK $(\theta > 0)$ with $\hat{i}_t = 0$

$$\Delta \hat{y}_{t+1} = \underbrace{\frac{1}{1-\theta\left(1-\alpha\right)}}_{\text{Keynesian multiplier}} \times \left(\underbrace{\alpha\theta}_{\text{real income}} + \underbrace{\left(1-\theta\right)\alpha\gamma}_{\text{intertemporal substitution}} - \underbrace{\frac{\alpha\chi}{1-\alpha}}_{\text{expenditure switching}} \right)\hat{i}_t^*$$

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- RANK with $\hat{i}_t = 0$:

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- HANK $(\theta > 0)$ with $\hat{i}_t = 0$ **with $\gamma = 1$**

$$\Delta \hat{y}_{t+1} = \underbrace{\frac{1}{1-\theta\left(1-\alpha\right)}}_{\text{Keynesian multiplier}} \times \left(\alpha - \frac{\alpha\chi}{1-\alpha}\right)\hat{i}_t^*$$

overall...

- ☐ thought provoking paper!
- ☐ however, both HANK and RANK can feature contractionary depreciation
- ☐ ... but not when monetary policy tries to keep $\widehat{r}_t = 0$, need small χ
- ☐ important to provide empirical support for small χ

END